Mass effects in the Higgs q_T spectrum

Rebecca von Kuk

DESY

TUM/MPP Collider seminar Nov. 12 2024

Outline.

Introduction

- q_T factorization and resummation in SCET
- Higgs q_T spectrum
- measurement of the Yukawa coupling

Quark initiated Higgs production

- N^{3}LL$'$ + aN^{3}LO prediction for $\bar{b}bH$, $\bar{c}cH$ and $\bar{s}sH$

$y_b\,y_t$ interference in gluon fusion

- state of the literature
- different regimes
- cancellation of endpoint divergences

Summary

Introduction.

Introduction.

Kinematic distributions

- kinematic distributions and differential cross sections are particularly interesting
- for Higgs production: most Higgs bosons are produced with small transverse momentum q_T
- in this kinematic region the fixed-order perturbative expansion is no longer valid
- **cross section diverges and needs to be resummed!**

Introduction.

Large logs

- consider cross section for $q_T \ll Q = m_H$

$$\sigma(q_T) \sim 1 + \boxed{\frac{\alpha_s}{4\pi}\left[c_{12}\ln^2_{q_T/Q} + c_{11}\ln_{q_T/Q} + c_{10}\right]}$$

NLO

$$+\boxed{\left(\frac{\alpha_s}{4\pi}\right)^2\left[c_{24}\ln^4_{q_T/Q} + c_{23}\ln^3_{q_T/Q} + c_{22}\ln^2_{q_T/Q} + \ldots\right]}$$

NNLO

$$\boxed{+\left(\frac{\alpha_s}{4\pi}\right)^3\left[c_{36}\ln^6_{q_T/Q} + c_{35}\ln^5_{q_T/Q} + c_{34}\ln^4_{q_T/Q} + \ldots\right]}$$

N^3LO

- for $q_T \to 0$ logs become large $\alpha_s \log^2(q_T/Q) \approx 1$
- switch from fixed-order to logarithmic counting

Introduction.

Large logs

- consider cross section for $q_T \ll Q = m_H$

$$
\begin{aligned}
\sigma(q_T) \sim 1 &+ \frac{\alpha_s}{4\pi}\Big[c_{12}\ln^2_{q_T/Q} + c_{11}\ln_{q_T/Q} + c_{10}\Big] \\
&+ \left(\frac{\alpha_s}{4\pi}\right)^2 \Big[c_{24}\ln^4_{q_T/Q} + c_{23}\ln^3_{q_T/Q} + c_{22}\ln^2_{q_T/Q} + \ldots\Big] \\
&+ \left(\frac{\alpha_s}{4\pi}\right)^3 \Big[c_{36}\ln^6_{q_T/Q} + c_{35}\ln^5_{q_T/Q} + c_{34}\ln^4_{q_T/Q} + \ldots\Big]
\end{aligned}
$$

LL NLL NNLL

- switch from fixed-order to logarithmic counting

Introduction.

Resummation from RGEs

① factorize cross section: $\sigma(Q, q_T) = H(Q,\mu) \times F(q_T,\mu)$ ➡ $\log\frac{q_T}{Q} = \log\frac{\mu}{Q} + \log\frac{q_T}{\mu}$

② Write down renormalization group c.f. running coupling

③ Solve RGE between $\mu_H = Q$ and $\mu_F = q_T$ (“running”)

Introduction.

Resummation from RGEs

1. factorize cross section: $\sigma(Q, q_T) = H(Q,\mu) \times F(q_T,\mu) \Rightarrow \log\frac{q_T}{Q} = \log\frac{\mu}{Q} + \log\frac{q_T}{\mu}$

2. Write down renormalization group c.f. running coupling $\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\alpha_s = \beta(\alpha_s)$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu} H(Q,\mu) = \left[\Gamma(\alpha_s)\log\frac{\mu}{Q} + \gamma(\alpha_s)\right] H(Q,\mu)$$

3. Solve RGE between $\mu_H = Q$ and $\mu_F = q_T$ ("running")

Introduction.

Resummation from RGEs

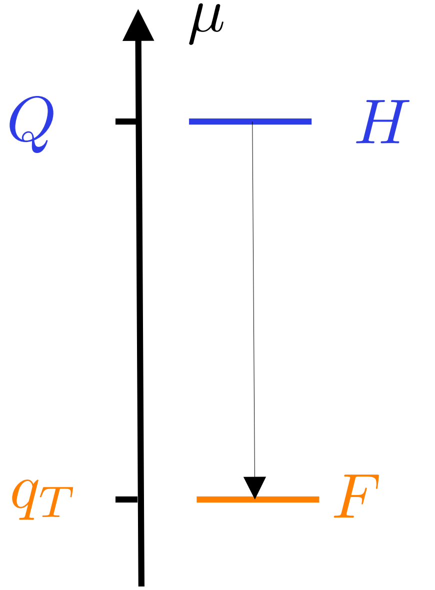
1. factorize cross section: $\sigma(Q, q_T) = H(Q,\mu) \times \; F(q_T,\mu) \;\Rightarrow\; \log\frac{q_T}{Q} = \log\frac{\mu}{Q} + \log\frac{q_T}{\mu}$

2. Write down renormalization group c.f. running coupling $\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\alpha_s = \beta(\alpha_s)$

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu} H(Q,\mu) = \left[\Gamma(\alpha_s)\log\frac{\mu}{Q} + \gamma(\alpha_s)\right] H(Q,\mu)$$

3. Solve RGE between $\mu_H = Q$ and $\mu_F = q_T$ ("running")

$$\sigma(Q, q_T) = H(Q, \mu_H)\exp\left[\int_{\mu_H}^{\mu_F} \mu\frac{\mathrm{d}}{\mathrm{d}\mu}(\ldots)\right] \times F(q_T, \mu_F)$$



➔ **Logs resummed to all orders by exponential**

- LL, NLL, etc. corresponds to loop orders of H, F, Γ and γ

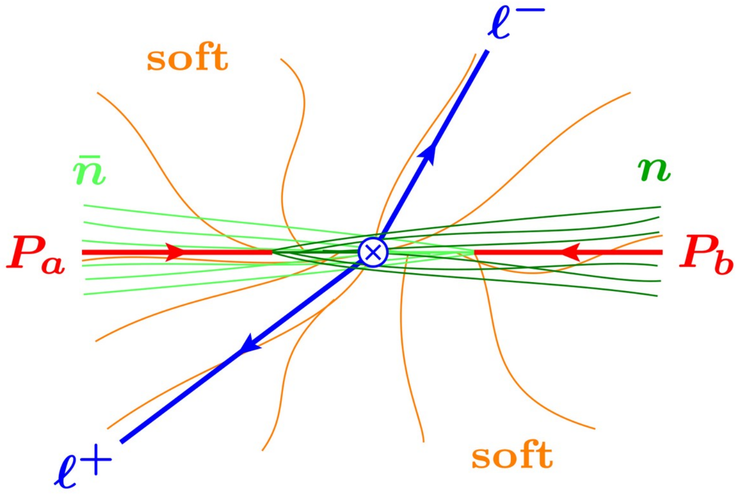
Introduction.

q_T factorization

- SCET factorization theorem separates scales at cross section level

$$\boxed{\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} = H(\mu_H) \times B(\mu_B) \otimes B(\mu_B) \otimes S(\mu_S) \left[1 + \mathcal{O}\left(\frac{q_T^2}{m_H^2}\right)\right]}$$

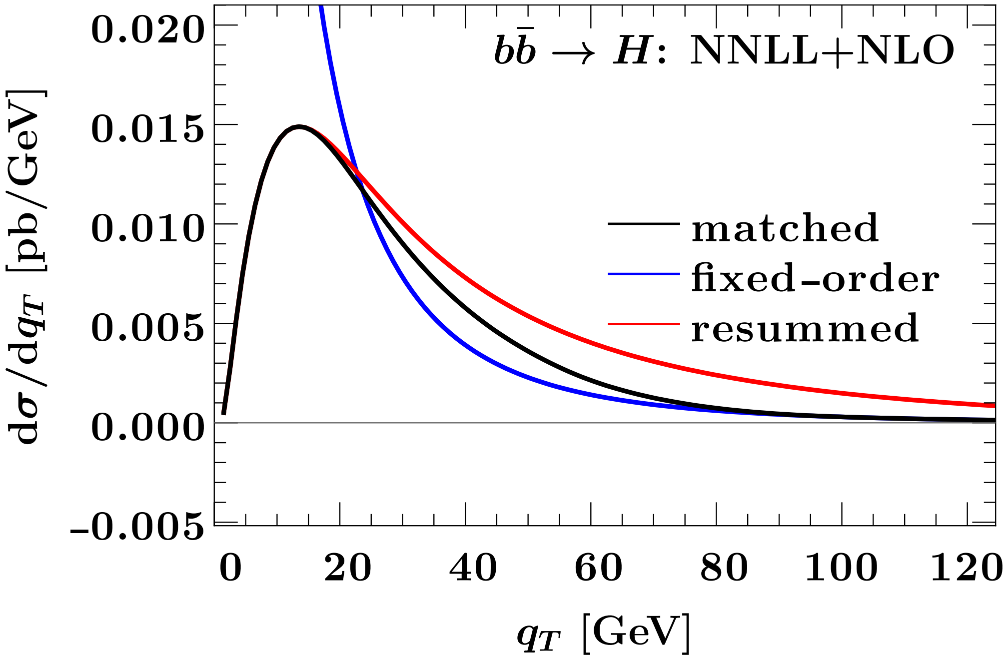
- Hard function: virtual contributions on hard scale
- Beam function: collinear radiation
- Soft function: soft, isotropic radiation



Introduction.

Resummed cross section

- solve RGEs for $H(\mu_H), B(\mu_B)$ and $S(\mu_S)$ to resum logs
- resummation generates Sudakov peak for $q_T \ll Q$
- for $q_T \sim Q$ the fixed-order prediction is sufficient
- transition connects fixed-order and resummed prediction



Motivation.

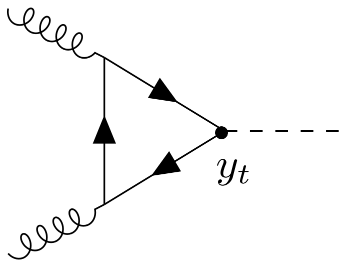
Higgs q_T spectrum

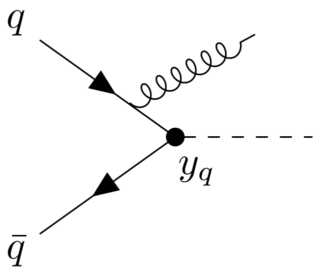
- allows to access quark Yukawa couplings from Higgs production
  - complementary to measuring it from the final state
- initial state discrimination [Ebert et al. ‘16, Bishara at al. ‘16]
  - the q_T spectra of gluon fusion and quark-initiated Higgs productions have different shapes
- **goal: combine different prediction and fit the Yukawa coupling**

$$\boxed{\frac{\mathrm{d}\sigma(pp \to H)}{\mathrm{d}q_T} = y_t^2 \frac{\mathrm{d}\sigma_{tt}}{\mathrm{d}q_T} + y_t y_b \frac{\mathrm{d}\sigma_{tb}}{\mathrm{d}q_T} + y_b^2 \frac{\mathrm{d}\sigma_{bb}}{\mathrm{d}q_T} + (y_b \to y_c)}$$

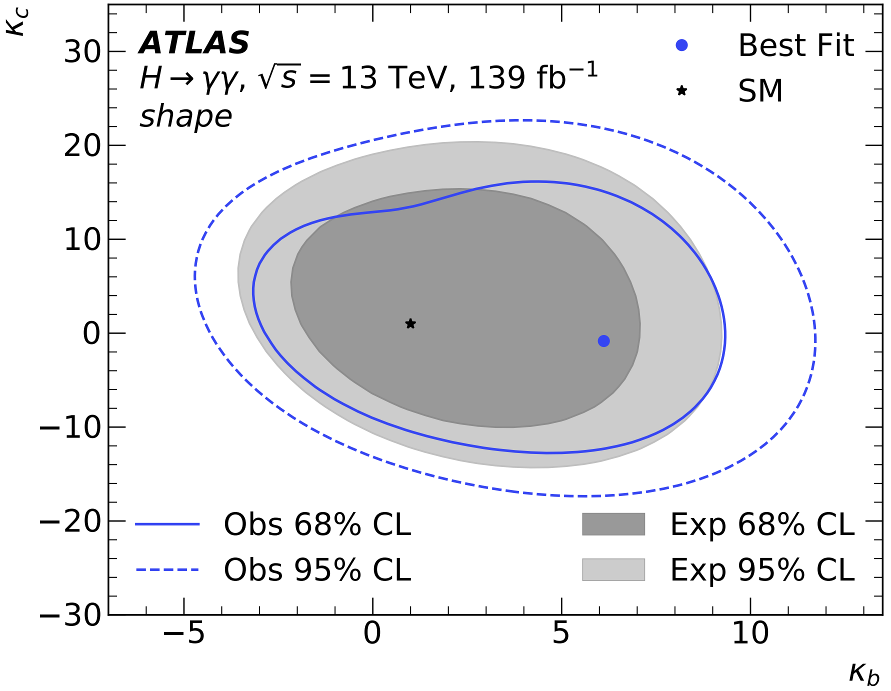
Motivation.

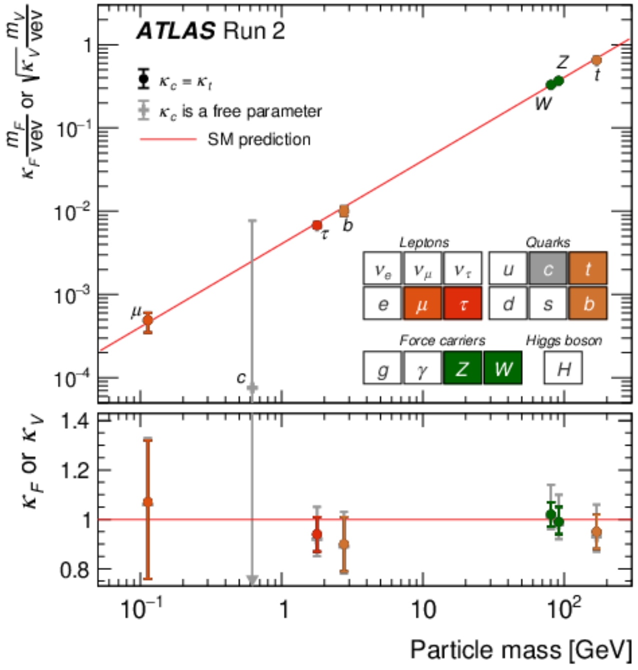
Yukawa coupling from Higgs production





- fit to the the $H \to \gamma\gamma$ q_T spectrum
- N^{3}LL$'$ prediction for gluon fusion
- No resummed prediction for quark initiated production!





[ATLAS Collaboration ‘22]

Motivation.

Higgs q_T spectrum

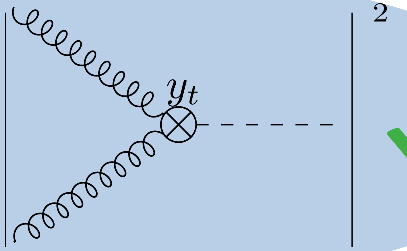
$$\boxed{\frac{\mathrm{d}\sigma(pp \to H)}{\mathrm{d}q_T} = y_t^2 \frac{\mathrm{d}\sigma_{tt}}{\mathrm{d}q_T} + y_t y_b \frac{\mathrm{d}\sigma_{tb}}{\mathrm{d}q_T} + y_b^2 \frac{\mathrm{d}\sigma_{bb}}{\mathrm{d}q_T} + (y_b \to y_c)}$$

Motivation.

Higgs q_T spectrum

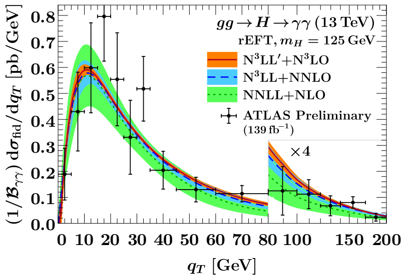
$gg \to H$

$\mathrm{N^3LL'} + \mathrm{N^3LO}$



✓

[Billis,Dehnadi, Ebert, Michel, Tackmann ‘21]

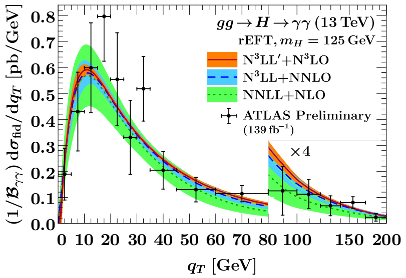


$$\frac{\mathrm{d}\sigma(pp \to H)}{\mathrm{d}q_T} = y_t^2 \frac{\mathrm{d}\sigma_{tt}}{\mathrm{d}q_T} + y_t y_b \frac{\mathrm{d}\sigma_{tb}}{\mathrm{d}q_T} + y_b^2 \frac{\mathrm{d}\sigma_{bb}}{\mathrm{d}q_T} + (y_b \to y_c)$$

Motivation.

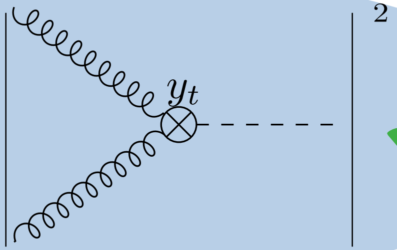
Higgs q_T spectrum

[Billis,Dehnadi, Ebert, Michel, Tackmann ‘21]



$gg \to H$

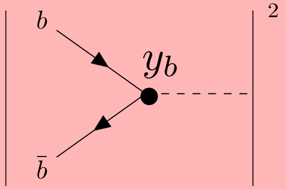
$\mathrm{N^3LL}' + \mathrm{N^3LO}$



✓

$$\frac{d\sigma(pp \to H)}{dq_T} = y_t^2 \frac{d\sigma_{tt}}{dq_T} + y_t y_b \frac{d\sigma_{tb}}{dq_T} + y_b^2 \frac{d\sigma_{bb}}{dq_T} + (y_b \to y_c)$$

$b\bar{b} \to H$

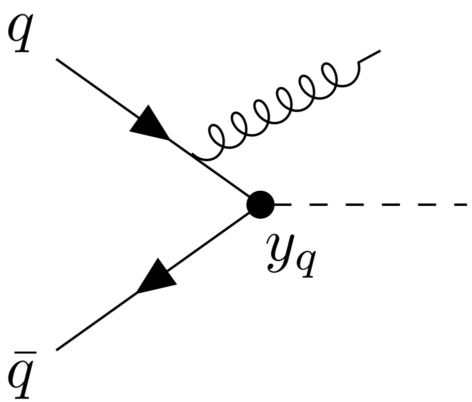


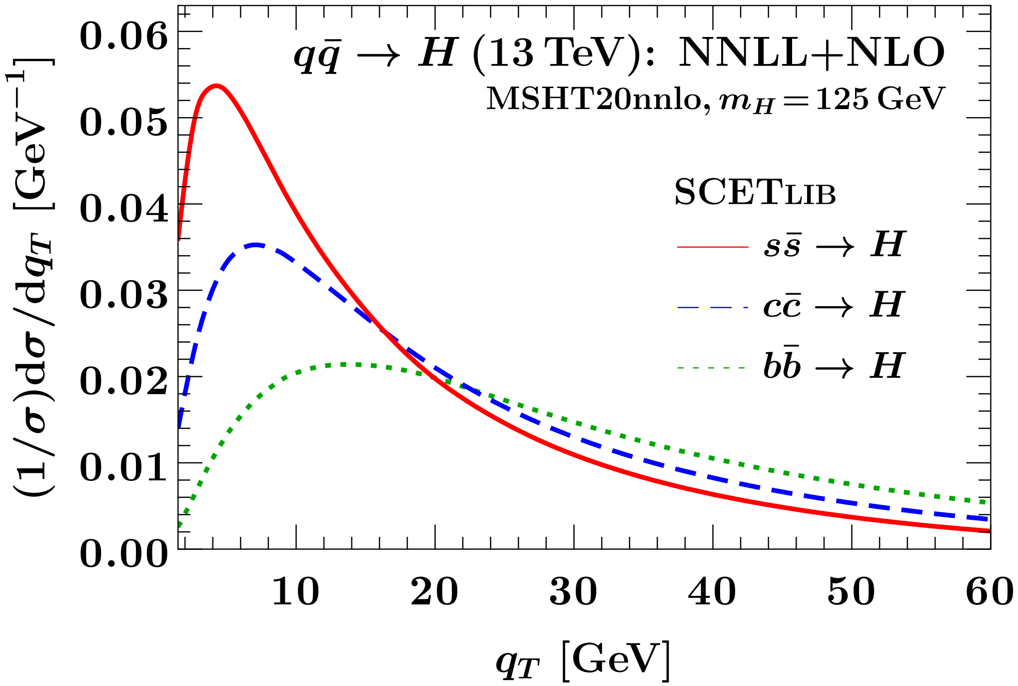
The q_T spectrum for quark initiated Higgs production.

Motivation.

measurement of y_b

- the q_T spectra of $\bar{b}bH, \bar{c}cH$ and $\bar{s}sH$ have different shapes
- precise prediction for $q\bar{q} \to H$ allows for Yukawa fit from the initial state for the quark induced channels
- for NNLL+NLO the uncertainties overlap!
  - ▸ Insufficient precision to distinguish them
- **goal: N^{3}LL$'$ + aN^{3}LO prediction**

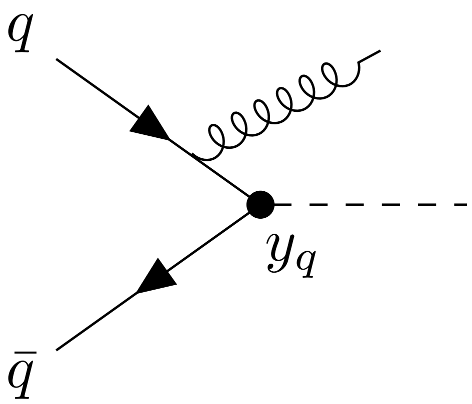


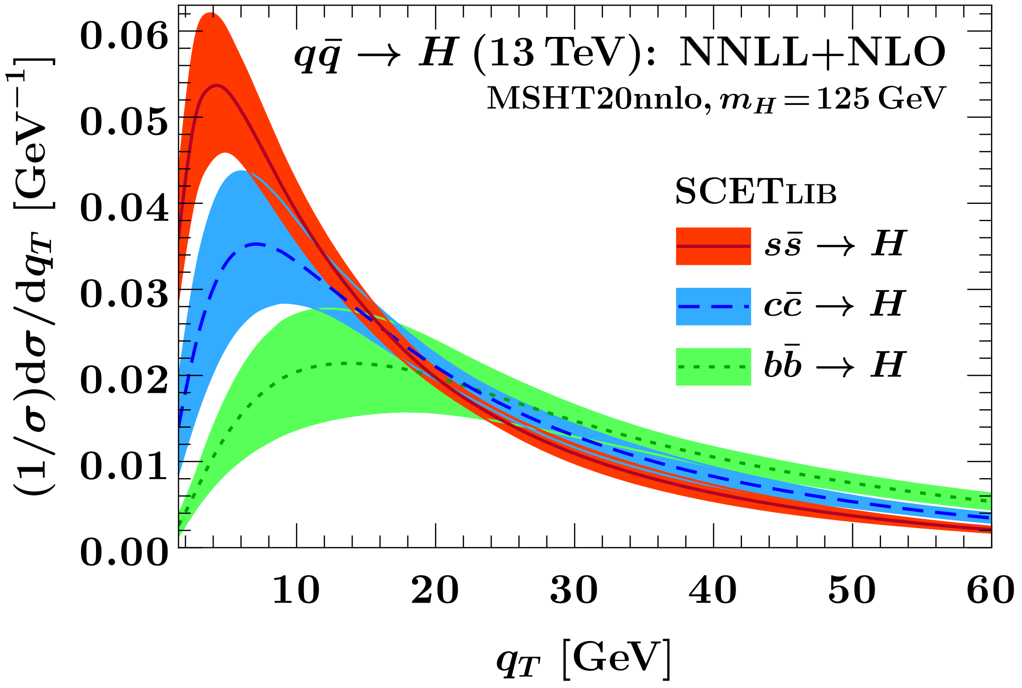


Motivation.

measurement of y_b

- the q_T spectra of $\bar{b}bH, \bar{c}cH$ and $\bar{s}sH$ have different shapes
- precise prediction for $q\bar{q} \to H$ allows for Yukawa fit from the initial state for the quark induced channels
- for NNLL+NLO the uncertainties overlap!
  - ▸ Insufficient precision to distinguish them
- **goal: N^{3}LL$'$ + aN^{3}LO prediction**





Resummation.

Resummation at N^{3}LL$'$

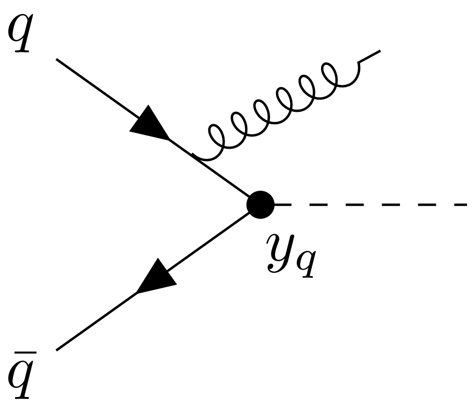
- resummation with `SCETlib` in b_T space [Billis, Ebert, Michel, Tackmann]
- ingredients for N^{3}LL$'$ resummation
  - Hard function at N^{3}LO [Gehrmann, Kara`14, Ebert, Michel, Tackmann `17]
  - Beam function at N^{3}LO [Luo,Yang, Zhu, Zhu`19, Ebert, Mistelberger, Vita `20]
  - Soft function at N^{3}LO [Liu, Zhu, Neill`16, Li, Zhu `16]
  - 4-loop cusp and 3-loop non-cusp anom. dim.

    [Henn, Korchemsiky, Mistelberger `20, v. Manteuffel, Panzer , Schabinger`20] [Li, Zhu `16, Valdimirov`16]
- for $q_T \sim m_H$ use hybrid profile scales to turn off resummation

Fixed order prediction.

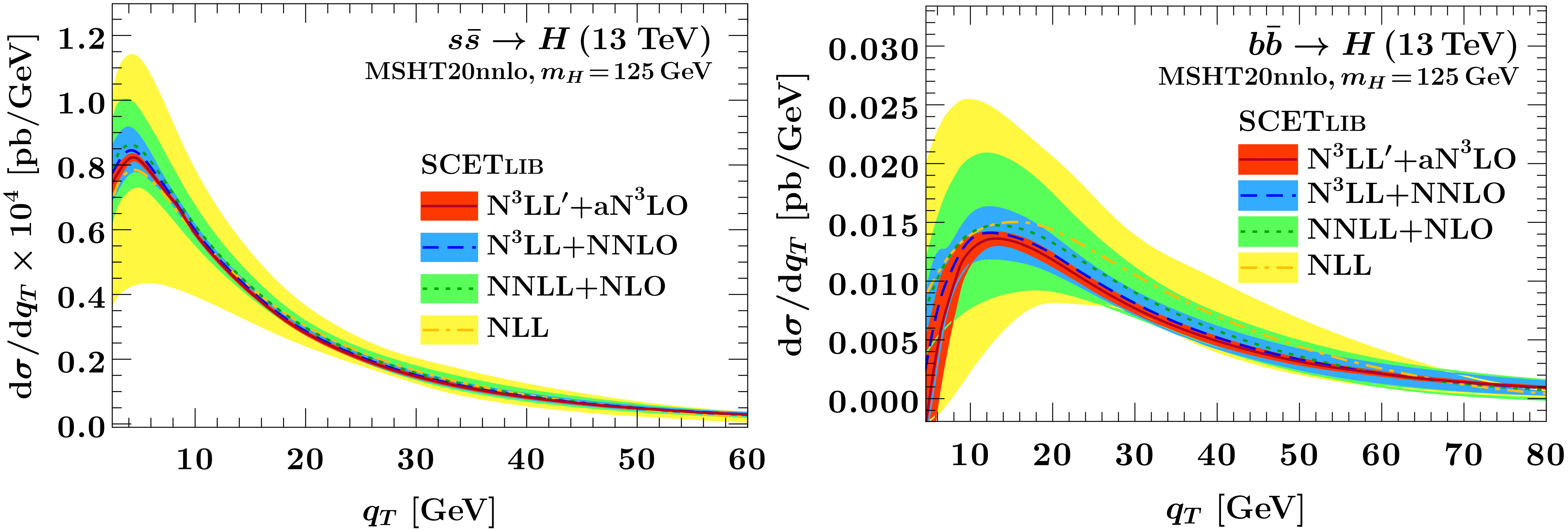
$qqH+$ jet prediction

- LO$_1$ analytic expression implemented in `SCETlib`
- NLO$_1$ implemented qqH in MC event generator `Geneva` [Alioli et al. '14]
  - Use OpenLoops matrix elements [Bucciconi et al. '19]
- aNNLO$_1$: approximate something that could be NNLO$_1$



Results.

N^{3}LL$'$ $+$ aN^{3}LO prediction for $\bar{q}qH$

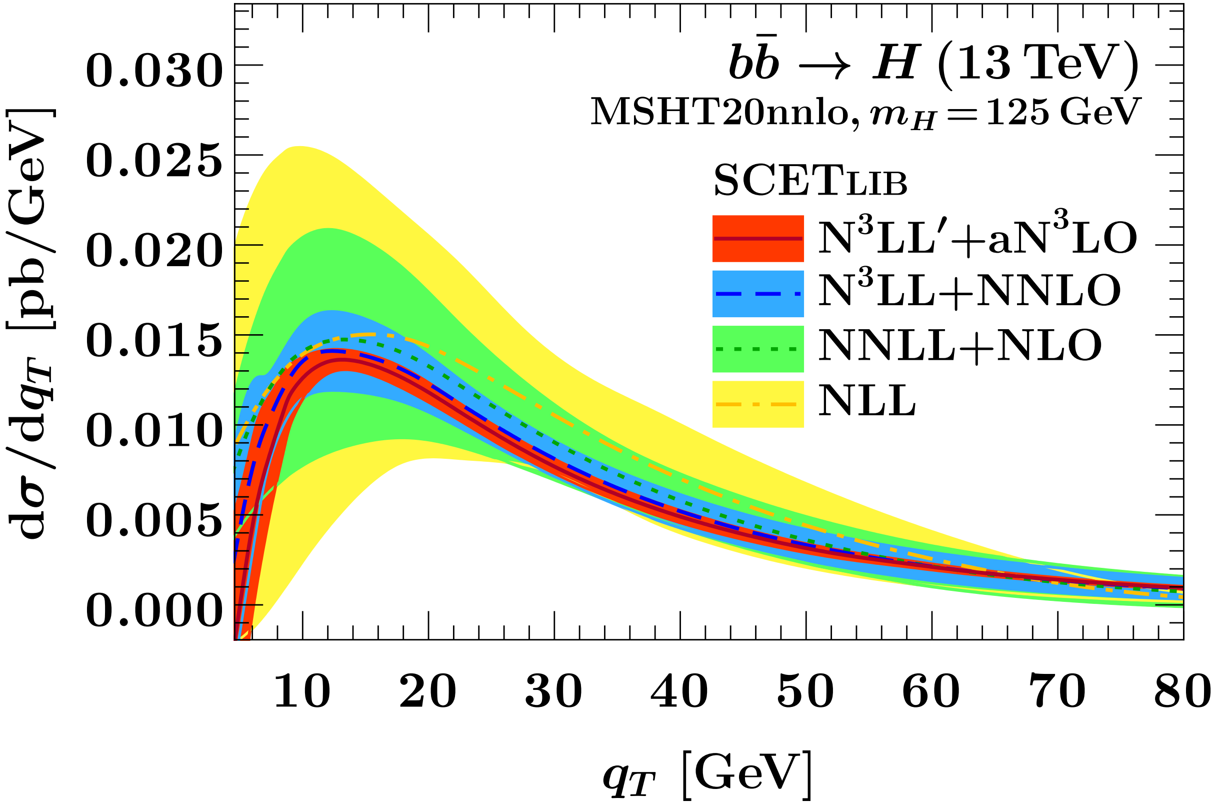


[Cal, RvK, Lim, Tackmann. ‘23]

Results.

$\mathrm{N^3LL'}$ + $\mathrm{aN^3LO}$ prediction for $\bar{q}qH$

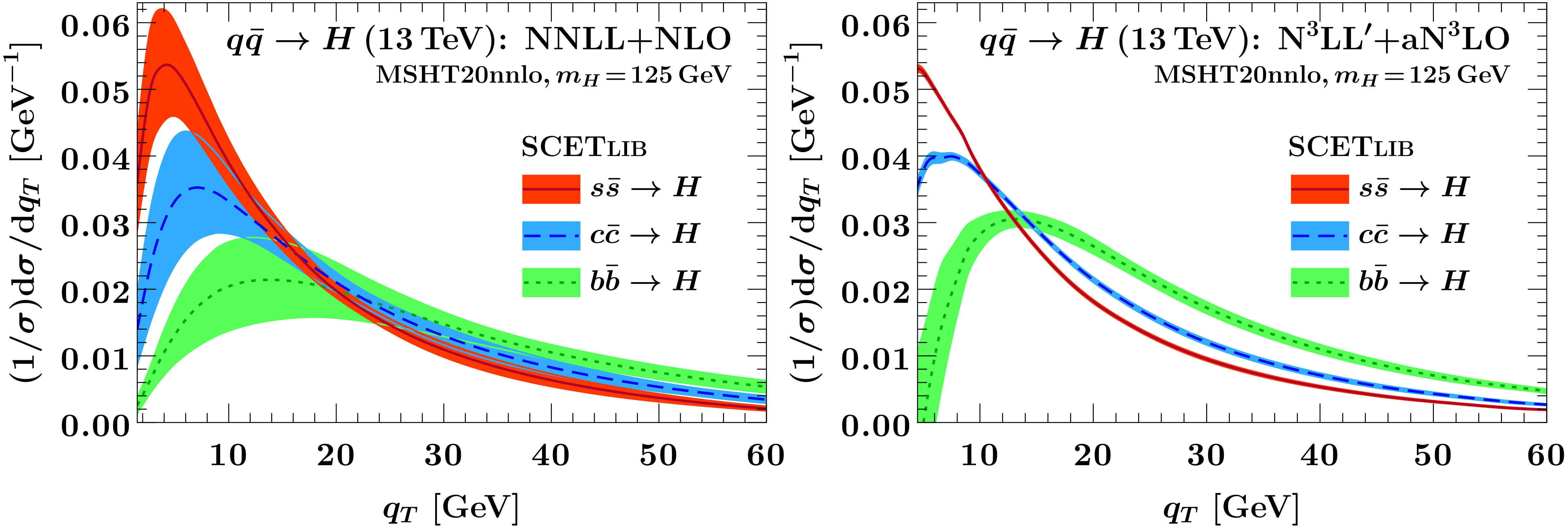
- note: plot is cut at 5 GeV
- using factorization theorem for massless quarks
  - b-quark mass effects become relevant
  - need to include mass effects!
- not an issue for c and s because they are much lighter



Results.

N^{3}LL$'$ + aN^{3}LO prediction for $\bar{q}qH$

[Cal, RvK, Lim, Tackmann. ‘23]



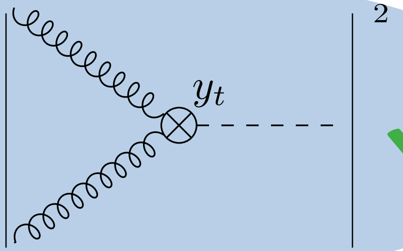
- theory precision high enough uncertainties to allow clear distinction!

Motivation.

Higgs q_T spectrum

$gg \to H$

$\mathrm{N^3LL' + N^3LO}$

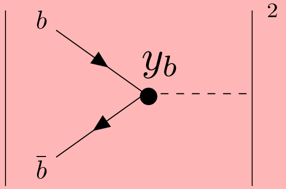


✓

$$\frac{d\sigma(pp \to H)}{dq_T} = y_t^2 \frac{d\sigma_{tt}}{dq_T} + y_t y_b \frac{d\sigma_{tb}}{dq_T} + y_b^2 \frac{d\sigma_{bb}}{dq_T} + (y_b \to y_c)$$

$b\bar{b} \to H$ ✓

$\mathrm{N^3LL' + aN^3LO}$

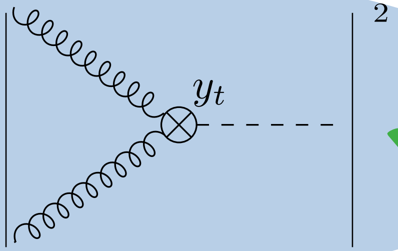


Motivation.

Higgs q_T spectrum

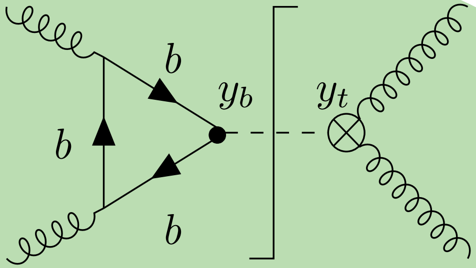
$gg \to H$

$\mathrm{N^3LL' + N^3LO}$



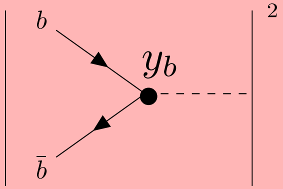
✓

$$\frac{d\sigma(pp \to H)}{dq_T} = y_t^2 \frac{d\sigma_{tt}}{dq_T} + y_t y_b \frac{d\sigma_{tb}}{dq_T} + y_b^2 \frac{d\sigma_{bb}}{dq_T} + (y_b \to y_c)$$



$b\bar{b} \to H$ ✓

$\mathrm{N^3LL' + aN^3LO}$

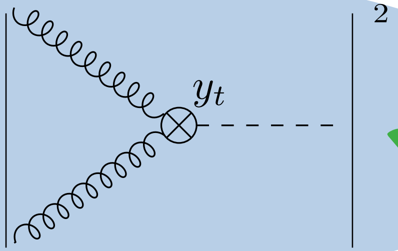


Motivation.

Higgs q_T spectrum

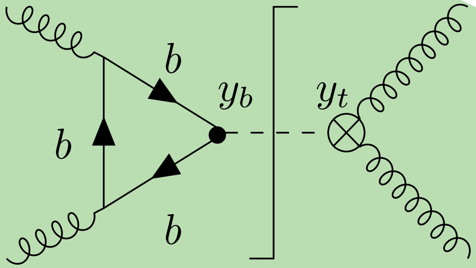
$gg \to H$

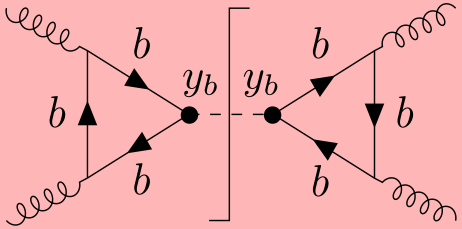
$\mathrm{N^3LL' + N^3LO}$



✓

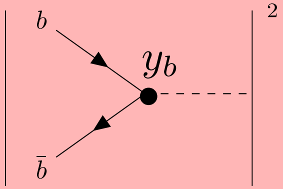
$$\frac{\mathrm{d}\sigma(pp \to H)}{\mathrm{d}q_T} = y_t^2 \frac{\mathrm{d}\sigma_{tt}}{\mathrm{d}q_T} + y_t y_b \frac{\mathrm{d}\sigma_{tb}}{\mathrm{d}q_T} + y_b^2 \frac{\mathrm{d}\sigma_{bb}}{\mathrm{d}q_T} + (y_b \to y_c)$$





$b\bar{b} \to H$ ✓

$\mathrm{N^3LL' + aN^3LO}$

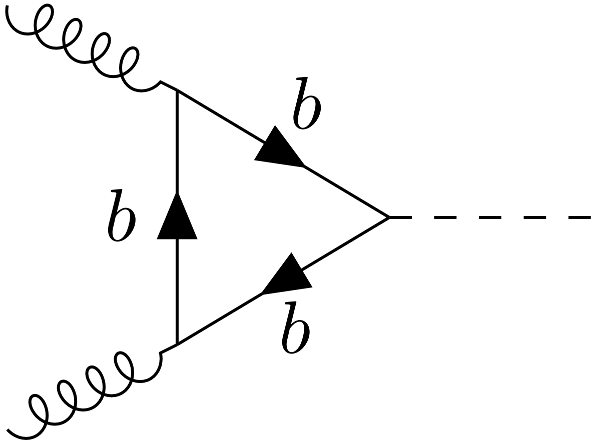


bottom-mass effects in $gg \to H$.

Motivation.

bottom mass effects in gluon fusion

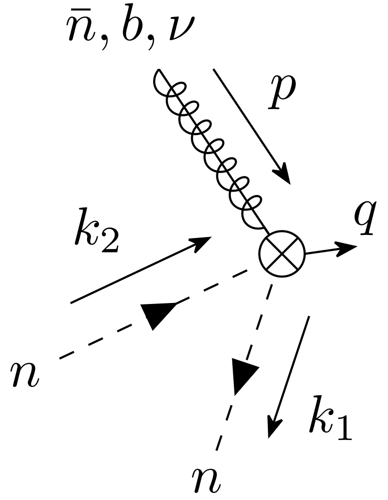
- **now:** consider $gg \to H$ with massive bottom-quark loop
- usually consider top-quark loop since $m_t \gg m_q$
- bottom loop gives $\mathcal{O}(5 - 10\%)$ contribution from interference with top-quark
- lighter quarks only make up for a few percent of the Higgs cross section



Notation and conventions.

Lightcone momenta

- use lightcone coordinates $p = (p^+, p^-, p_\perp)$
- power-counting: small parameter $\lambda = m_b/m_H \ll 1$
  - collinear $p^\mu \sim (\lambda^2, 1, \lambda)$
  - anti-collinear $p^\mu \sim (1, \lambda^2, \lambda)$
  - soft $p^\mu \sim (\lambda, \lambda, \lambda)$
- Higgs minus momentum $q^- = \omega_n$
- fraction of total minus momentum: $\xi = k_2^- / \omega_n$



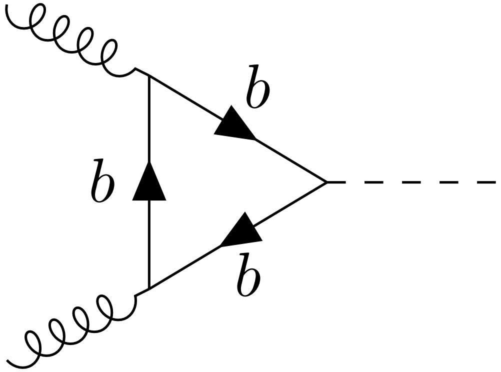
$$\boxed{n^\mu = (1,0,0,1), \quad \bar{n}^\mu = (1,0,0,-1), \quad p^\mu = \frac{n^\mu}{2} p^- + \frac{\bar{n}^\mu}{2} p^+ + p_\perp^\mu, \quad p^- = \bar{n} \cdot p, \quad p^+ = n \cdot p}$$

Mass effects in $gg \rightarrow H$.

so far: form factor $F(m_b, m_H)$

- subleading power factorization and resummation of form factor for $m_q \ll Q$

[Liu, Neubert '19,Liu, Mecaj, Neubert, Wang '20, Liu, Neubert, Schnubel, Wang '22]

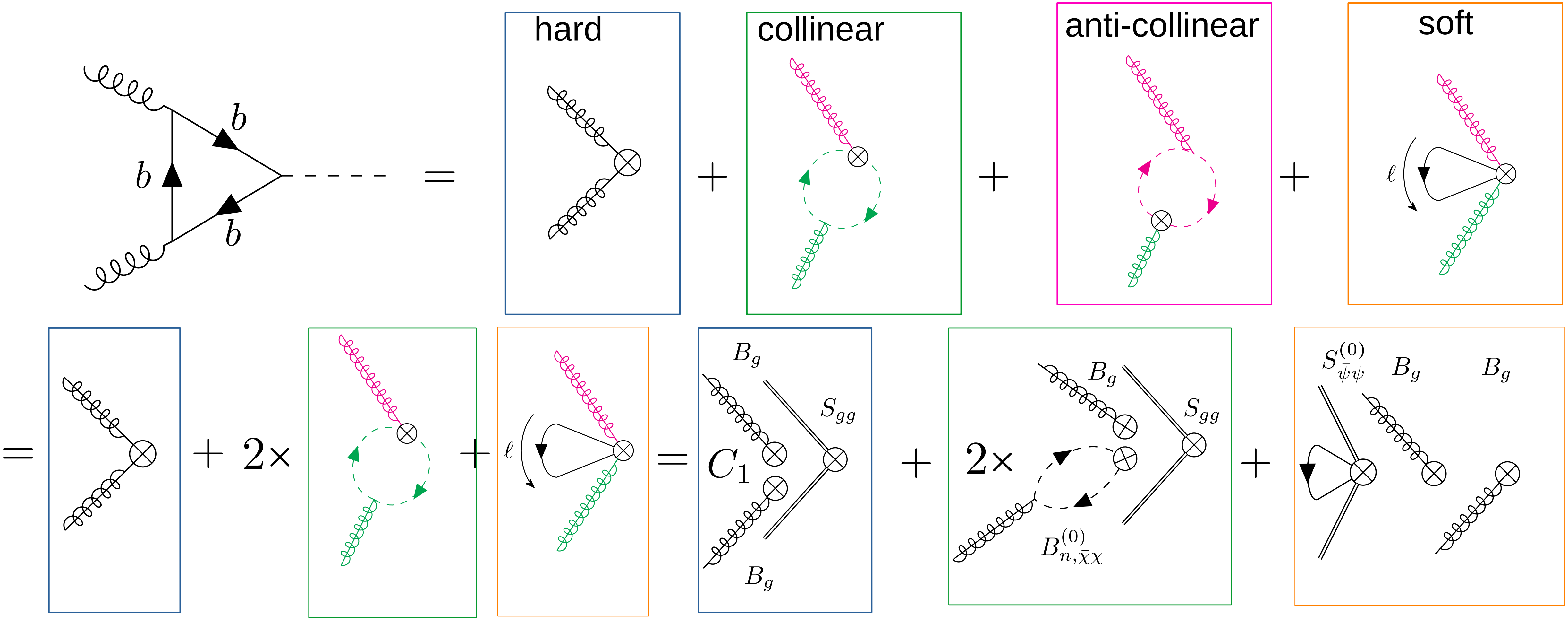


- $F(m_b, m_H)$ depends on two scales
- renormalization and treatment of endpoint divergences is active field of research

[Beneke, Ji, Wang '24]

Mass effects in $gg \to H$.

Notation LO NLP diagram

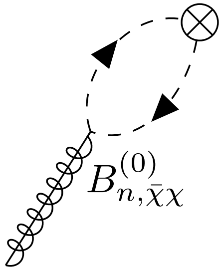


Endpoint divergences at LO.

- regulate endpoint divergences just like rapidity divergences
- example: LO NLP collinear contribution:

$$C^{(0)}_{bbg}(\xi) = \frac{1}{\xi} + \frac{1}{1-\xi}$$

$$\to \int \mathrm{d}\xi \left(\frac{1}{\xi} \left| \frac{\xi \omega_n}{\nu} \right|^{-\eta} + \frac{1}{1-\xi} \left| \frac{(1-\xi)\omega_n}{\nu} \right|^{-\eta} \right)$$



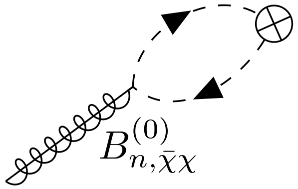
$$\propto \frac{1}{\eta} + \mathcal{O}(\eta^0)$$

- $\frac{1}{\eta}$ is **not** a rapidity divergence!
- the “true” rapidity divergence (related to q_T spectrum) comes later from the phase space integral over k!

Endpoint divergences.

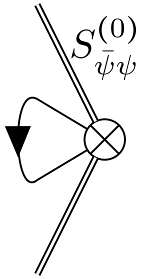
LO NLP contribution

$$\int \mathrm{d}\xi \left(\frac{1}{\xi} \left| \frac{\xi \omega_n}{\nu} \right|^{-\eta} + \frac{1}{1-\xi} \left| \frac{(1-\xi)\omega_n}{\nu} \right|^{-\eta} \right)$$



$\times 2$

$$+ \int \mathrm{d}\ell^+ \mathrm{d}\ell^- \frac{1}{\ell^+\ell^-} \left| \frac{\ell^+\ell^-}{\nu} \right|^{-\frac{\eta}{2}} \left|\sinh y_\ell\right|^{-\eta}$$



$= \mathcal{O}(\eta^0)$

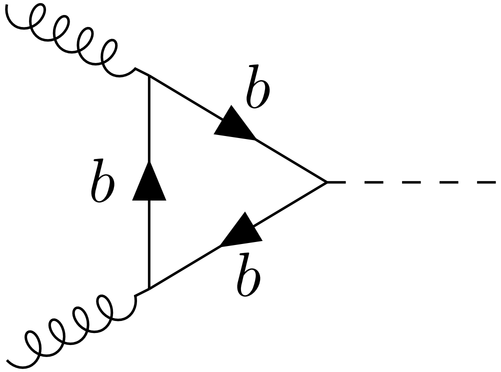
- all endpoint divergences cancel between soft and collinear contributions!

Mass effects in $gg \to H$.

so far: form factor $F(m_b, m_H)$

- subleading power factorization and resummation of form factor for $m_q \ll Q$

[Liu, Neubert ‘19,Liu, Mecaj, Neubert, Wang ‘20,
Liu, Neubert, Schnubel, Wang ‘22]



- $F(m_b, m_H)$ depends on two scales
- renormalization and treatment of endpoint divergences is active field of research

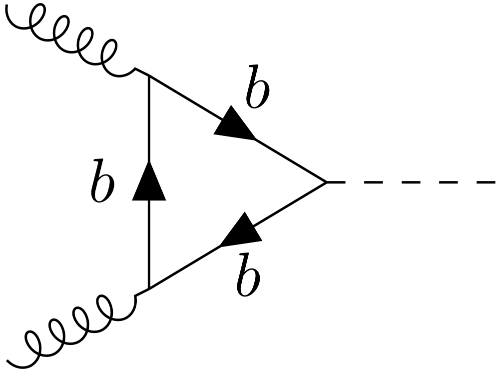
[Beneke, Ji, Wang ‘24]

Mass effects in $gg \rightarrow H$.

so far: form factor $F(m_b, m_H)$

- subleading power factorization and resummation of form factor for $m_q \ll Q$

[Liu, Neubert ‘19,Liu, Mecaj, Neubert, Wang ‘20, Liu, Neubert, Schnubel, Wang ‘22]



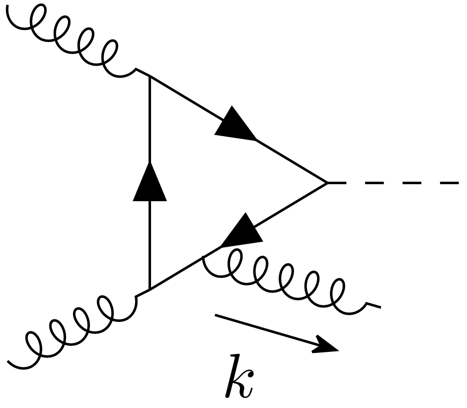
- $F(m_b, m_H)$ depends on two scales
- renormalization and treatment of endpoint divergences is active field of research

[Beneke, Ji, Wang ‘24]

now: q_T spectrum d$\sigma(q_T, m_b, m_H)$

- q_T measurment adds additional scale

➡ three scale problem!



- add emission $k_T \sim q_T$
- still have $m_q \ll Q$, but k_T can have different scalings

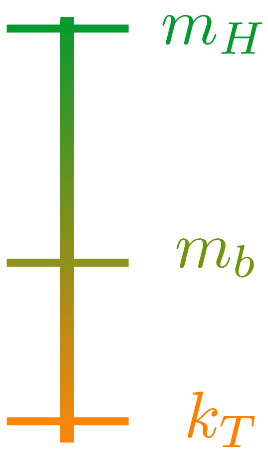
Different regimes.

consider different scalings of k_T

- emission k_T introduces additional scale to the calculation

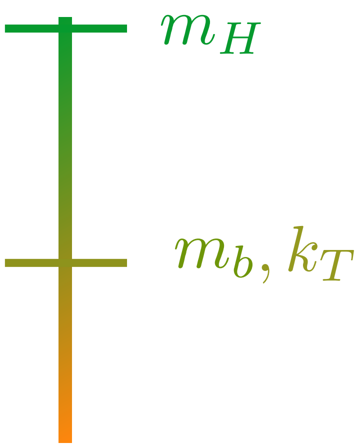
I

$$k_T \ll m_b \ll m_H$$



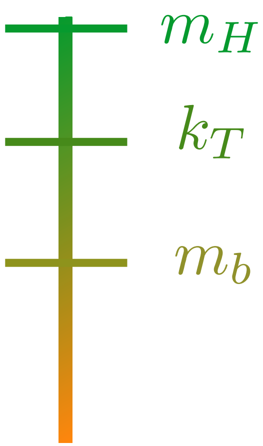
II

$$k_T \sim m_b \ll m_H$$



III

$$m_b \ll k_T \ll m_H$$



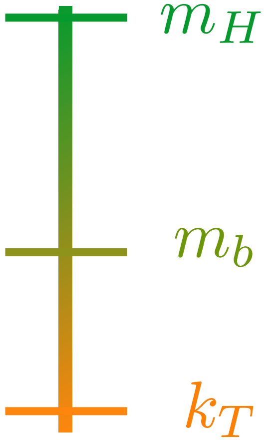
Different regimes.

consider different scalings of k_T

- emission k_T introduces additional scale to the calculation

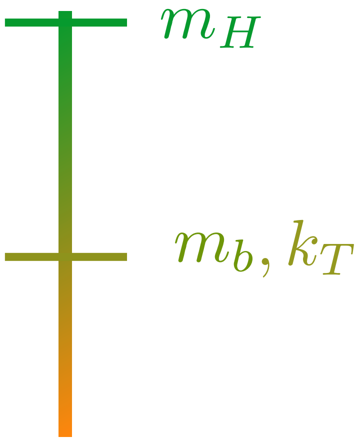
I

$$k_T \ll m_b \ll m_H$$



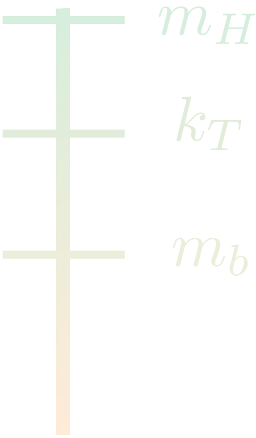
II

$$k_T \sim m_b \ll m_H$$



III

$$m_b \ll k_T \ll m_H$$

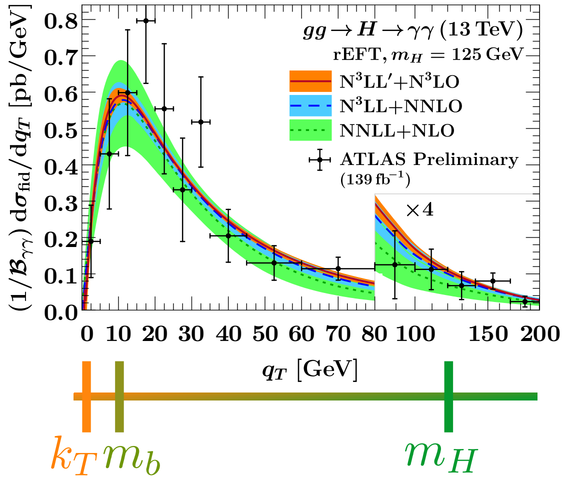


Regime I.

Factorization theorem

- only valid in a very small region of the q_T spectrum
- use standard factorization for q_T resummation with $n_f = 4$ massless flavors

$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = 2\mathrm{Re}[C^*_{ggt}(m_H)\, C_{ggb}(m_b, m_H)]\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$

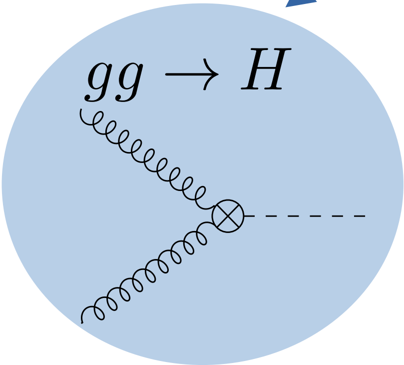


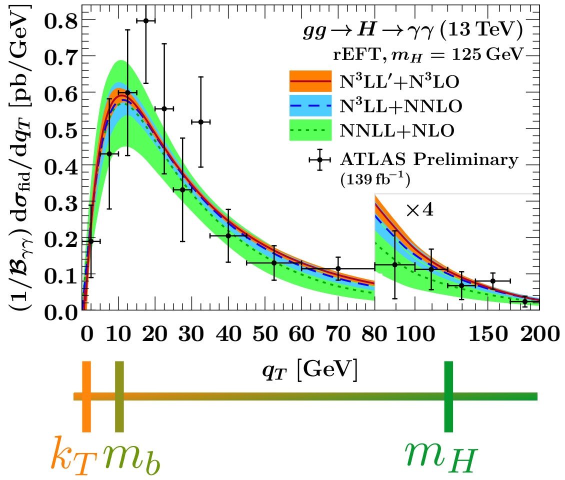
Regime I.

Factorization theorem

- only valid in a very small region of the q_T spectrum
- use standard factorization for q_T resummation with $n_f = 4$ massless flavors

$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = 2\mathrm{Re}[C^*_{ggt}(m_H)\, C_{ggb}(m_b, m_H)]\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$



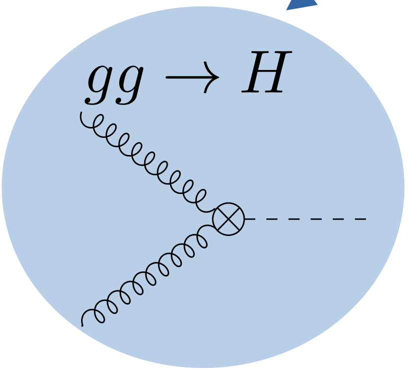


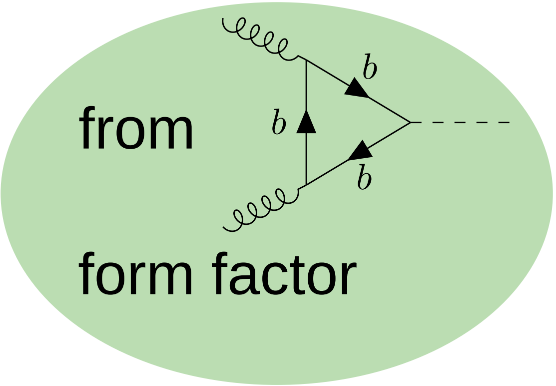
Regime I.

Factorization theorem

- only valid in a very small region of the q_T spectrum
- use standard factorization for q_T resummation with $n_f = 4$ massless flavors

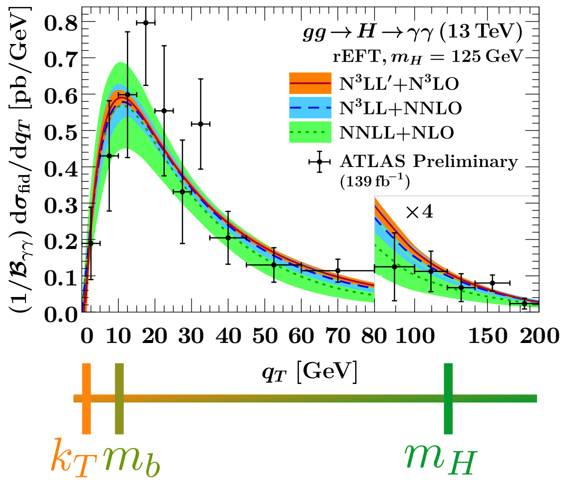
$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = 2\mathrm{Re}[C^*_{ggt}(m_H)\, C_{ggb}(m_b, m_H)]\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$





from

form factor



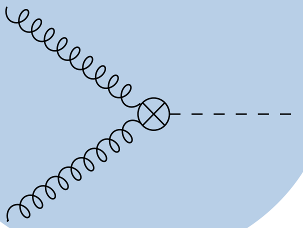
Regime I.

Factorization theorem

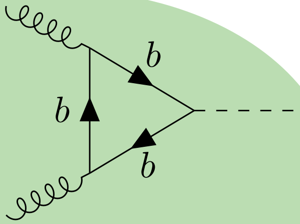
- only valid in a very small region of the q_T spectrum
- use standard factorization for q_T resummation with $n_f = 4$ massless flavors

$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = 2\mathrm{Re}[C^*_{ggt}(m_H)\, C_{ggb}(m_b, m_H)]\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$

$gg \to H$

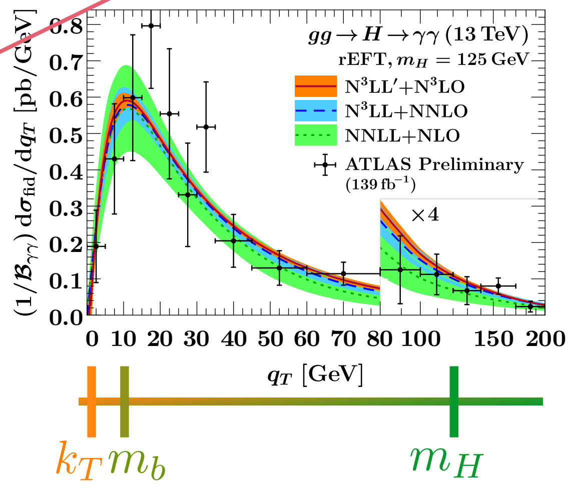


from



form factor

q_T resummation



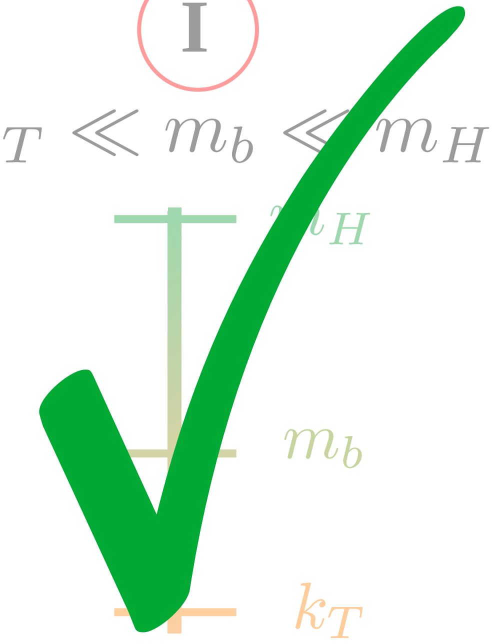
Different regimes.

Consider different scalings of k_T

- emission k_T introduces additional scale to the calculation

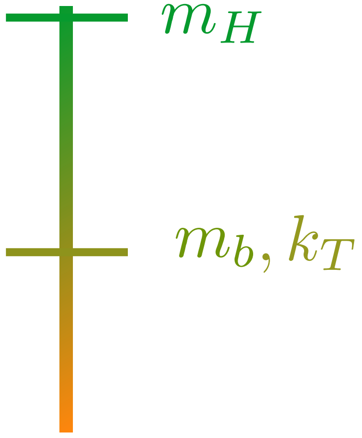
I

$k_T \ll m_b \ll m_H$



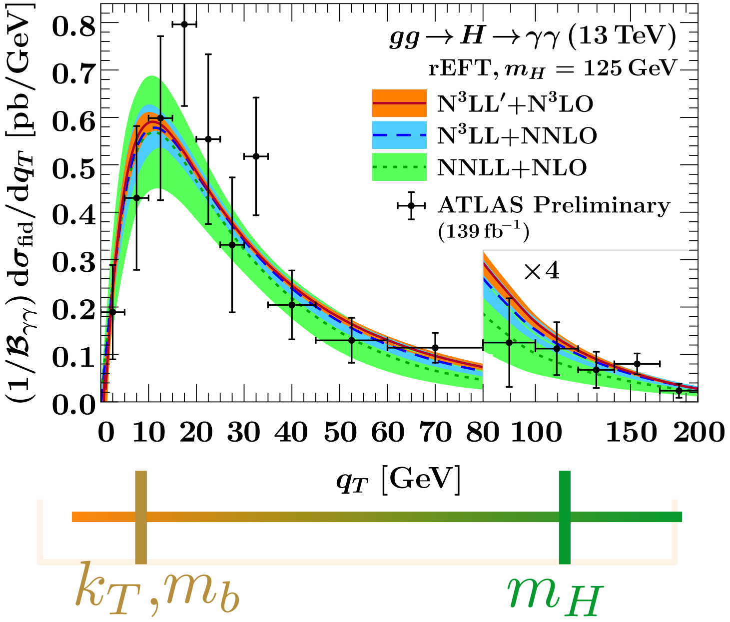
II

$k_T \sim m_b \ll m_H$



III

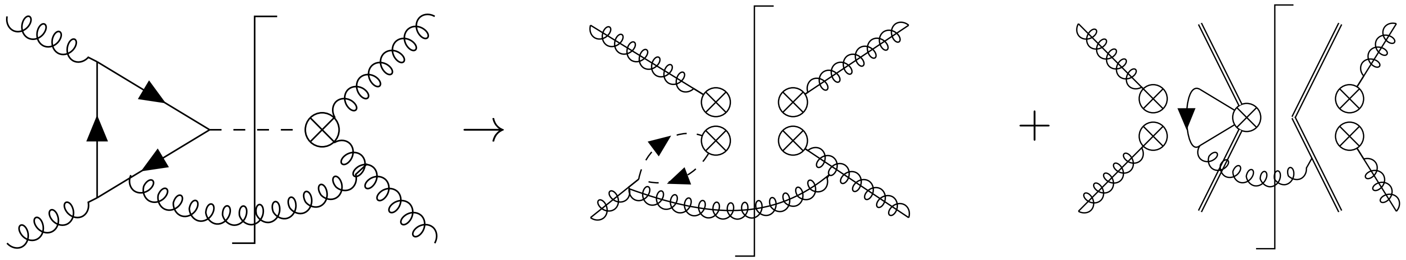
$m_b \ll k_T \ll m_H$



Regime II.

Bare factorization theorem $k_T \sim m_b \ll m_H$

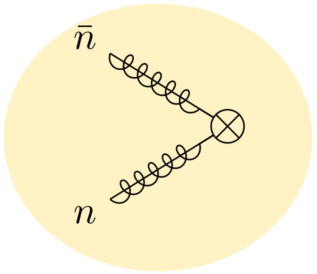
$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$
$$+ \int \mathrm{d}\xi H_{bbg}(\xi)\, \left[B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T)\right.$$
$$\left. + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)\right]$$
$$+ \int \mathrm{d}\ell^+ \mathrm{d}\ell^- H_{bbgg} \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)$$



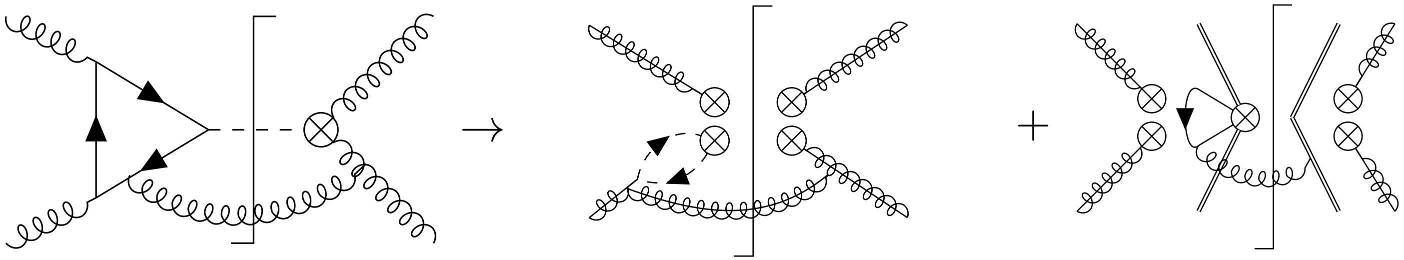
operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

Regime II.

Bare factorization theorem $k_T \sim m_b \ll m_H$



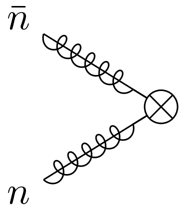
$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} &= H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&+ \int \mathrm{d}\xi H_{bbg}(\xi)\, [B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&\qquad + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)] \\
&+ \int \mathrm{d}\ell^+ \mathrm{d}\ell^- H_{bbgg} \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)
\end{aligned}
$$



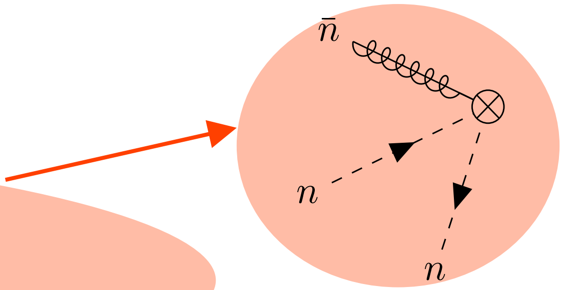
operators: [M. Stahlhofen’s SCET talk ‘ 15, Liu, Neubert ‘19]

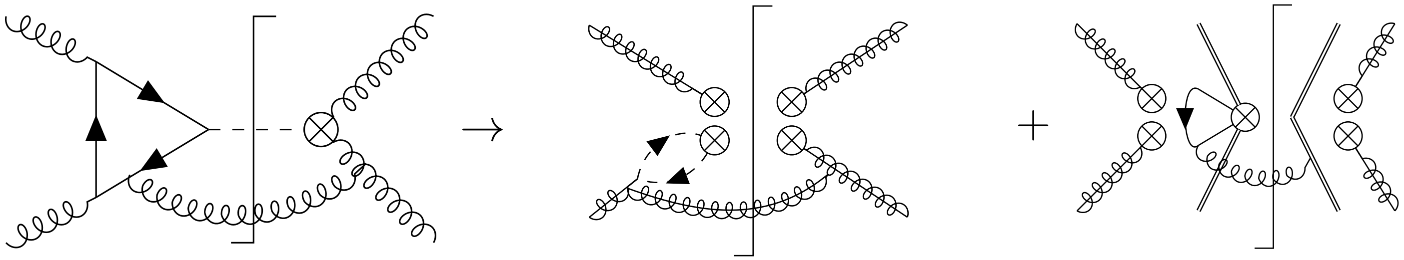
Regime II.

Bare factorization theorem $k_T \sim m_b \ll m_H$



$$
\begin{aligned}
\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} &= H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&+ \int \mathrm{d}\xi H_{bbg}(\xi)\, [B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
&\qquad + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)] \\
&+ \int \mathrm{d}\ell^+ \mathrm{d}\ell^- H_{bbgg} \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)
\end{aligned}
$$

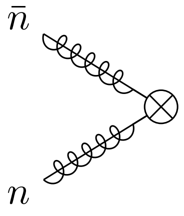


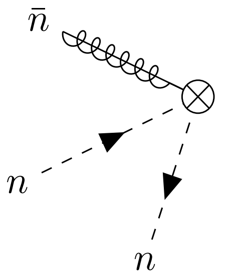


operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

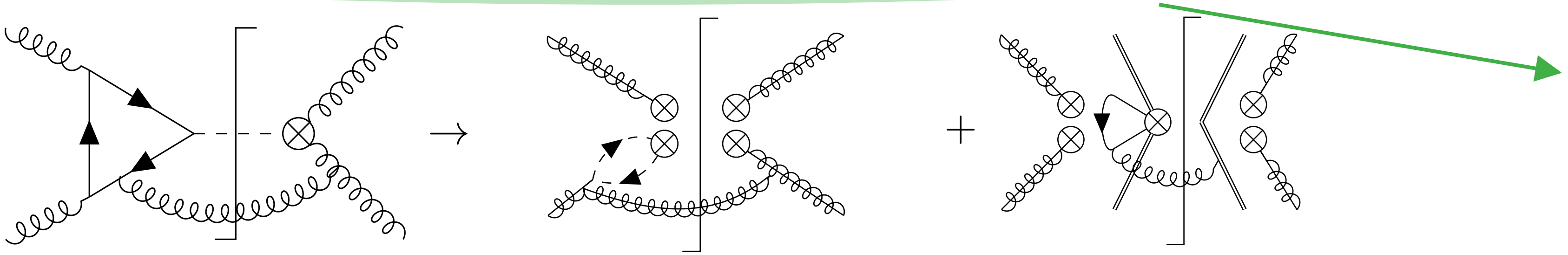
Regime II.

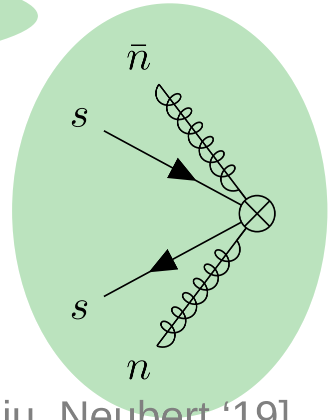
Bare factorization theorem $k_T \sim m_b \ll m_H$





$$
\begin{aligned}
\frac{d\sigma_{y_t y_b}}{dq_T} = {} & H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
& + \int d\xi H_{bbg}(\xi)\, [B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) \\
& \qquad + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)] \\
& + \int d\ell^+ d\ell^- H_{bbgg} \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)
\end{aligned}
$$





operators: [M. Stahlhofen's SCET talk ' 15, Liu, Neubert '19]

Regime II.

Bare factorization theorem $k_T \sim m_b \ll m_H$

$$\frac{\mathrm{d}\sigma_{y_t y_b}}{\mathrm{d}q_T} = H_{gg}(m)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{gg}(q_T)$$

$$+ \int \boxed{\mathrm{d}\xi} H_{bbg}(\xi)\, \left[B_{n,\bar{\chi}\chi}(\xi, q_T, m) \otimes B_g(q_T) \otimes S_{gg}(q_T) + B_g(q_T) \otimes B_{\bar{n},\bar{\chi}\chi}(\xi, q_T, m) \otimes S_{gg}(q_T)\right]$$

$$+ \int \boxed{\mathrm{d}\ell^+ \mathrm{d}\ell^-} H_{bbgg}\, \mathcal{J}(\ell^+) \mathcal{J}(\ell^-)\, B_g(q_T) \otimes B_g(q_T) \otimes S_{\bar{\psi}\psi}(\ell^+, \ell^-, q_T, m)$$

$$\boxed{\frac{1}{\xi} + \frac{1}{1-\xi}}$$

$$\boxed{\frac{1}{\ell^+ \ell^-}}$$

lead to endpoint divergences! (just as for form factor)

operators: [M. Stahlhofen’s SCET talk ‘ 15, Liu, Neubert ‘19]

Endpoint divergences.

- expect endpoint divergences in soft and collinear contribution
- form factor $F(m_b,m_H)$ depends on two scales
- the spectrum introduces an additional scale k_T

$$\boxed{\frac{1}{\xi} f_n\left(\frac{m}{k_T}\right) \quad \Longleftrightarrow \quad \frac{1}{\ell^+\ell^-} f_s\left(\frac{m}{k_T}\right)}$$

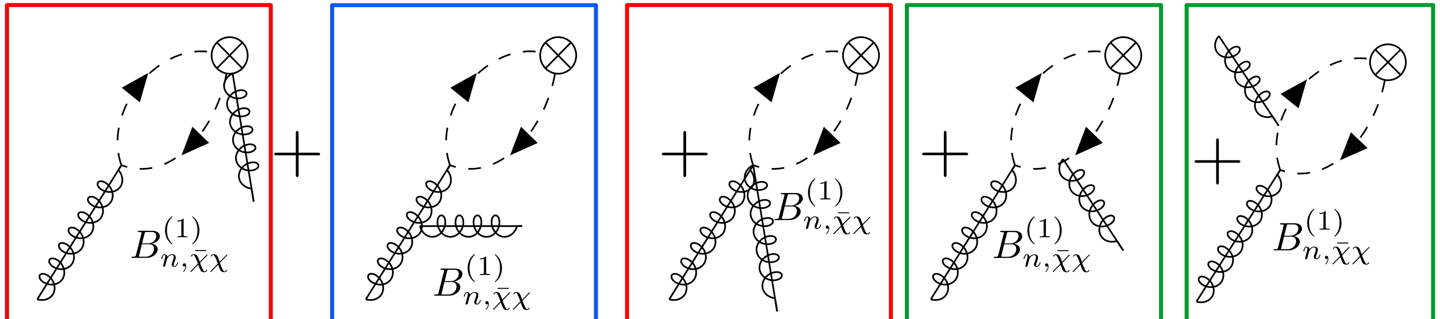
- **how does the additional scale affect the structure of the endpoint divergences?**
- in general $f_n(m/k_T)$ and $f_s(m/k_T)$ can be non-trivial functions of m/k_T

Endpoint divergences.

collinear NLP one-gluon contribution

- Now: add emission k_T to contribution from collinear loop

$$\int \mathrm{d}\xi \left(\frac{1}{\xi} \left| \frac{\xi}{\nu} \right|^{-\eta} + \frac{1}{1-\xi} \left| \frac{1-\xi}{\nu} \right|^{-\eta} \right) \Bigg(\quad + \quad + \quad + \quad + \quad \Bigg)$$



$$= -\frac{A(k^-)}{4\eta\epsilon} - \frac{B(k^-)}{2\eta\epsilon} - \frac{A(k^-)}{4\eta\epsilon} \underbrace{- \frac{C(k^-)}{2\eta\epsilon} + \frac{C(k^-)}{2\eta\epsilon}}_{= \mathcal{O}(\eta^0)}$$

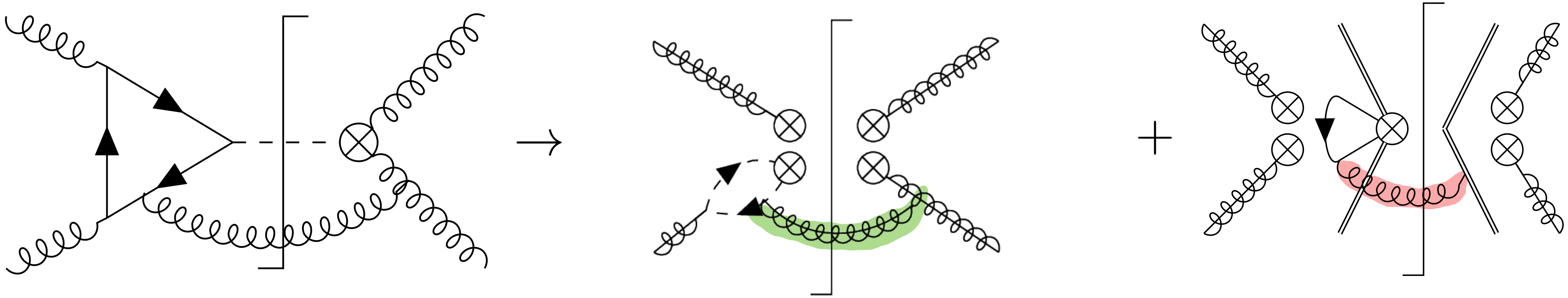
$$= -\frac{A(k^-)}{2\eta\epsilon} - \frac{B(k^-)}{2\eta\epsilon}$$

- endpoint divergences partially cancel within the beam function but there are left-over poles

Emission k_T.

soft vs. collinear emission

- emitted gluon can be soft or collinear
- endpoint divergences **have to cancel** within the same sector
- consider both sectors separately

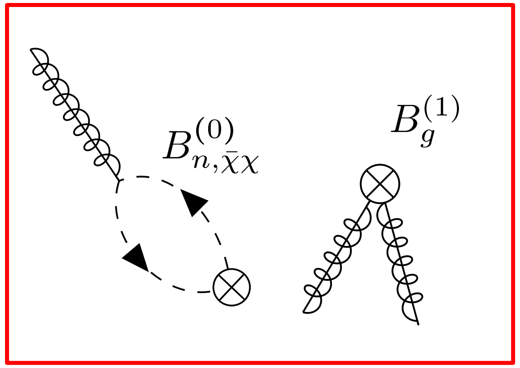


Endpoint divergences.

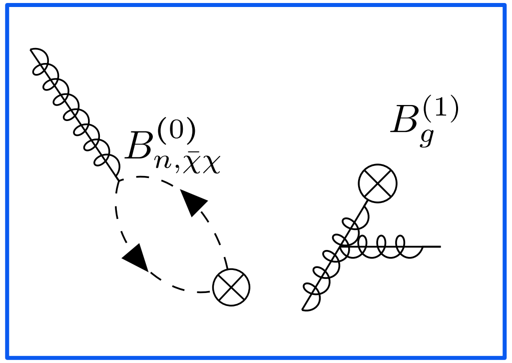
collinear LP one-gluon contribution

- contribution from anti-collinear loop can have LP gluon emission

$$\int \mathrm{d}\xi \left(\frac{1}{\xi}\left|\frac{\xi}{\nu}\right|^{-\eta} + \frac{1}{1-\xi}\left|\frac{1-\xi}{\nu}\right|^{-\eta}\right)\Bigg(\quad\cdots\quad + \quad\cdots\quad\Bigg)$$



$+$

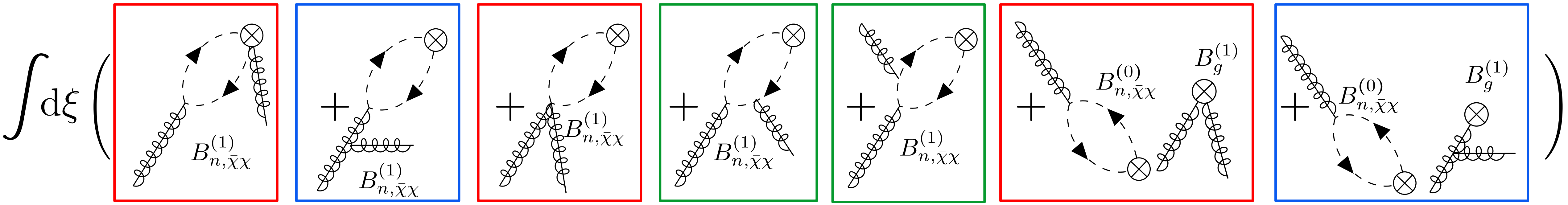


$$= -\frac{A(k^{-})}{2\eta\epsilon} - \frac{B(k^{-})}{2\eta\epsilon}$$

- endpoint divergences have the same sign as NLP collinear emission

Endpoint divergences.

collinear NLP and LP emissions



$$= -\frac{A(k^-)}{\eta\epsilon} - \frac{B(k^-)}{\eta\epsilon}$$

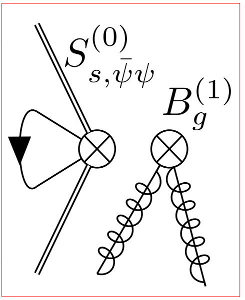
- **uncanceled endpoint divergences for collinear and anti-collinear loops!**

Endpoint divergences.

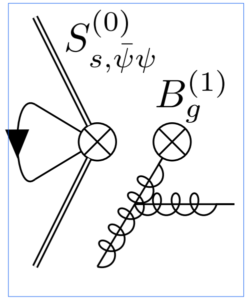
collinear emission

- has to be canceled by collinear LP emission and soft LO NLP!

$$\int \mathrm{d}\ell^+\mathrm{d}\ell^- \left|\frac{\ell^+\ell^-}{\nu}\right|^{-\frac{\eta}{2}} |\sinh y_\ell|^{-\eta} \Bigg($$



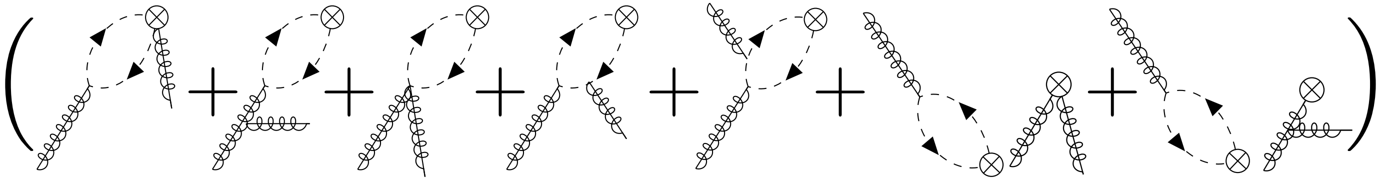
$+$



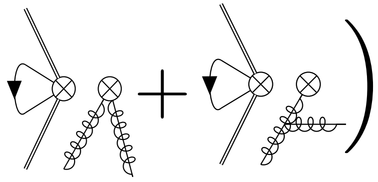
$$\Bigg) = +\frac{A(k^-)}{\eta\epsilon} + \frac{B(k^-)}{\eta\epsilon}$$

- endpoint divergences cancel between diagrams with collinear emission!

$\int \mathrm{d}\xi \Bigg($



$\Bigg) + \int \mathrm{d}\ell^+\mathrm{d}\ell^- \Bigg($



$\Bigg) = \mathcal{O}(\eta^0)$

- **divergences cancel against LP collinear emission!**
- **mass and k_T dependence are factorized!**

Endpoint divergences.

soft emission

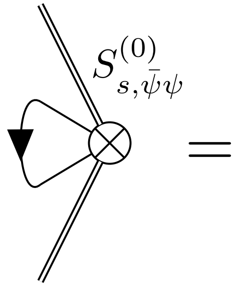
- sum of diagrams with collinear emission is finite

➔ **sum of diagrams with soft emission has to be finite as well!**

➔ free to choose regulator to regulate endpoint divergences in this subset of diagrams!

- LO NLP example for pure rapidity regulator [Ebert, Moult, Stewart,Tackmann, Vita, Zhu ‘18]

$$\int \mathrm{d}\ell^+ \mathrm{d}\ell^- \left(\frac{\ell^+}{\ell^-}\right)^{-\eta}$$



$$= 0$$

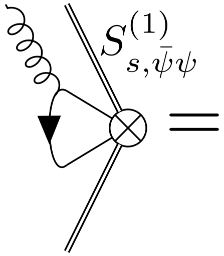
(scaleless)

Endpoint divergences.

soft emission

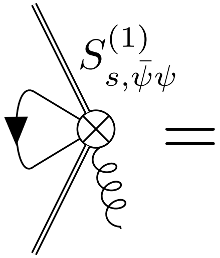
- use the pure rapidity regulator for NLP soft diagram

$$\int \mathrm{d}\ell^+\mathrm{d}\ell^- \left(\frac{\ell^+}{\ell^-}\right)^{-\eta}$$



$$= \mathcal{O}(\eta^0)$$

$$\int \mathrm{d}\ell^+\mathrm{d}\ell^- \left(\frac{\ell^+}{\ell^-}\right)^{-\eta}$$



$$= \mathcal{O}(\eta^0)$$

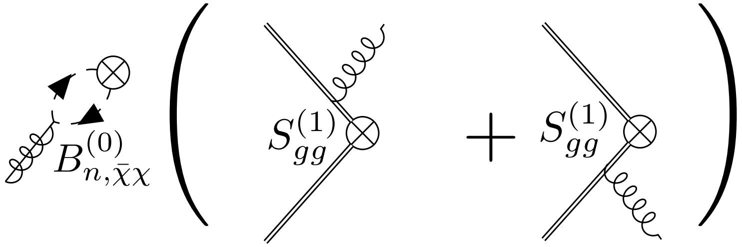
- **both contributions are individually finite!**

Endpoint divergences.

soft emission

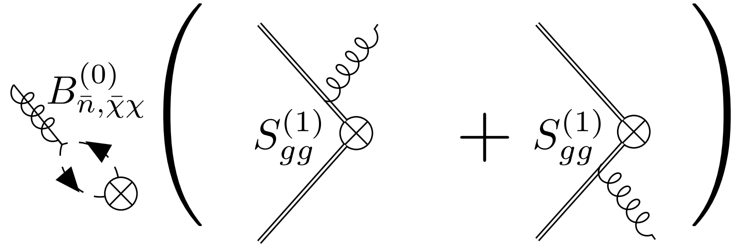
- what about the collinear LO NLP × soft LP emission?

$$\int \mathrm{d}\xi \left(\frac{1}{\xi} + \frac{1}{1-\xi}\right)\xi^{-\eta}(1-\xi)^{-\eta}$$



$$= \frac{D(k^-)}{\eta}$$

$$\int \mathrm{d}\xi \left(\frac{1}{\xi} + \frac{1}{1-\xi}\right)\xi^{-\eta}(1-\xi)^{-\eta}$$

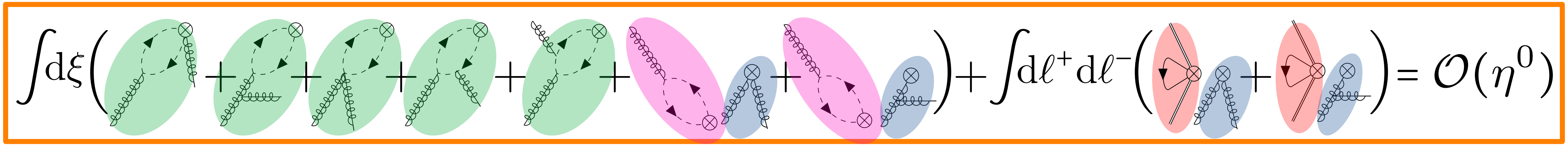


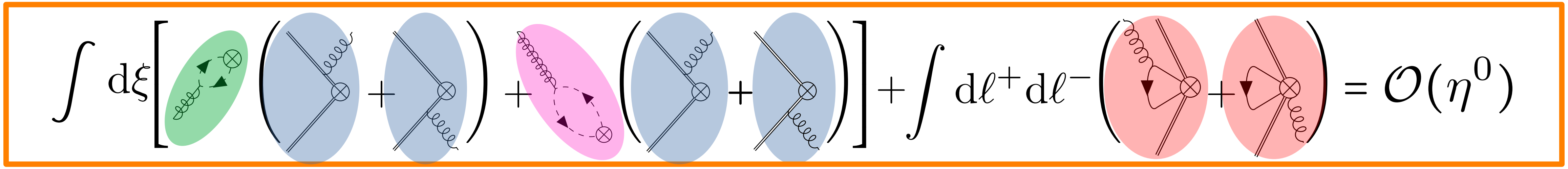
$$= -\frac{D(k^-)}{\eta}$$

- **endpoint divergences from n- and $\bar{n}$- collinear sector cancel!**

Endpoint divergences.

Summary





- **all endpoint divergences cancel!**
- **m and k_T dependence factorizes!**

Mass effects in ggH.

Next steps

- endpoint divergences cancel ✓
- calculate finite parts of the integrals ✓
- compare against full QCD amplitude in respective limit [Bauer, Glover 1989]
- phase space integral over emission k
- write paper!

Outlook and summary.

Outlook.

Yukawa fits

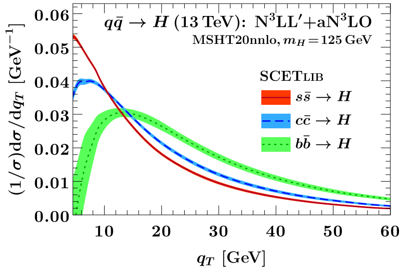
- include finite mass effects for qqH
- combine qqH and bottom-mass effects in gluon fusion

$$\boxed{\frac{\mathrm{d}\sigma(pp \to H)}{\mathrm{d}q_T} = y_t^2 \frac{\mathrm{d}\sigma_{tt}}{\mathrm{d}q_T}\checkmark + y_t y_b \frac{\mathrm{d}\sigma_{tb}}{\mathrm{d}q_T}\checkmark + y_b^2 \frac{\mathrm{d}\sigma_{bb}}{\mathrm{d}q_T}\checkmark + (y_b \to y_c)}$$

- fit the bottom and charm Yukawa couplings from Higgs production!

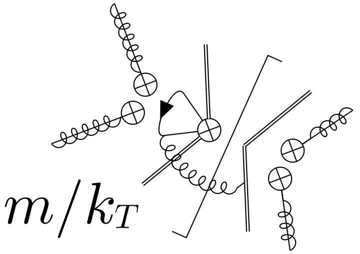
Summary.

N^{3}LL$'$ + aN^{3}LO prediction for qqH

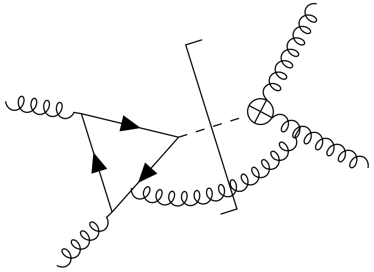


- new prediction for quark initiated Higgs production
- at N^{3}LL$'$ + aN^{3}LO: uncertainties no longer overlap!

m_b effects in gluon fusion



- the emission k_T adds an extra scale to the problem
- coefficient functions of endpoint divergences could be non- trivial functions of m/k_T
- m and k_T dependence factorizes!
- endpoint divergences from soft and collinear emissions cancel separately



Outlook

- put everything together and fit the Yukawa coupling

Thank you!

Acknowledgments.

This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation program (Grant agreement No. 101002090 COLORFREE).

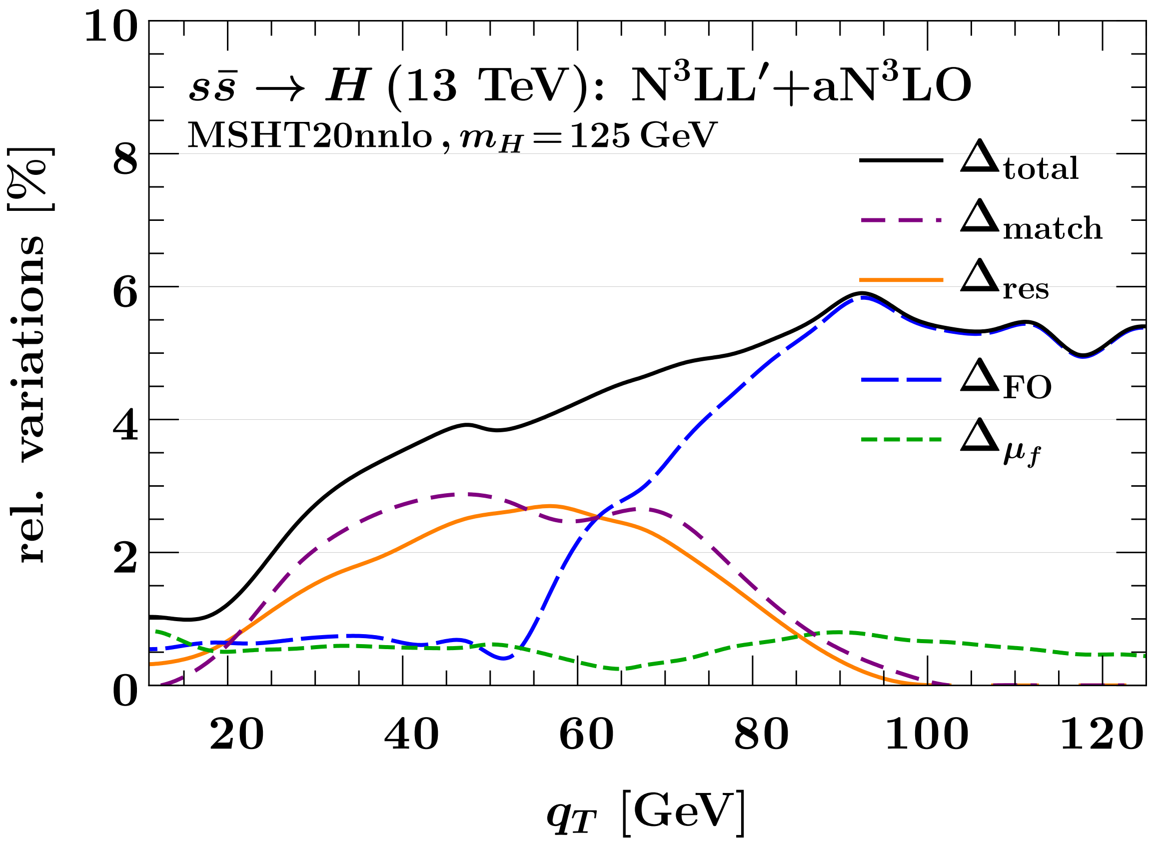
erc

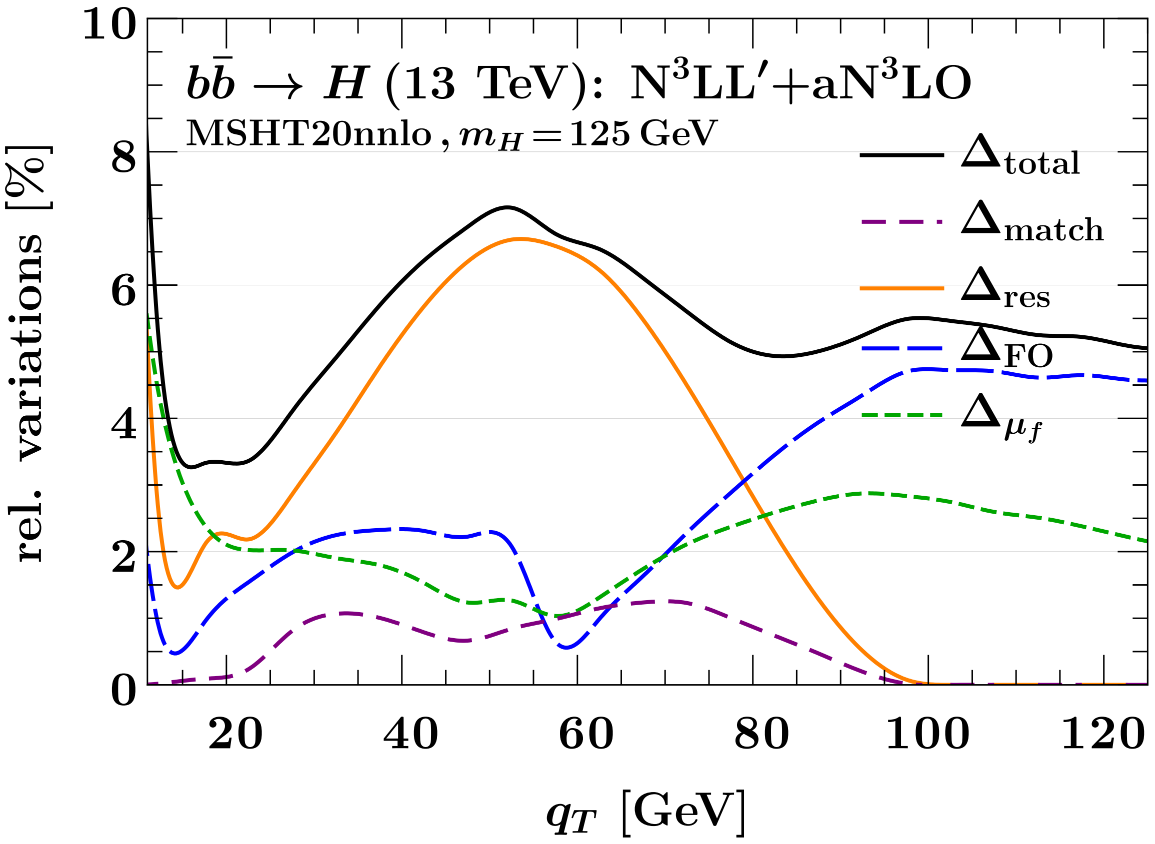
European Research Council

Established by the European Commission

Back up.

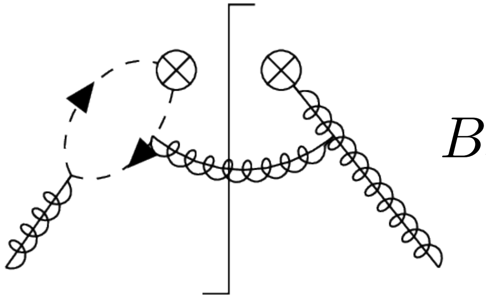
Impact plots.





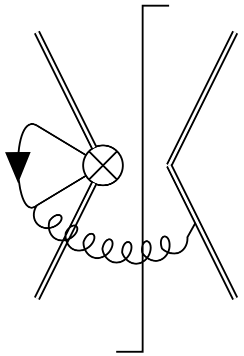
Matrix element definition.

Beam function



$$B_{n,\chi\bar{\chi}}(x = \frac{\omega}{P_N^-}) = \sum\!\!\!\!\!\!\!\int \langle N| \mathcal{B}_\perp^{a,\mu} \delta^2(k_\perp - P_{X,\perp}) |X\rangle \langle X| \bar{\chi}_{n,\omega z} T^a \gamma_{\perp\mu} \chi_{n,\omega(1-z)} |N\rangle$$

Soft function

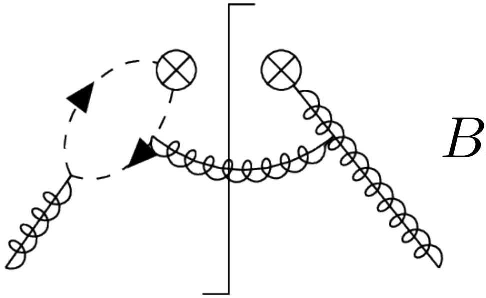


$$\mathcal{O}^{ab}_{S,\bar{\psi}\psi}(\ell^+\ell^-) = \frac{1}{\ell^+\ell^-}[\bar{\psi} S_n \delta(\ell^+ - n\cdot\mathcal{P}])[\gamma_{\perp,\mu} T^a \frac{\bar{\not{n}}\not{n}}{4} S_n^\dagger S_{\bar{n}} \gamma_\perp^\mu T^b][S_{\bar{n}}^\dagger \psi \delta(\ell^- - \bar{n}\cdot\mathcal{P}])$$

$$S_{\bar{\psi}\psi}(\ell^+, \ell^-, k_T, m) = \sum\!\!\!\!\!\!\!\int \frac{1}{N_c^2 - 1} \langle 0| \mathcal{O}_s^{(0)\,ab} \delta^2(k_\perp - P_{X,\perp}) |X\rangle \langle X| \mathcal{O}^{ba}_{s,\bar{\psi}\psi} |0\rangle$$

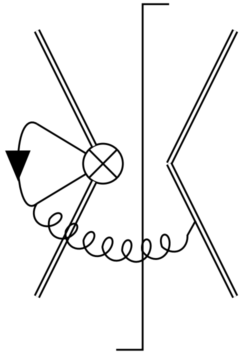
Matrix element definition.

Beam function



$$B_{n,\chi\bar{\chi}}(x = \frac{\omega}{P_N^-}) = \sum\!\!\!\!\!\!\!\int \langle N| \mathcal{B}_\perp^{a,\mu} \delta^2(k_\perp - P_{X,\perp}) |X\rangle \langle X| \bar{\chi}_{n,\omega z} T^a \gamma_{\perp\mu} \chi_{n,\omega(1-z)} |N\rangle$$

Soft function



$$\mathcal{O}^{ab}_{S,\bar{\psi}\psi}(\ell^+\ell^-) = \frac{1}{\ell^+\ell^-}[\bar{\psi} S_n \delta(\ell^+ - n\cdot\mathcal{P}])[\gamma_{\perp,\mu} T^a \frac{\bar{\not{n}}\not{n}}{4} S_n^\dagger S_{\bar{n}} \gamma_\perp^\mu T^b][S_{\bar{n}}^\dagger \psi \delta(\ell^- - \bar{n}\cdot\mathcal{P}])$$

$$S_{\bar{\psi}\psi}(\ell^+, \ell^-, k_T, m) = \sum\!\!\!\!\!\!\!\int \frac{1}{N_c^2 - 1} \langle 0| \mathcal{O}_s^{(0)\,ab} \delta^2(k_\perp - P_{X,\perp}) |X\rangle \langle X| \mathcal{O}^{ba}_{s,\bar{\psi}\psi} |0\rangle$$