TUM/MPP PHENOMENOLOGY SEMINAR

9TH DECEMBER 2025

ANOMALOUS SCALING OF LINEAR POWER CORRECTIONS

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University of Oxford

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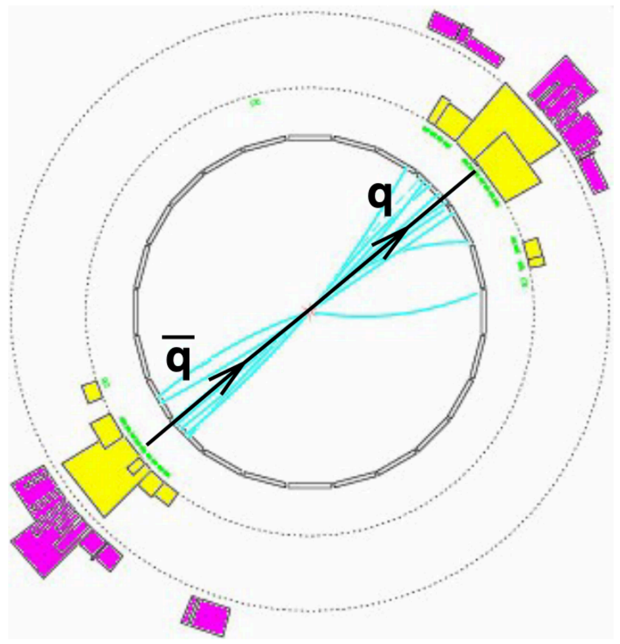
Based on **2507.18696**, in collaboration with C. Farren-Colloty, J. Helliwell, R. Patel, and G.P. Salam

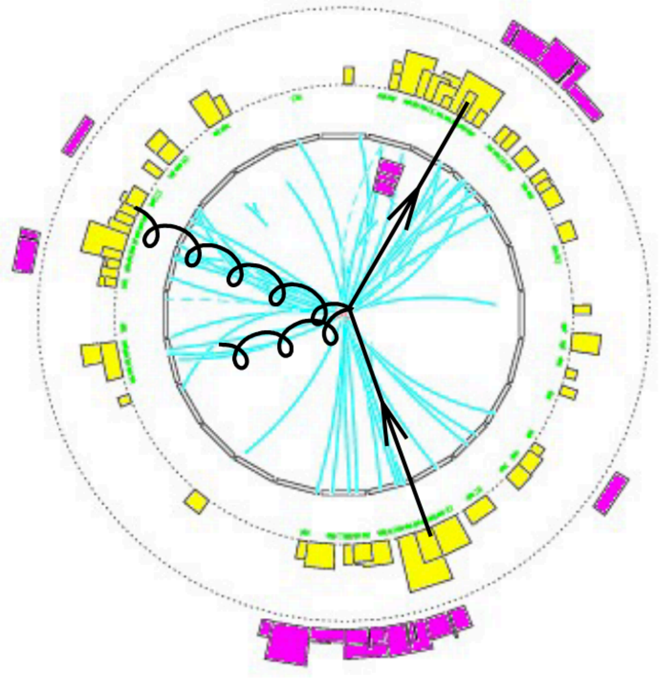
EVENT SHAPES

Used mostly in the context of e^+e^- collisions, they provide information on the geometry of an event.

Example: THRUST

$$\tau = 1 - T = \max_{\vec{n}_T} \left(\frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|} \right)$$





pencil-like ⟷ spherical

$\tau = 0$ ⟷ $\tau = 1$

NICE PROPERTIES

- Very clean environment for experimental measurements.
- IR safe, so they can be computed in perturbative QCD.
- Known at high orders in QCD, including resummation.

↓

USED TO TEST QCD AND ITS DYNAMICS

e.g. they are a widely used dataset for the determination of α_S.

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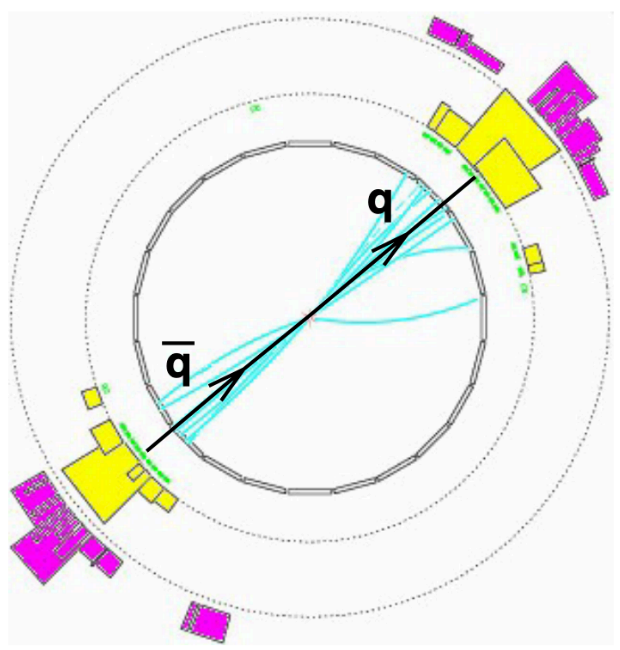

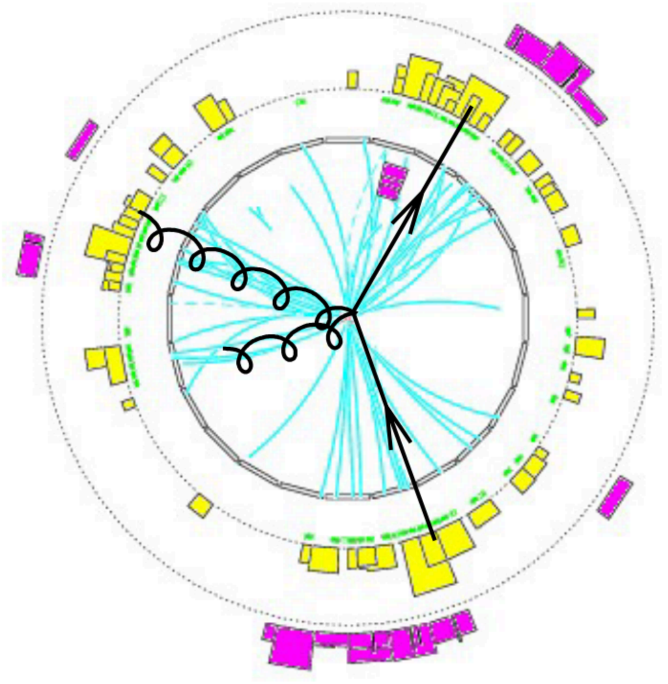

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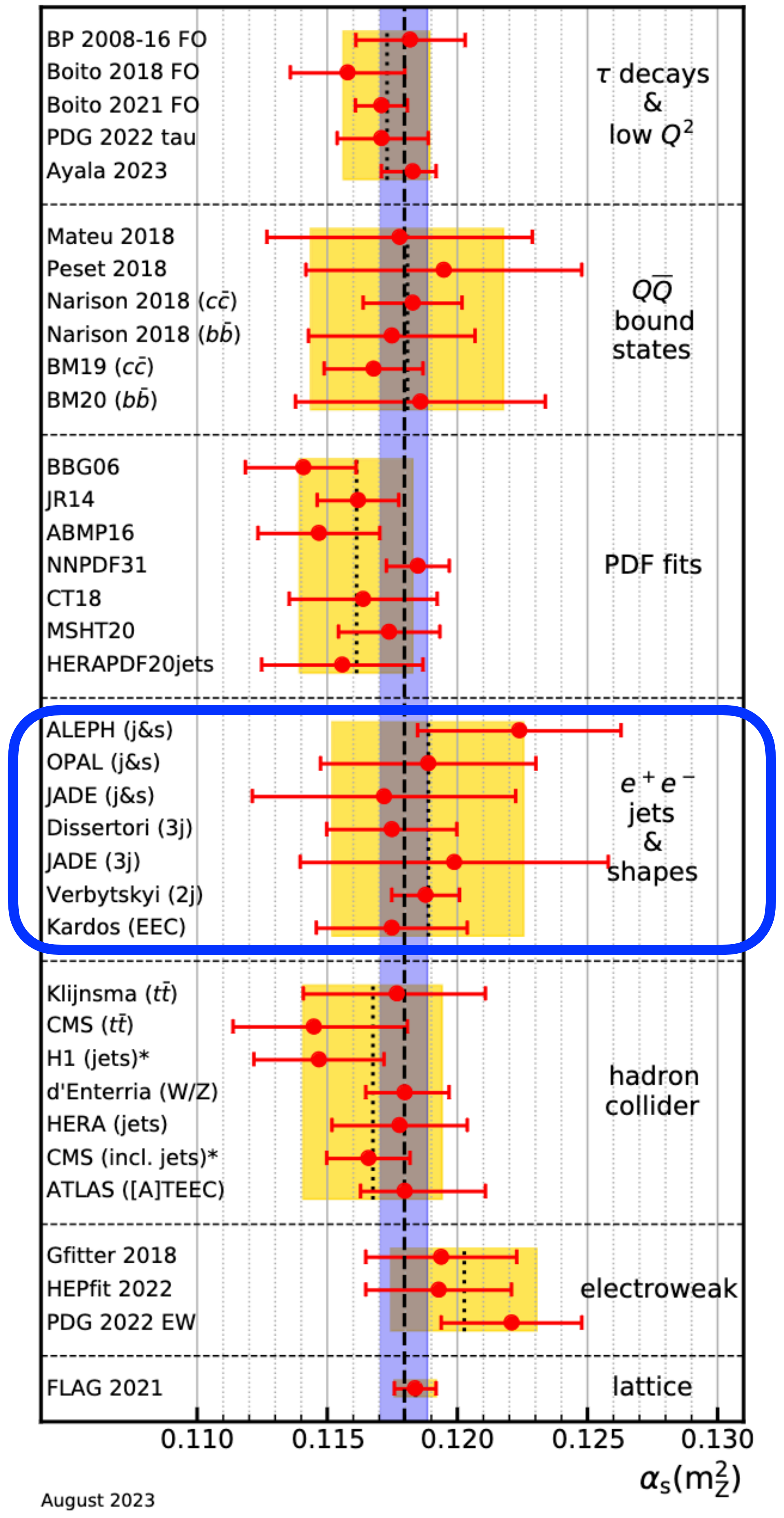
AFFECTED BY LARGE NON-PERTURBATIVE POWER CORRECTIONS

$\sim \Lambda/Q$ **(linear)**

DETERMINATION OF α_S

Hadronic final state of e^+e^- collisions:

PDG 2023



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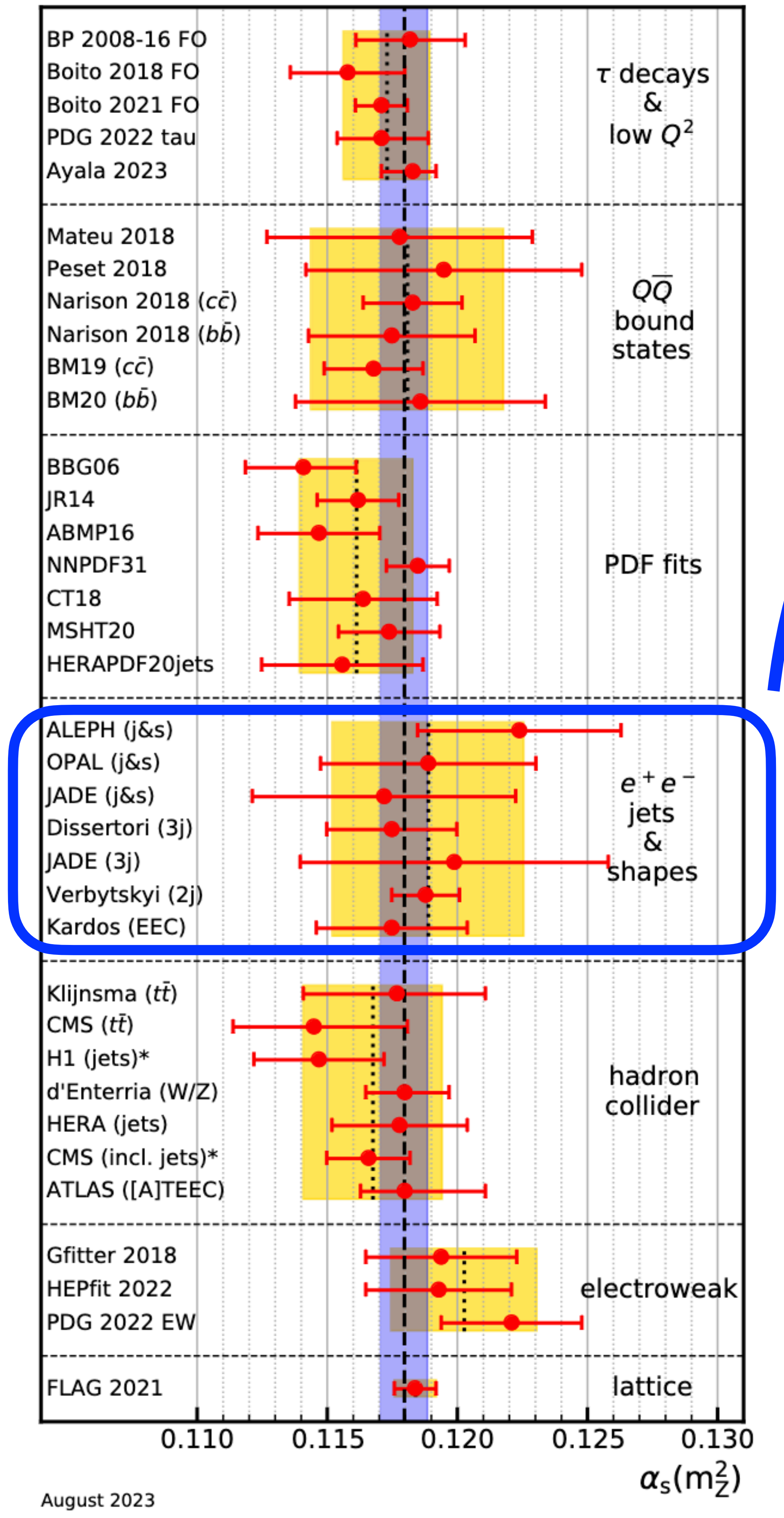

NON-PERTURBATIVE (=HADRONISATION) CORRECTIONS CAN BE OBTAINED IN TWO DIFFERENT WAYS:

1 - FROM A MC GENERATOR

✓ Very practical: construct a migration matrix describing the parton to hadron transition, and apply it in the data/theory comparison.

✗ No clean relation between hadronisation models and QCD first principles.

[* see also 2412.15164, 2502.12253]

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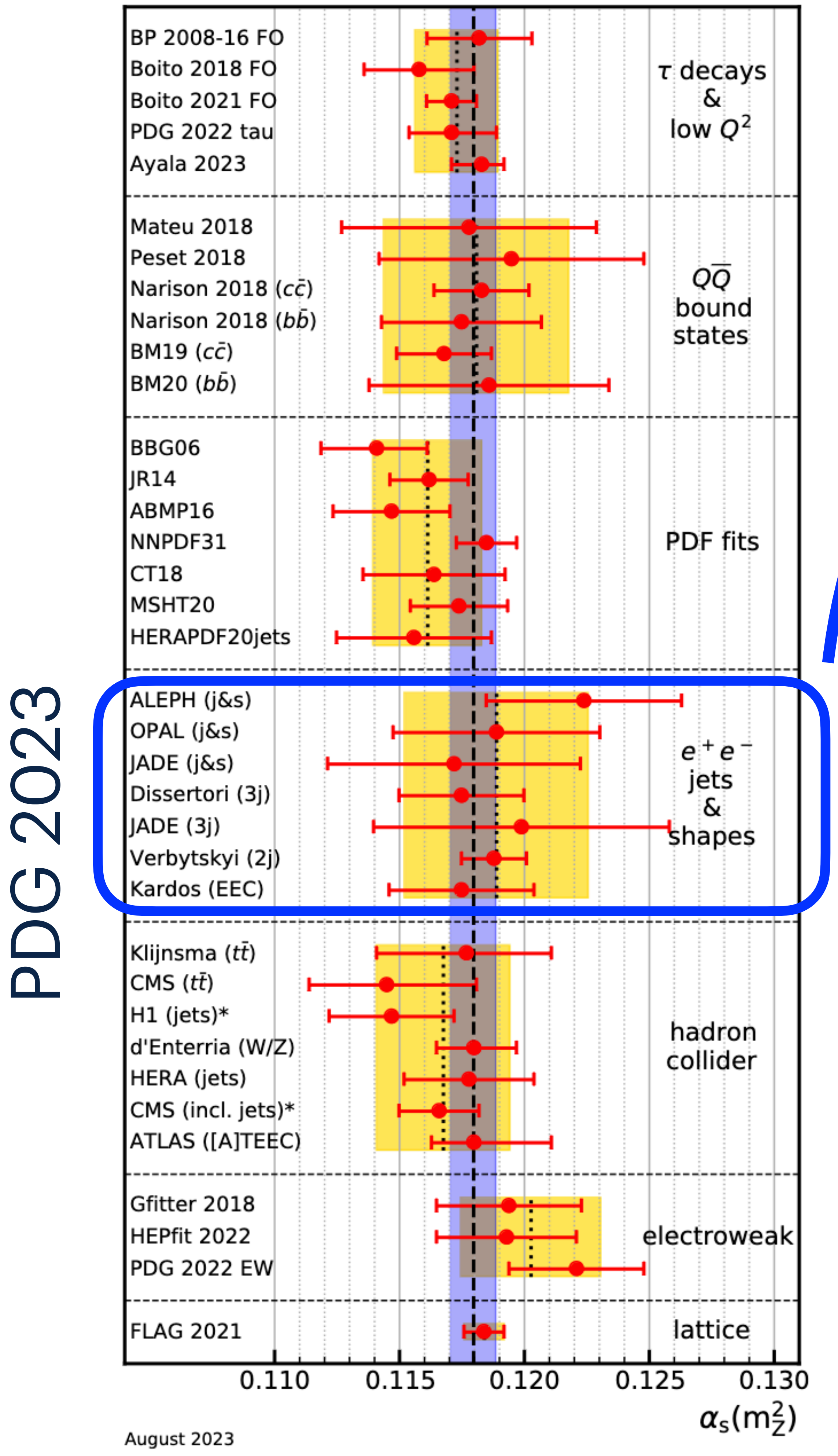

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✗ No clean relation between hadronisation models and QCD first principles.

2 - FROM ANALYTIC MODELS

Fit of thrust data: $\alpha_s(m_Z^2) = 0.1135 \pm 0.0011$ [1]

$\alpha_s(m_Z^2) = 0.1134^{+0.0031}_{-0.0025}$ [2]

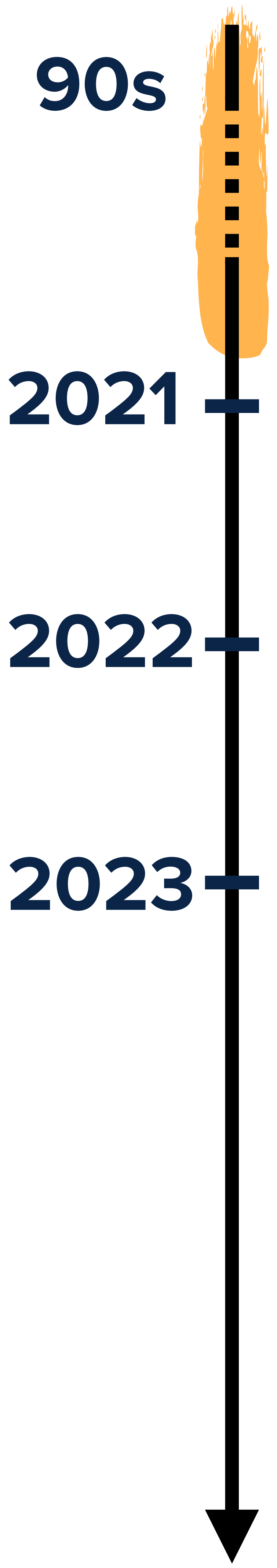
Fit of C-parameter data: $\alpha_s(m_Z^2) = 0.1123 \pm 0.0015$ [3]

?

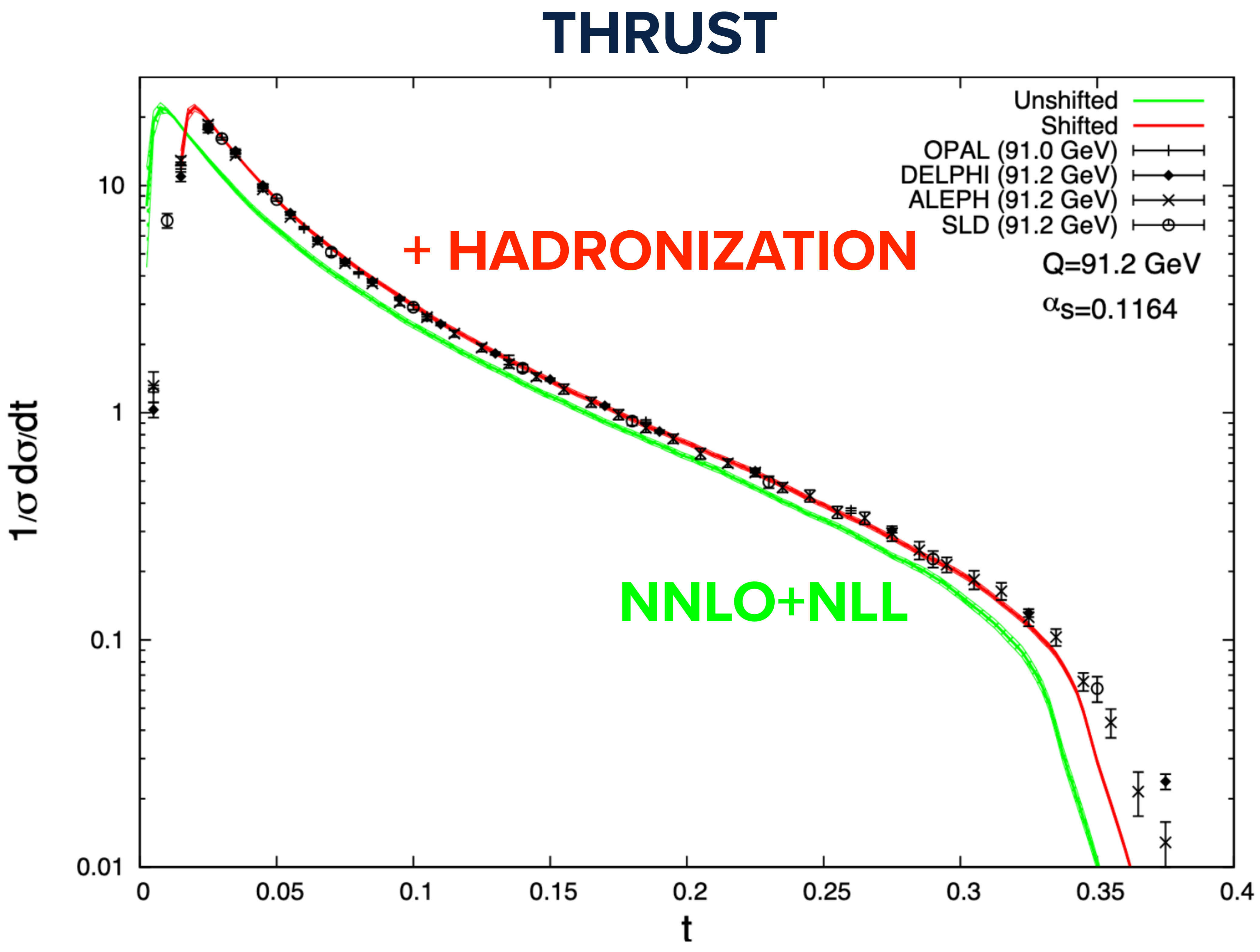
World average: $\alpha_s(m_Z^2) = 0.1180 \pm 0.0009$

[1: Abbate et al. 1006.3080] [2: Gehrmann, Luisoni, Monni 1210.6945][3: * Hoang et al. 1501.04111]

TIMELINE OF ANALYTIC MODELS



Analytic models are based on a 2-jet limit analysis, **assuming that non perturbative effects are constant in the entire kinematic spectrum.**

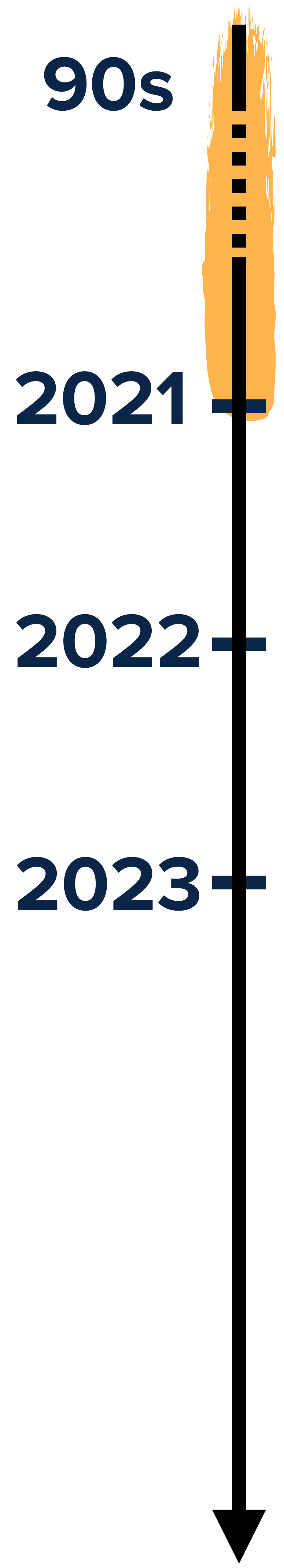


$$\sigma^{\text{had}}(\tau) = \sigma^{\text{part}}(\tau + \delta\tau)$$

$\delta\tau$ = constant 1/Q shift

[Davison, Webber 0809.3326]

TIMELINE OF ANALYTIC MODELS



Analytic models are based on a 2-jet limit analysis, **assuming that non perturbative effects are constant in the entire kinematic spectrum.**

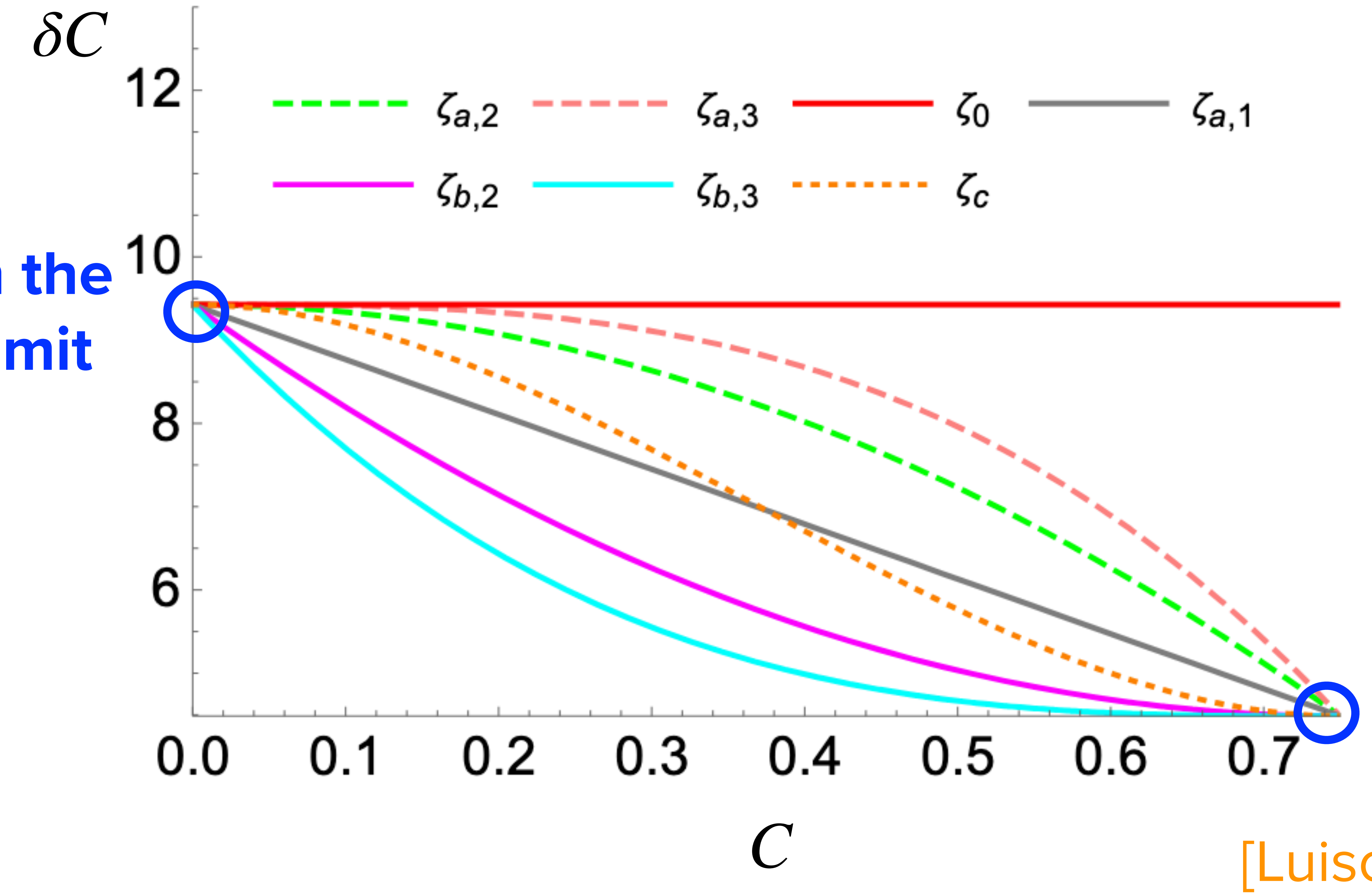
The previous assumption is proven to be wrong calculating the leading non-perturbative correction to the *C*-parameter in *C*=0 and *C*=3/4.

$$\Theta^{\alpha\beta}\frac{1}{\sum_i |p_i|}\sum_i \frac{p_i^\alpha p_i^\beta}{|\vec{p}_i|},$$

$$C = 3\cdot(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3)$$

SHIFT C-PARAMETER

Shift in the 2-jet limit



Previous assumption of constant shift

Shift in the symmetric 3-jet limit

[Luisoni, Monni, Salam 2012.00622]

TIMELINE OF ANALYTIC MODELS

90s Analytic models are based on a 2-jet limit analysis, **assuming that non perturbative effects are constant in the entire kinematic spectrum.**

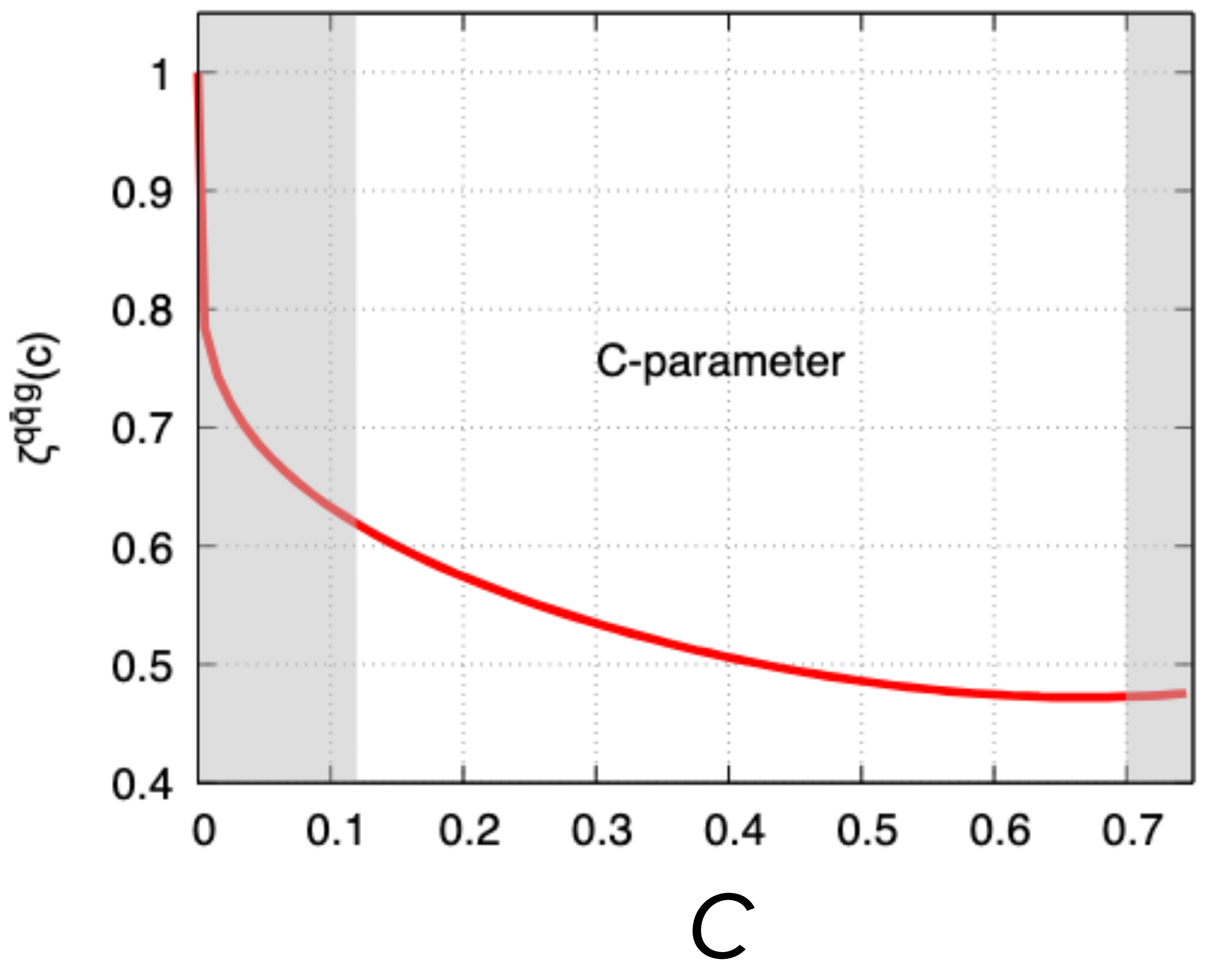
2021 **The previous assumption is proven to be wrong** calculating the leading non-perturbative correction to the *C*-parameter in *C*=0 and *C*=3/4.

2022 **Full calculation in the 3-jet limit** for the thrust and the *C*-parameter.*

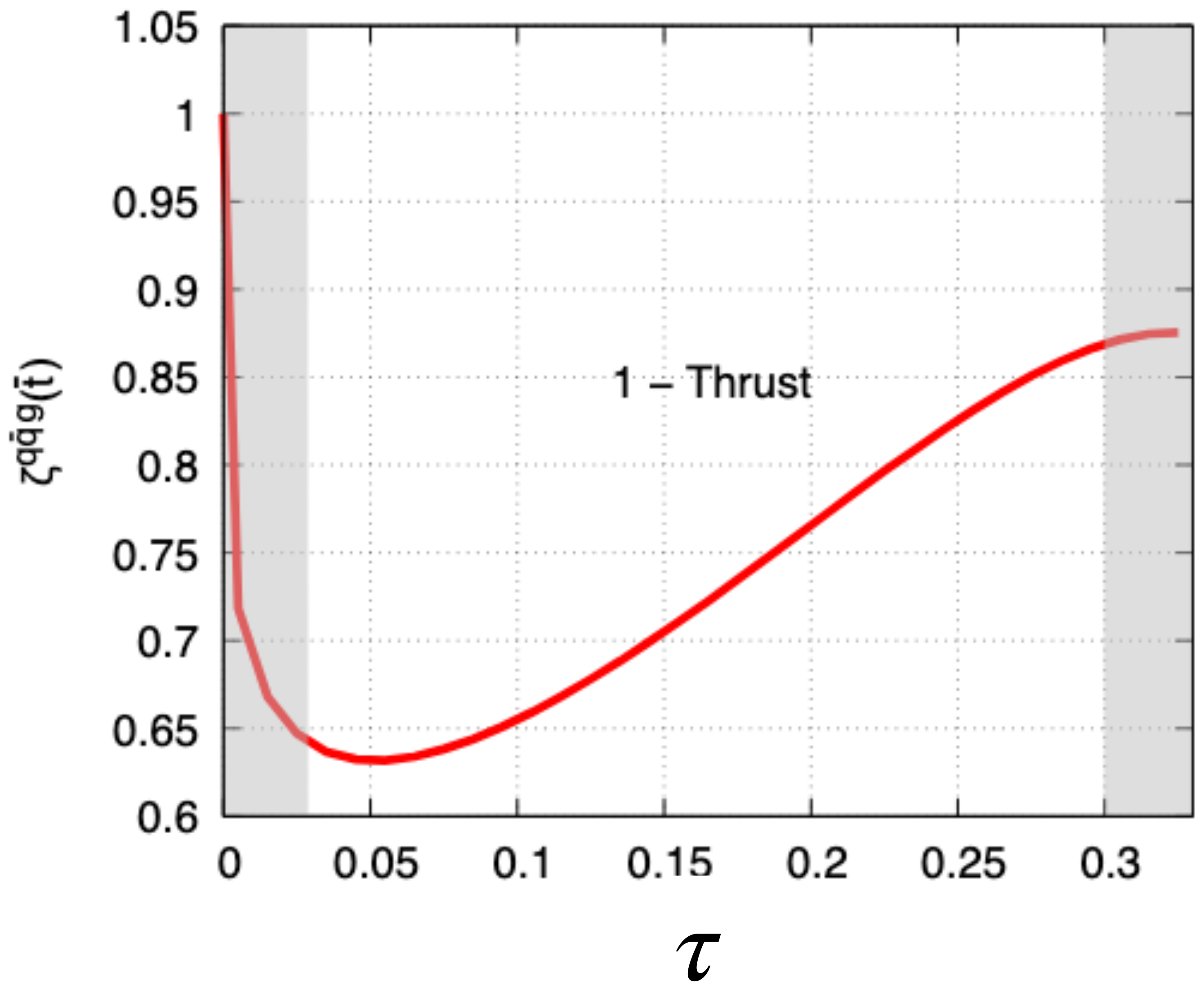
* Large n_f-limit of QCD + no gluons at Born level.

2023

SHIFT *C*-PARAMETER



SHIFT THRUST



NON TRIVIAL SHAPE!

[Caola, Ferrario Ravasio, Limatola, Melnikov, Nason 2108.08897; +Ozcelik 2204.02247]

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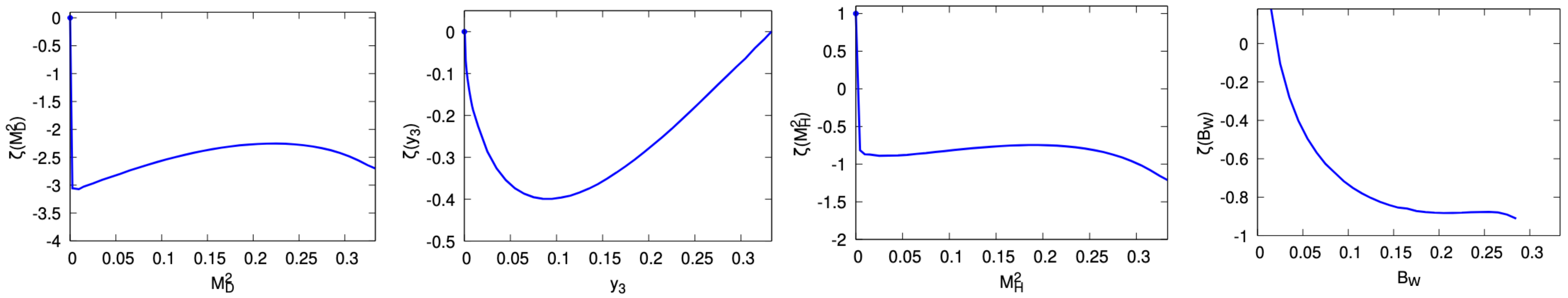
2021 **The previous assumption is proven to be wrong** calculating the leading non-perturbative correction to the C-parameter in C=0 and C=3/4.

2022 **Full calculation in the 3-jet limit** for the thrust and the C-parameter.

2023 **Extension of the calculation to other event shapes,** confirming the previous results.



[Nason, Zanderighi 2301.03607, 2501.18173]

THE DOKSHITZER-WEBBER MODEL

[Dokshitzer, Webber hep-ph/9504219]

HADRONISATION ≡ emission of soft $k_T \sim \Lambda$ non-perturbative gluon (= "*gluer*")

The divergent behaviour of the running coupling at low scales is cured by an effective coupling that is finite:

Intrinsic ambiguity of pQCD
"renormalons picture"

$$\int_0^Q \mathrm{d}k\,\alpha_s(k) = \int_0^{\mu_I} \mathrm{d}k\,\alpha_s(k) + \int_{\mu_I}^Q \mathrm{d}k\,\alpha_s(k) \longrightarrow \mu_I\,\bar{\alpha}_0(\mu_I) + \int_{\mu_I}^Q \mathrm{d}k\,\alpha_s(k)$$

Matching scale $\mathcal{O}(\mathrm{GeV})$

IR finite and universal coupling

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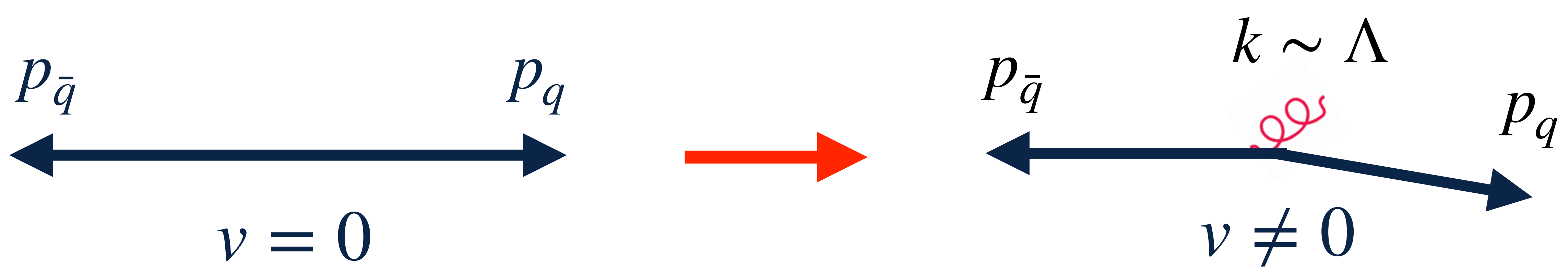
1 Start from a 2-jet configuration and emit a gluer:



2 Calculate the shift in the event shape: $\delta V_{q\bar{q}} = V(p_q, p_{\bar{q}}, k_n) - V(p_q, p_{\bar{q}}) = V(p_q, p_{\bar{q}}, k_n)$

3 Average this shift over the gluer emission probability: $\langle \delta V_{\mathrm{np}} \rangle_{q\bar{q}} = \int_0^{\mu_{\mathrm{np}}} \frac{dk_{tn}}{k_{tn}}\, d\eta_n\, \frac{2 C_F\, \alpha_s^{(\mathrm{eff})}(k_{tn})}{\pi}\, \delta V_{q\bar{q}}$

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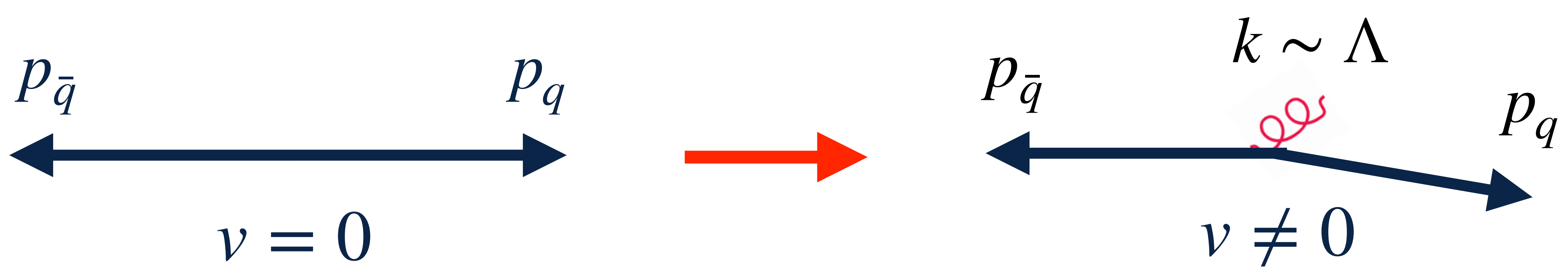

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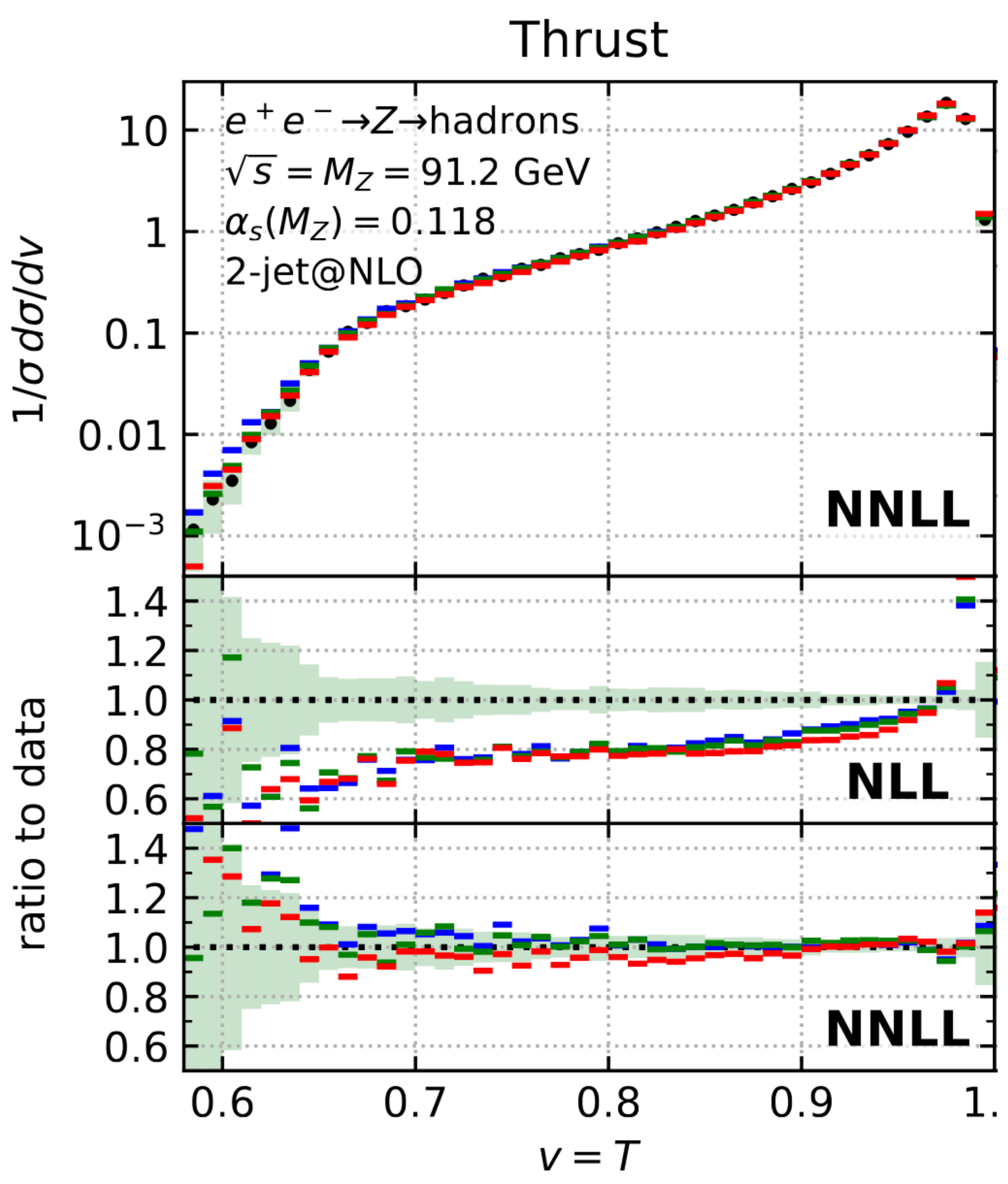
Can we reproduce known DW results with the PanScales showers?

WHAT IS A PANSCALES SHOWER?

PanScales is a project to bring logarithmic understanding and accuracy to parton showers.

CURRENT MEMBERS:

Melissa van Beekveld (NIKHEF)
Mrinal Dasgupta (Manchester)
Basem El-Menoufi (Torino)
Silvia Ferrario Ravasio (Torino)
Keith Hamilton (Uni. College London)
Jack Helliwell (Monash)
Petr Jakubcik (Oxford)
Alexander Karlberg (Oxford)
Pier Monni (CERN)
Gavin P. Salam (Oxford)
Nicolas Schalch (Oxford)
Ludovic Scyboz (Monash)
Alba Soto-Ontoso (Granada)
Gregory Soyez (IPhT, Saclay)
Silvia Zanoli (Oxford)



[2406.02661]

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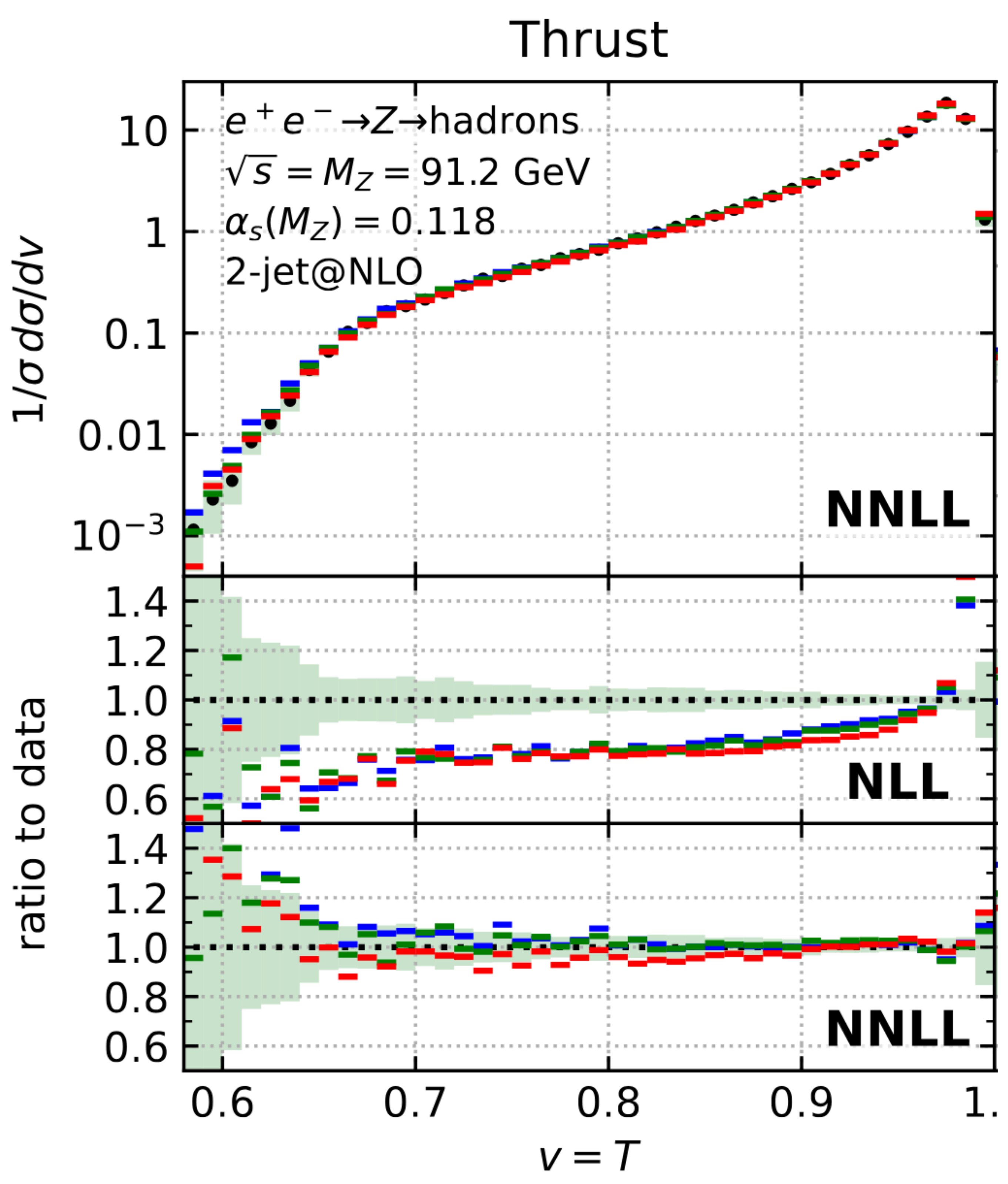


[2406.02661]

First shower at NNLL accuracy for event shapes.

For the study I am presenting today, we do not need high logarithmic accuracy in the parton shower. However, **an excellent control on numerics is mandatory.**

Panscales and related PanScales techniques offer a unique framework.

NP CORRECTIONS IN THE 3-JET REGION

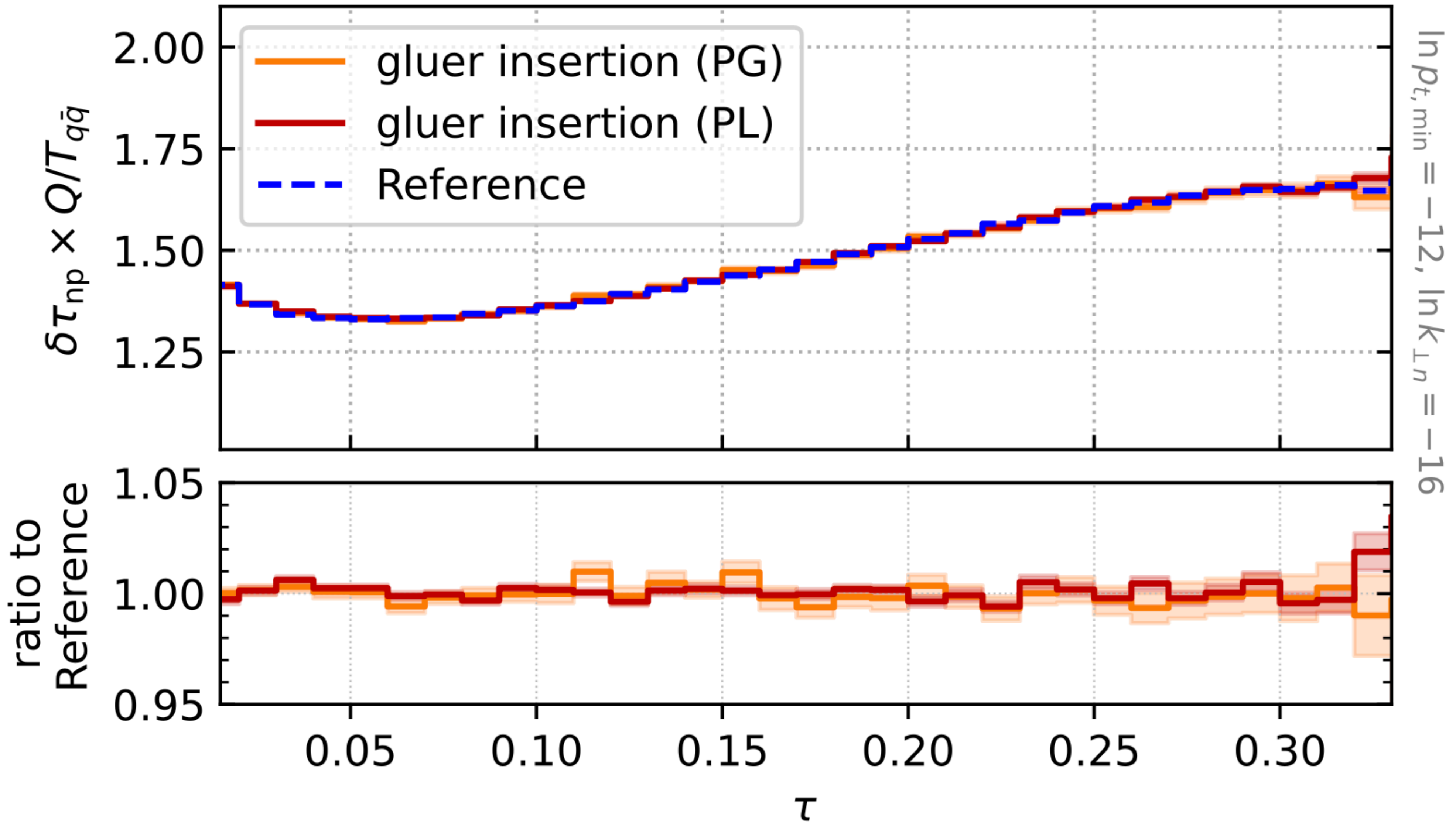
FIRST VALIDATION OF OUR METHODOLOGY

Non perturbative shift in the 3-jet region ($q\bar{q}g$ event) known in literature: reproduce it within Panscales.

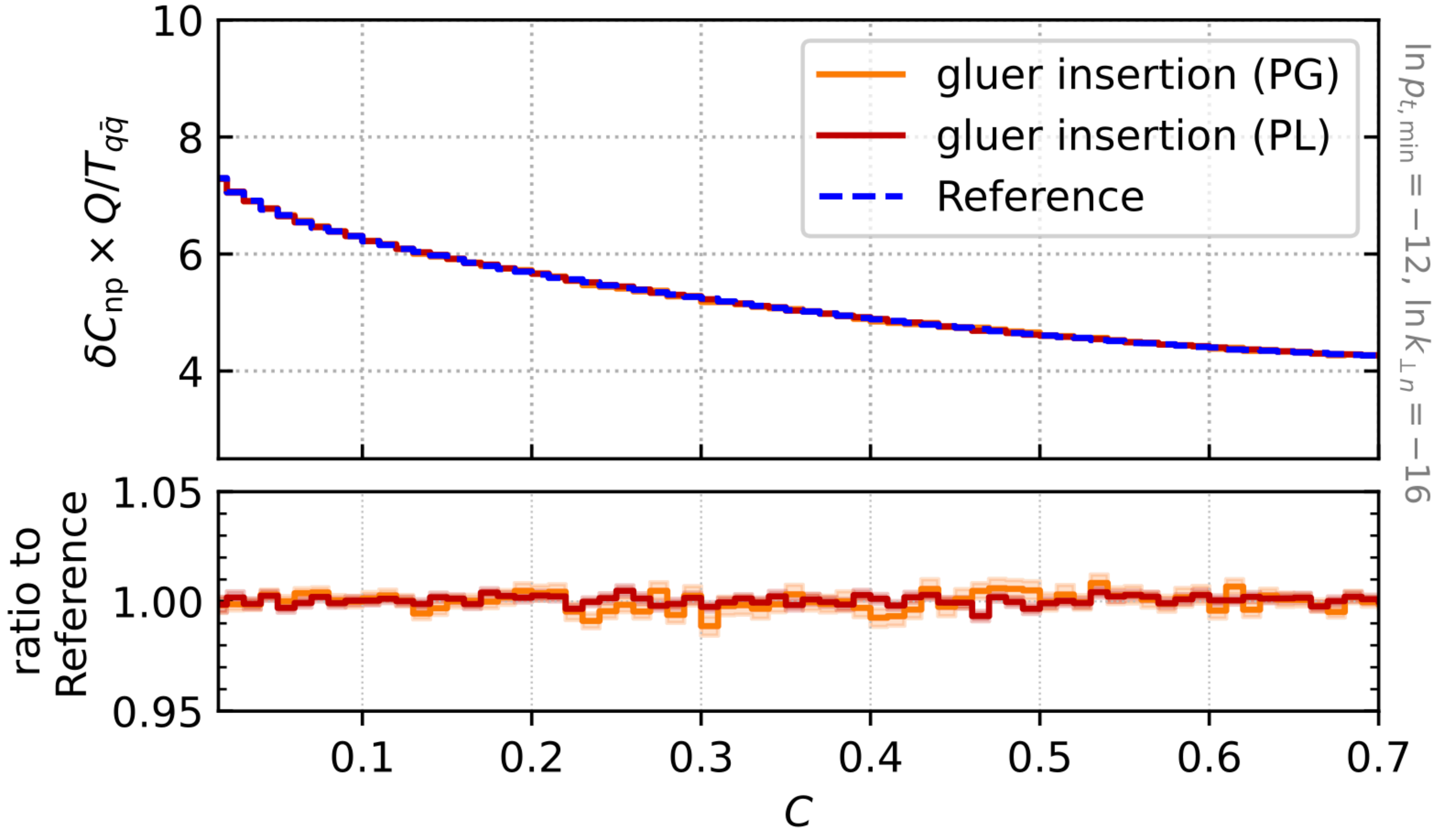
1. Generate a $q\bar{q}g$ event with the correct tree level ME.
2. Insert manually a soft gluer ($\ln k_{tn} = -16$).
3. Obtain the non-perturbative shift $\delta V = V(p_q, p_{\bar{q}}, p_g, k_n) - V_{qqg}(p_q, p_{\bar{q}}, p_g)$, and average over the gluer insertion.

Note: for these tests, we take the large-N_C limit.

AVG. SHIFT, THRUST



AVG. SHIFT, C-parameter

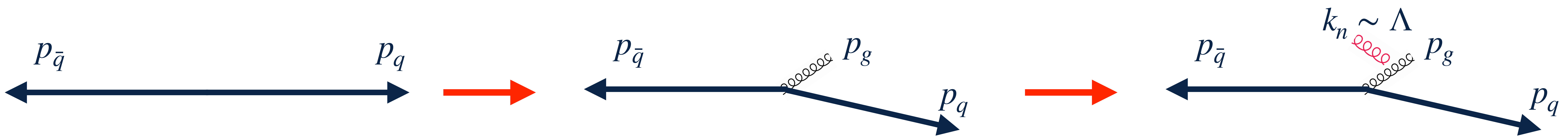


REFERENCE = [Caola, Ferrario Ravasio, Limatola, Melnikov, Nason, Ozcelikd 2022] + [Nason, Zanderighi 2023]

PERTURBATIVE EVOLUTION IN Q

[Dasgupta, Hounat 2411.16867]

The DW model has been extended to NLO accuracy dressing a $q\bar{q}$ system with a soft perturbative gluon.



$\mathcal{O}(\alpha_s)$, **formally beyond accuracy** wrt DW model

SOFT ENHANCED GLUON + STRONGLY ORDERED EMISSIONS

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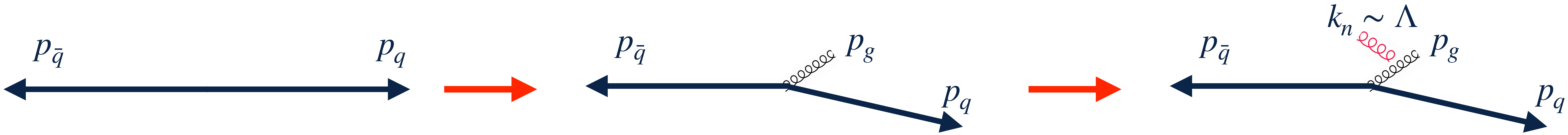


$\mathcal{O}(\alpha_s)$, **formally beyond accuracy** wrt DW model

SOFT ENHANCED GLUON + STRONGLY ORDERED EMISSIONS

The interplay between the soft emission and the gluer leads to a **large logarithmic correction governed by an anomalous dimension.**

Calculated in a semi-analytic way for Thrust and C-parameter:

$$\langle \delta V_{\rm np} \rangle = c_V \frac{1}{Q} \int_0^{\mu_{\rm np}} dk_{tn} \frac{2C_F\, \alpha_s^{(\rm eff)}(k_{tn})}{\pi} \left(1 - \mathcal{S}_1 C_A \frac{\alpha_s}{2\pi} \ln \frac{Q}{k_{tn}} + \dots \right)$$

$$\sim \frac{\Lambda}{Q} \left(1 - \mathcal{S}_1 \alpha_s \ln \frac{Q}{\Lambda} + \dots \right)$$

NUMERICAL PREDICTION OF $\mathcal{S}_1$

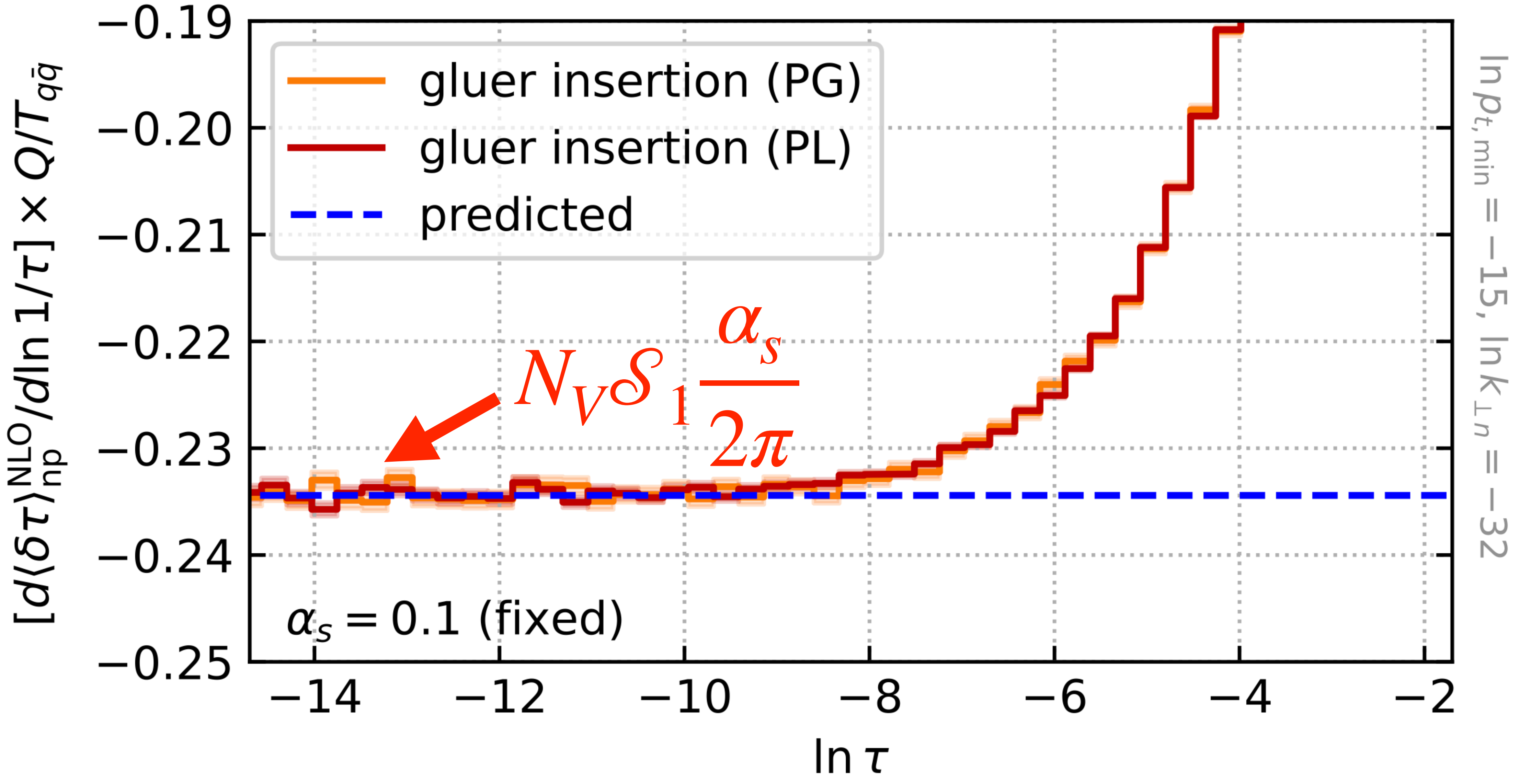
SECOND VALIDATION OF OUR METHODOLOGY

1. Generate a $q\bar{q}$ event and add a soft gluon sampled randomly by the shower algorithm ($\ln p_t^{\text{cut}} = -15$).
2. Insert manually a soft gluer ($\ln k_{tn} = -32$).

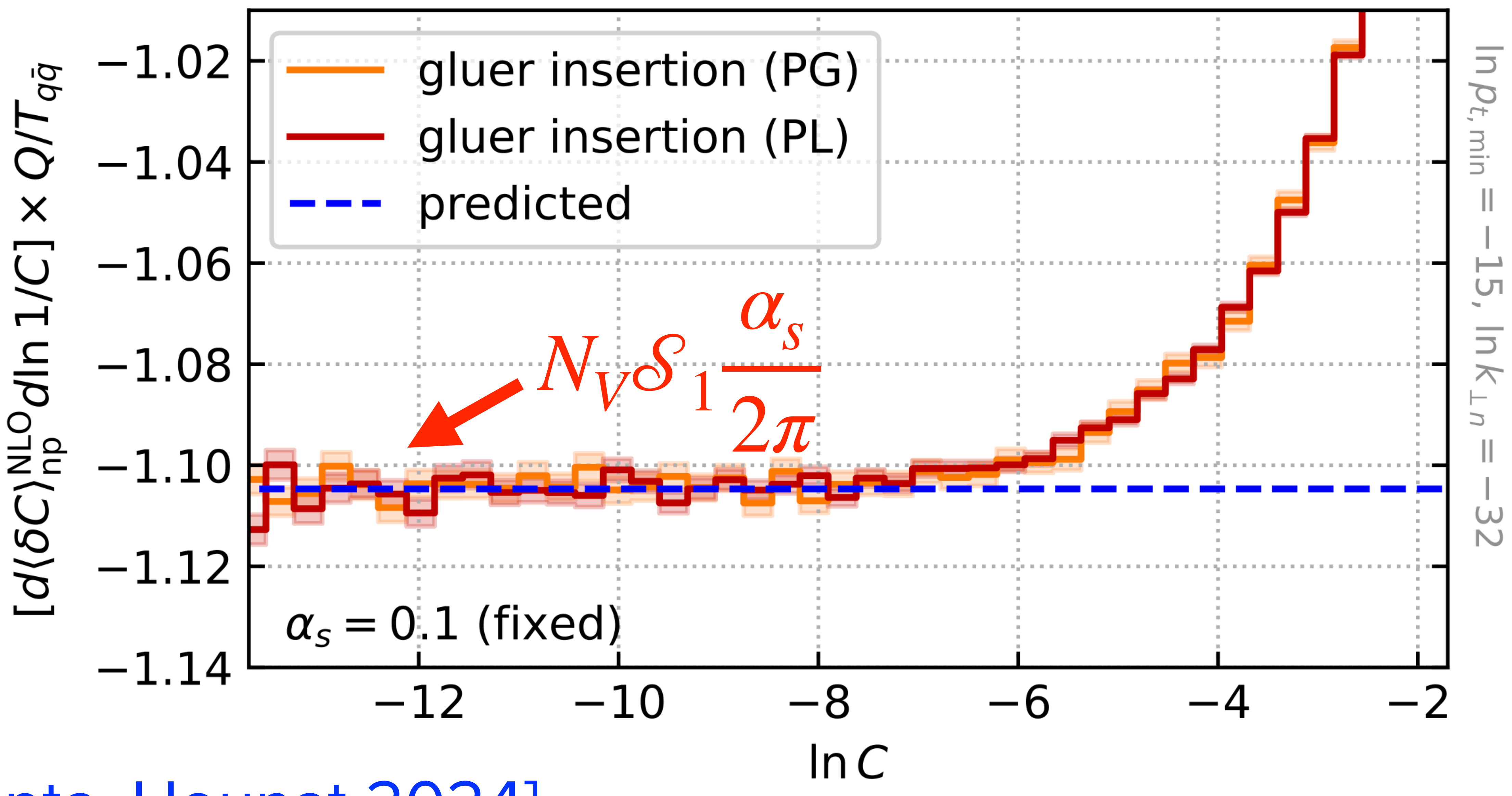
$$\langle \delta V_{\text{np}} \rangle^{\text{NLO}} = \int d\ln V' \frac{1}{\sigma}\frac{d\sigma}{d\ln V'} \left(\langle \delta V_{\text{np}} \rangle (V') - \langle \delta V_{\text{np}} \rangle^{\text{Born}} \right)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\ln V} \overset{V\to 0}{\simeq} -\ln V \qquad \langle \delta V_{\text{np}} \rangle (V) - \langle \delta V_{\text{np}} \rangle^{\text{Born}} \sim N_V \frac{\mathcal{S}_1}{\ln V}\frac{\alpha_s}{2\pi}$$

THRUST



C-parameter



PREDICTED = [Dasgupta, Hounat 2024]

ANALYTICAL CALCULATION OF $\mathcal{S}_1$

We can calculate analytically the anomalous dimension $\mathcal{S}_1$ for linear event shapes:

$$\delta V_{q\bar{q}} = \sum_i \frac{k_{ti}}{Q} f(\eta_i) + \mathcal{O}\left(\frac{k_{ti}^2}{Q^2}\right)$$

with i running over all soft massless partons emitted from a $q\bar{q}$ system, with transverse momentum and rapidity k_{ti} and η_i

e.g. thrust, C-parameter and EEC

If we consider a gluer insertion in a $q\bar{q}$ event we have:

$$\langle \delta V_{\rm np} \rangle_{q\bar{q}} = \int_0^{\mu_{\rm np}} \frac{dk_{tn}}{k_{tn}} d\eta_n \frac{2C_F \alpha_s^{(\rm eff)}(k_{tn})}{\pi} \delta V_{q\bar{q}} \longrightarrow \langle \delta V_{\rm np} \rangle_{q\bar{q}} = c_V \frac{T_{q\bar{q}}}{Q}$$

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EFFECTIVE NON-PERTURBATIVE TRANSVERSE MOMENTUM PER UNIT RAPIDITY INDUCED BY THE GLUER INSERTION

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$$T_{q\bar{q}} = \int_0^{\mu_{\rm np}} dk_{tn} \frac{2C_F \alpha_s^{(\rm eff)}(k_{tn})}{\pi}$$

EFFECTIVE NON-PERTURBATIVE TRANSVERSE MOMENTUM PER UNIT RAPIDITY INDUCED BY THE GLUER INSERTION

How does the transverse momentum per unit rapidity change when the non-perturbative gluer is emitted from a *generic system* $qg_1 \ldots g_m\bar{q}$?

$\mathcal{S}_1$ FROM A $qg\bar{q}$ EVENT

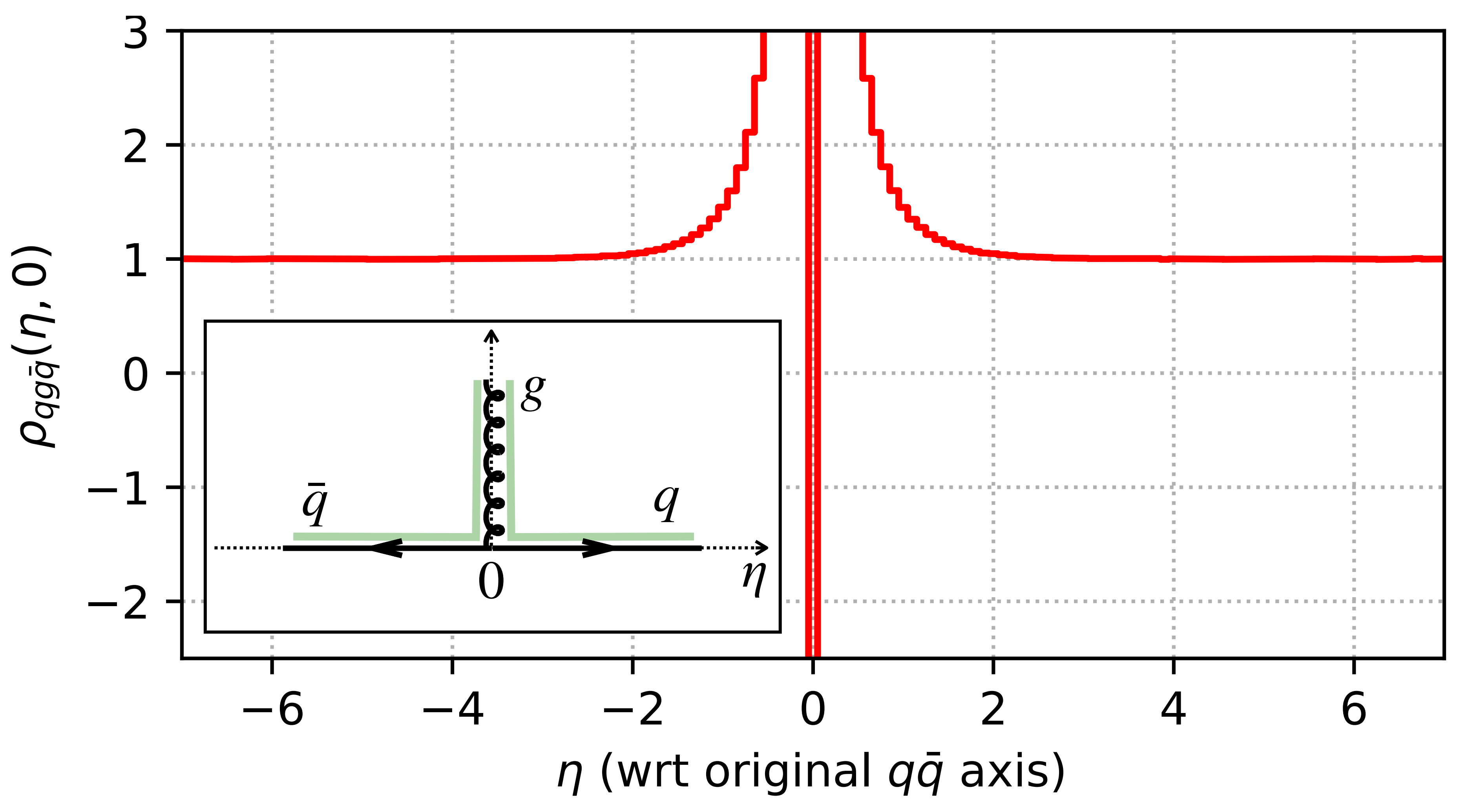
We start considering a $q\bar{q}g$ event, and we insert a gluer:

$$T_{qg\bar{q}}(\eta,\eta_g) = \frac{C_A}{\pi}\sum_{qg,g\bar{q}}\int \frac{dz_+}{z_+}\frac{dk_{\perp n}}{k_{\perp n}}\frac{d\Phi_n}{2\pi}\left[k_{tn}\,\delta(\eta-\eta_n) + (p_{tg}-\tilde{p}_{tg})\,\delta(\eta-\eta_g)\right]\alpha_s^{\mathrm{eff}}(k_{\perp n})$$

gluer variables (wrt emitting dipole)

change in scalar transverse momentum due to gluer insertion and gluon recoil (wrt original $q\bar{q}$ direction)

$$\rho(\eta,\eta_g) = \frac{T_{qg\bar{q}}(\eta,\eta_g)}{T_{q\bar{q}}}$$



$\mathcal{S}_1$ FROM A $qg\bar{q}$ EVENT

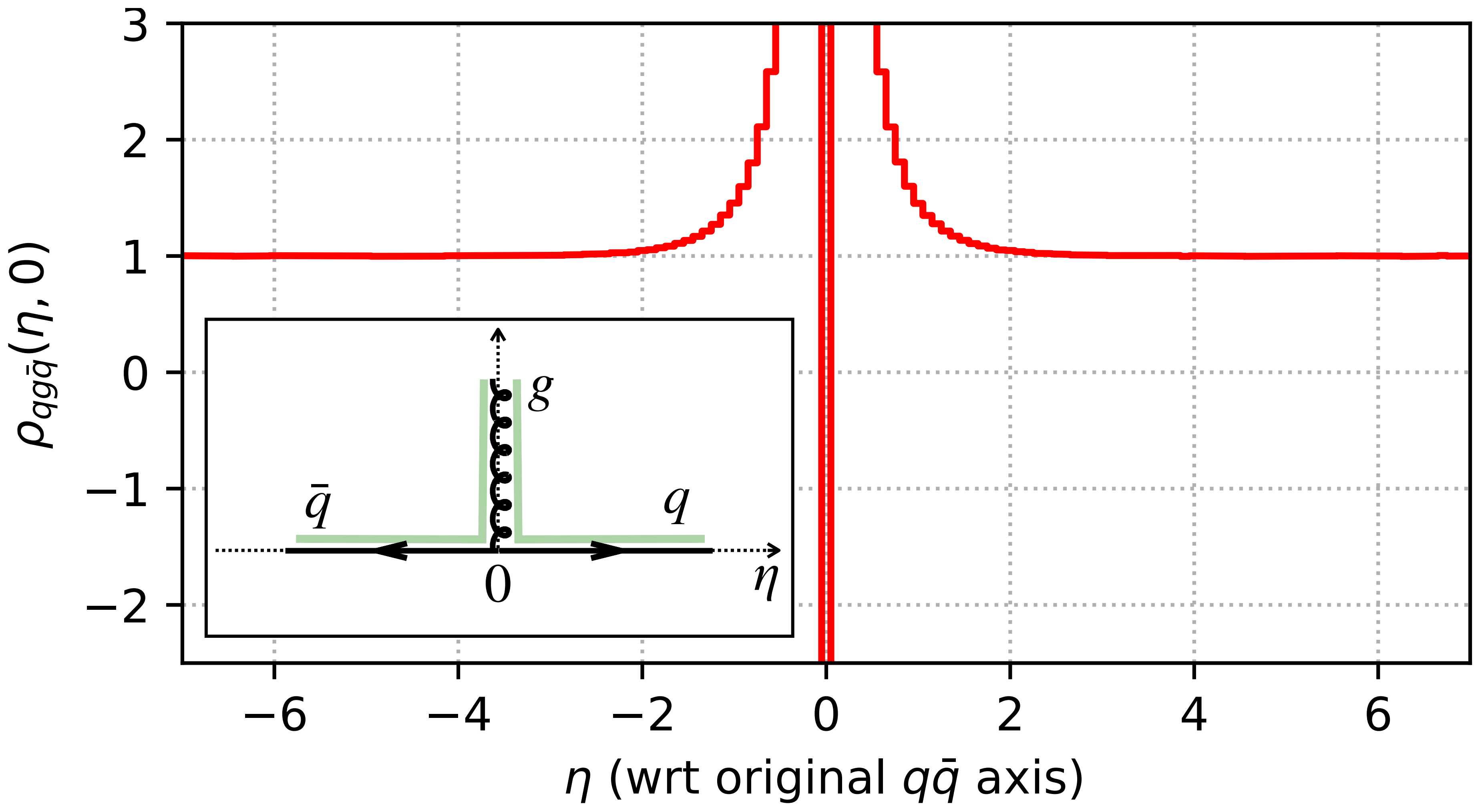
We start considering a $q\bar{q}g$ event, and we insert a gluer:

$$T_{qg\bar{q}}(\eta, \eta_g) = \frac{C_A}{\pi} \sum_{qg, g\bar{q}} \int \frac{dz_+}{z_+} \frac{dk_{\perp n}}{k_{\perp n}} \frac{d\Phi_n}{2\pi} \left[k_{tn}\, \delta(\eta - \eta_n) + (p_{tg} - \tilde{p}_{tg})\, \delta(\eta - \eta_g)\right] \alpha_s^{\text{eff}}(k_{\perp n})$$

gluer variables (wrt emitting dipole)

change in scalar transverse momentum due to gluer insertion and gluon recoil (wrt original $q\bar{q}$ direction)

$$\rho(\eta, \eta_g) = \frac{T_{qg\bar{q}}(\eta, \eta_g)}{T_{q\bar{q}}}$$



The NLO factor that multiplies the average transverse momentum density is:

$$\mathscr{R}^{\text{NLO}} = 1 + \int d\eta_g \int_{\mu_{\text{np}}}^{Q} \frac{d\tilde{p}_{tg}}{\tilde{p}_{tg}} \frac{2C_F\, \alpha_s(\tilde{p}_{tg})}{\pi} [\rho(\eta, \eta_g) - 1]$$

integral over gluon kinematics

real corrections

virtual corrections

$\mathcal{S}_1$ FROM A $qg\bar{q}$ EVENT

$$\mathscr{R}^{\mathrm{NLO}} = 1 + \int d\eta_g \int_{\mu_{\mathrm{np}}}^{Q} \frac{d\tilde{p}_{tg}}{\tilde{p}_{tg}} \frac{2C_F\,\alpha_s(\tilde{p}_{tg})}{\pi} \left[\rho(\eta,\eta_g) - 1\right]$$

$$\int_{-\infty}^{+\infty} d\eta_g \left[\rho_{qg\bar{q}}(\eta,\eta_g) - 1\right] = -4(1 - \ln 2)$$

We use a kinematic map where the recoil of the hard partons is a linear function of the soft gluer's momentum.*

We use the PanGlobal and PanLocal mappings from PanScales showers (**the longitudinal recoil is local**)

* = [Caola, Ferrario Ravasio, Limatola, Melnikov, Nason 2021, +Ozcelikd 2022]

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We use a kinematic map where the recoil of the hard partons is a linear function of the soft gluer's momentum.*

We use the PanGlobal and PanLocal mappings from PanScales showers (**the longitudinal recoil is local**)

In the large-NC limit we obtain:

$$\mathcal{R}^{\text{NLO}} = 1 - \lambda(Q,\mu_{\text{np}}) \frac{C_A \mathcal{S}_1}{2\pi} \qquad \lambda(Q,\mu_{\text{np}}) = \int_{\mu_{\text{np}}}^{Q} \frac{dp_t}{p_t} \alpha_s(p_t) \qquad \mathcal{S}_1 = 8(1 - \ln 2) \approx 2.4548226$$

The anomalous dimension $\mathcal{S}_1$ can be compared to Dasgupta,Hounat (2024): $\mathcal{S}_1 = 2.455$

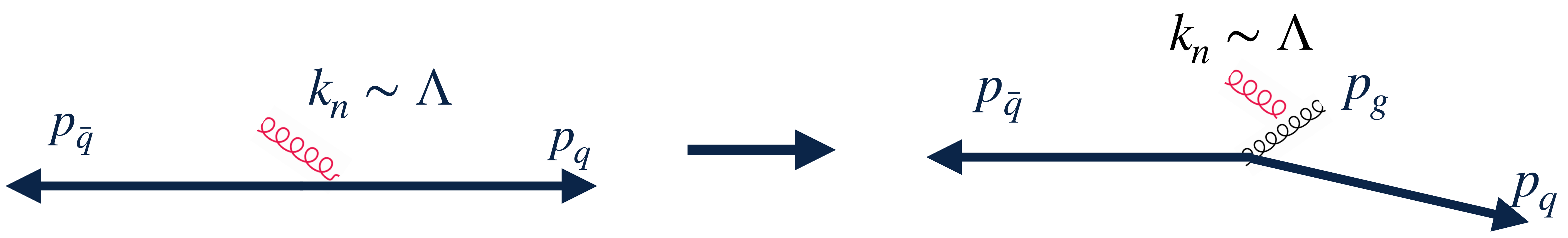
Our result has been obtained in a fully analytic way for any linear event shape.

* = [Caola, Ferrario Ravasio, Limatola, Melnikov, Nason 2021, +Ozcelikd 2022]

$\mathcal{S}_1$ RESUMMED TO ALL ORDERS

LO: $\dfrac{\Lambda}{Q}$

NLO: $\dfrac{\Lambda}{Q} \cdot \left(\mathcal{S}_1 \alpha_s \ln \dfrac{Q}{\Lambda} \right)$



$\mathcal{S}_1$ calculated fully analytically for linear event shapes

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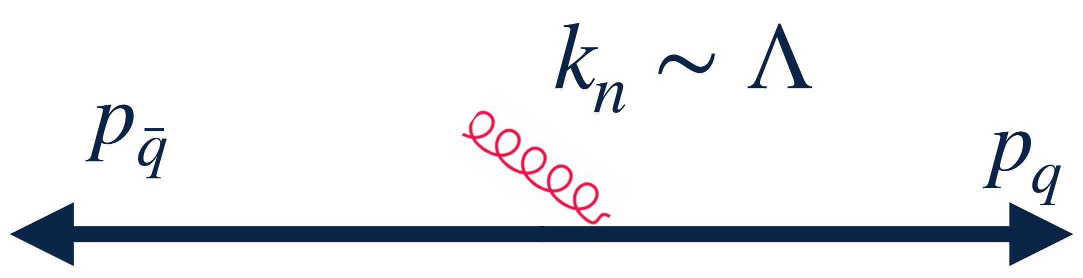

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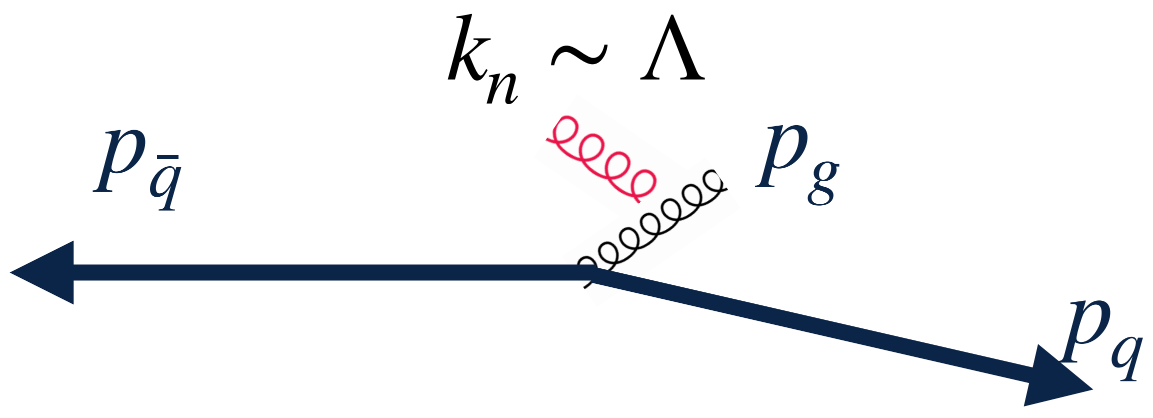
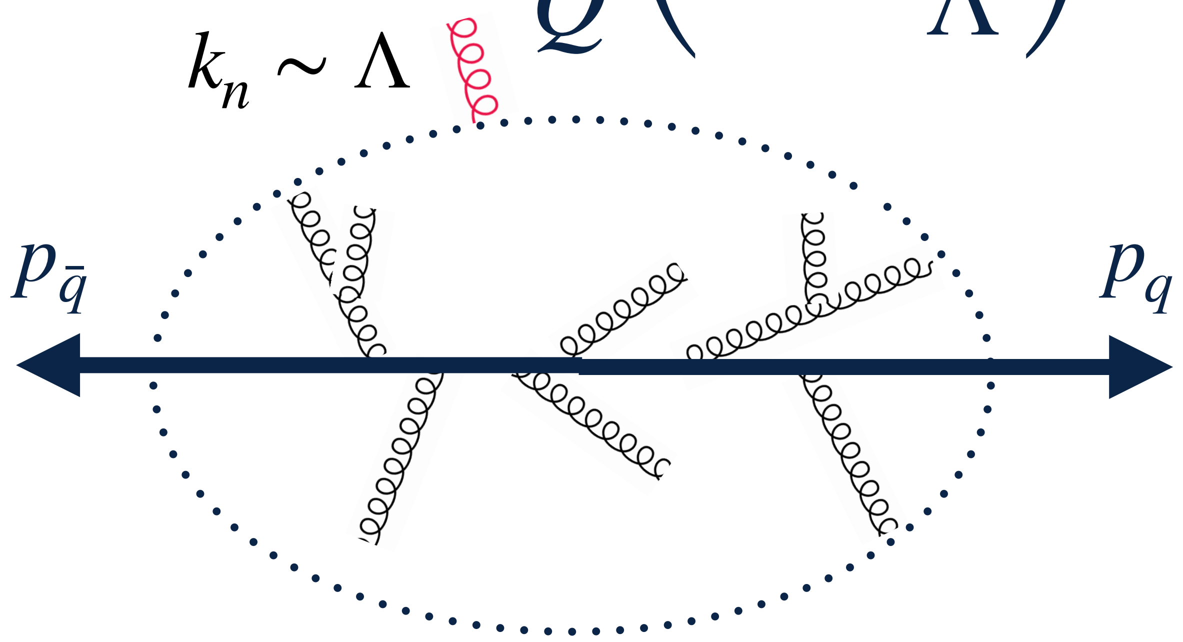


$\mathcal{S}_1$ calculated fully analytically for linear event shapes

ALL ORDER: $\frac{\Lambda}{Q} \left(\alpha_s \ln \frac{Q}{\Lambda} \right)^n$



highly non trivial (non-global resummation)

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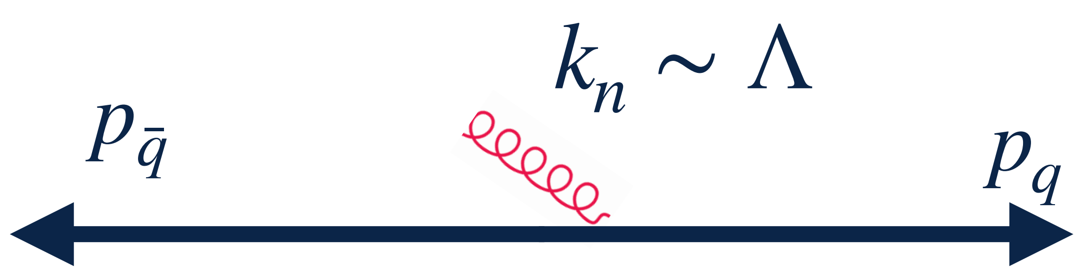

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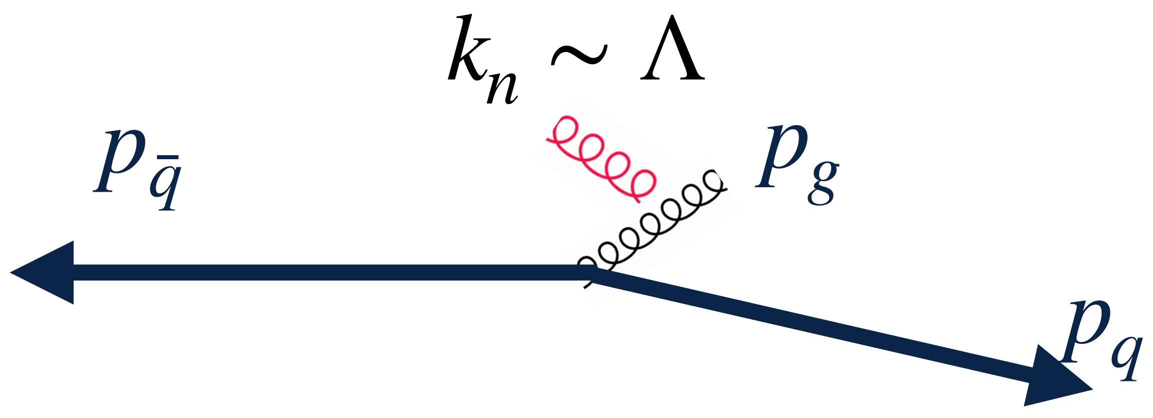

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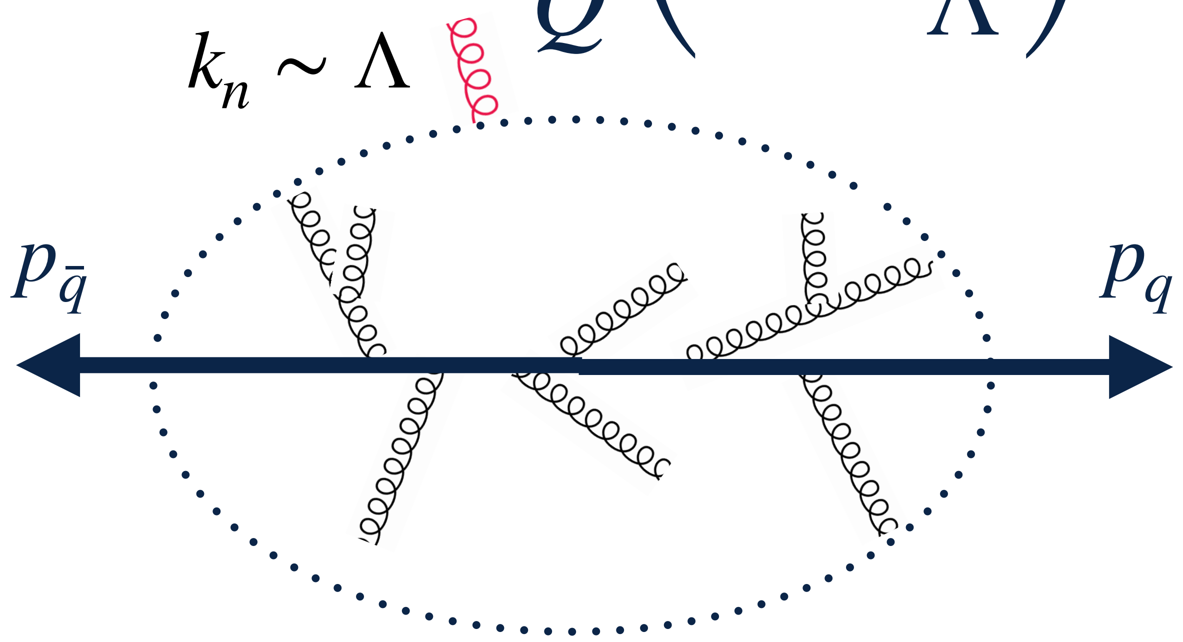


highly non trivial (non-global resummation)

Our analytic calculation can be easily extended to all orders, giving an impressively simple result.

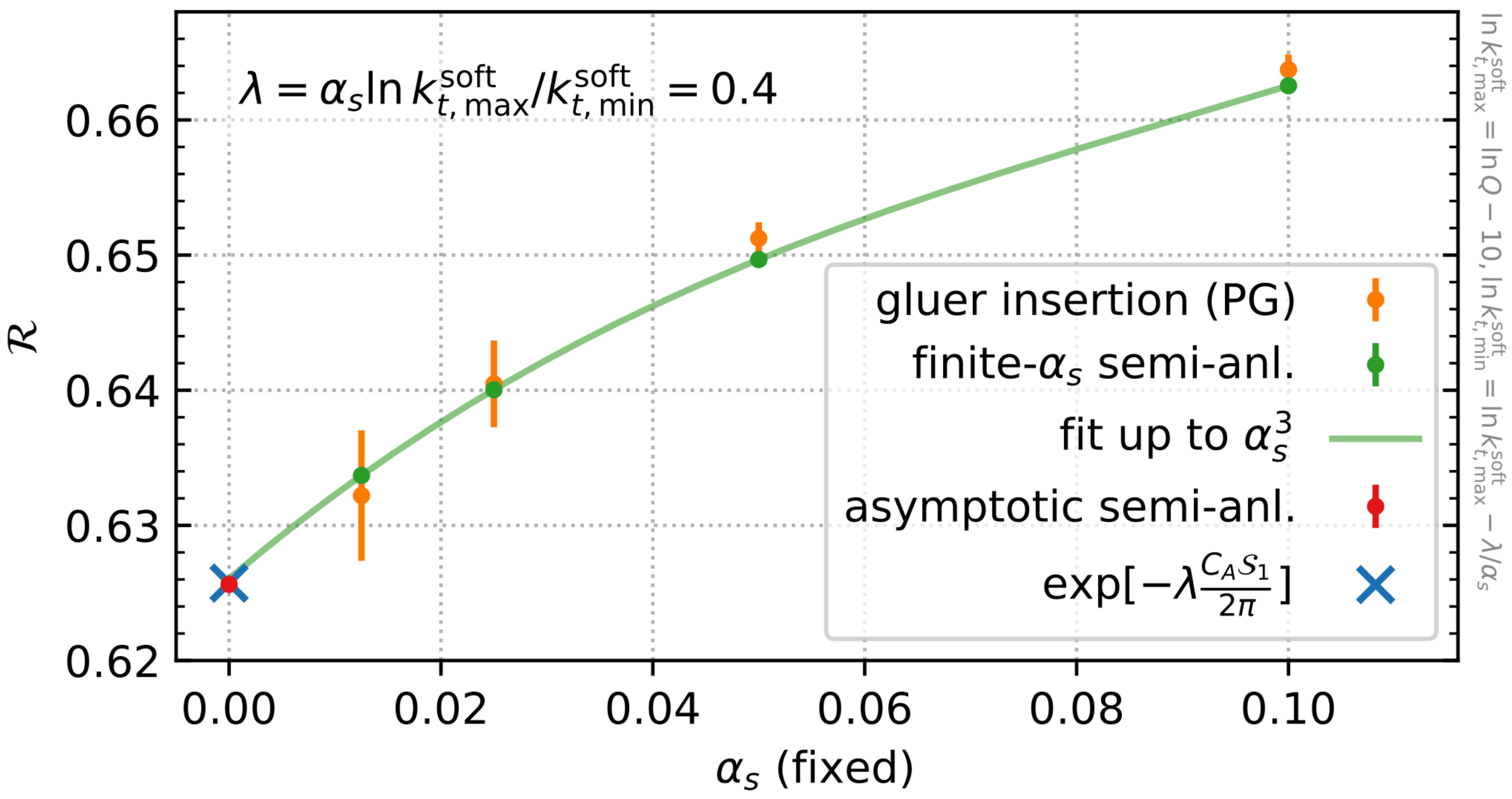
Consider a generic dipole configuration which is now described by the all-order version of $\mathscr{R}$, dependent on Q, $\mathscr{R}(Q)$. When increasing the scale, $Q \to (1+\epsilon)Q$, we allow for extra configurations with extra soft gluons. These configurations have the same pattern of dressed qg and $g\bar{q}$ dipoles, namely $\mathscr{R}(Q)T_{q\bar{q}}$.

$$\frac{d\mathscr{R}(Q)}{d\ln(Q)} = \frac{2C_F\alpha_s(Q)}{\pi} \int d\eta_g \left[\rho^{qg\bar{q}}(\eta,\eta_g) - 1\right] \mathscr{R}(Q) \quad \longrightarrow \quad \mathscr{R}(Q) = \exp\left[-\lambda(Q,\mu_{\rm np}) \frac{C_A \mathscr{S}_1}{2\pi} \right]$$

ALL ORDER RESULT

$$\mathscr{R}(Q) = \exp\left[-\lambda(Q,\mu_{\rm np})\frac{C_A\mathcal{S}_1}{2\pi}\right] \longrightarrow \langle \delta V_{\rm np}\rangle = c_V\,\frac{T_{\rm all-order}(Q)}{Q} \qquad T_{\rm all-order}(Q) = T_{q\bar{q}}\,\mathscr{R}(Q)$$

This is impressively simple, as in general soft gluon resummation does not exponentiate. **We can test this result numerically.**



- **Gluer insertion (PG)**
  Generate a $q\bar{q}$ event and shower it with PG. Add then a soft gluer, averaging over all possible insertions.
- **Finite-α_s semi-analytic (+ fit)**
  Tabulate the non perturbative shift, binning in the separation $(\Delta y, \Delta\phi)$ of the edges of a generic dipole. Interpolate the grid and sum over every dipole to get the final result.
- **Asymptotic semi-analytic**
  Like green but running the shower in an asymptotic regime with $\alpha_s \sim 10^{-9}$
- **X Analytic result**

ALL ORDER RESULT

$$\mathscr{R}(Q) = \exp\left[-\lambda(Q,\mu_{\rm np})\frac{C_A \mathscr{S}_1}{2\pi}\right] \longrightarrow \langle \delta V_{\rm np}\rangle = c_V \frac{T_{\rm all-order}(Q)}{Q} \qquad T_{\rm all-order}(Q) = T_{q\bar{q}}\,\mathscr{R}(Q)$$

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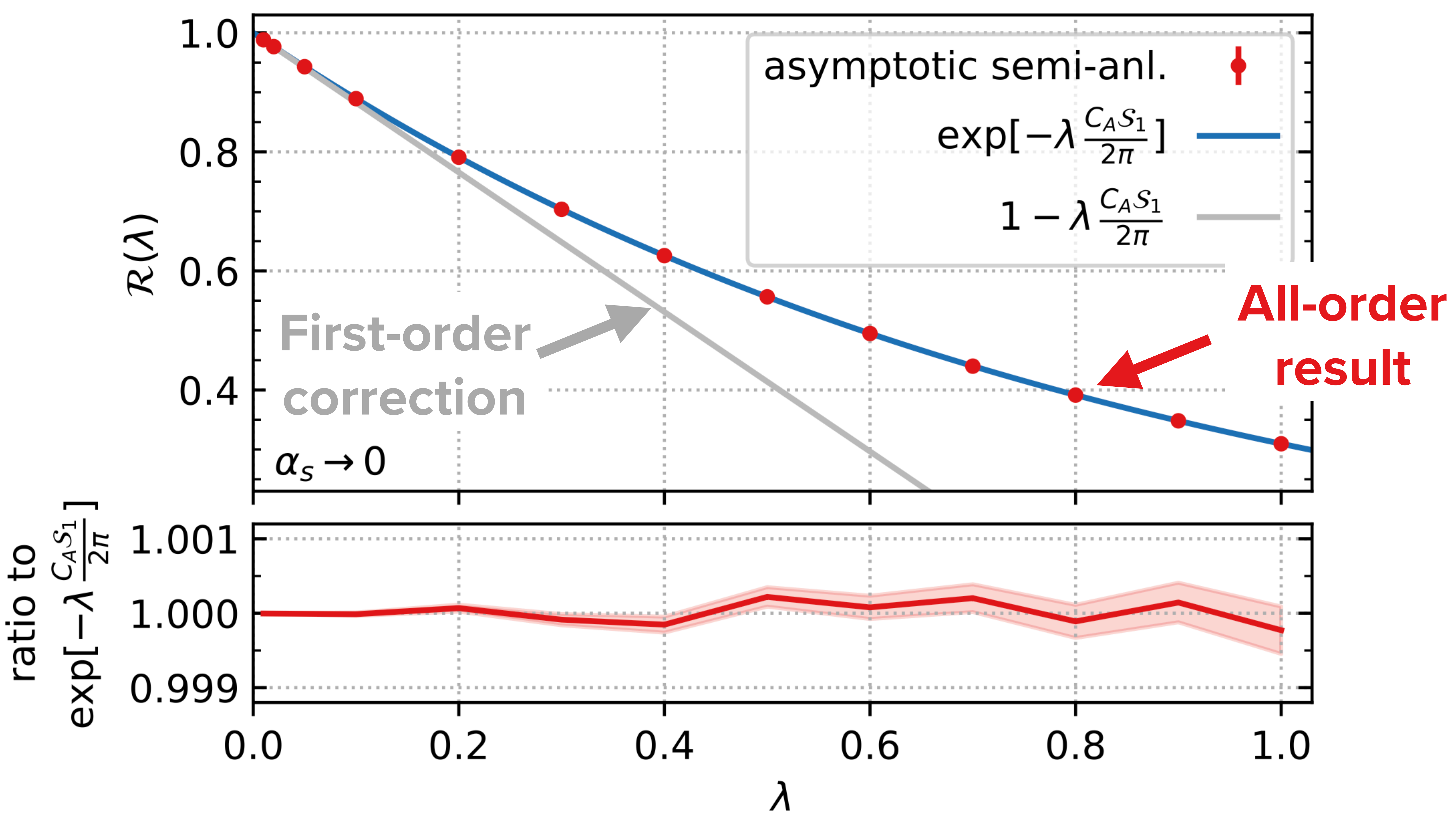
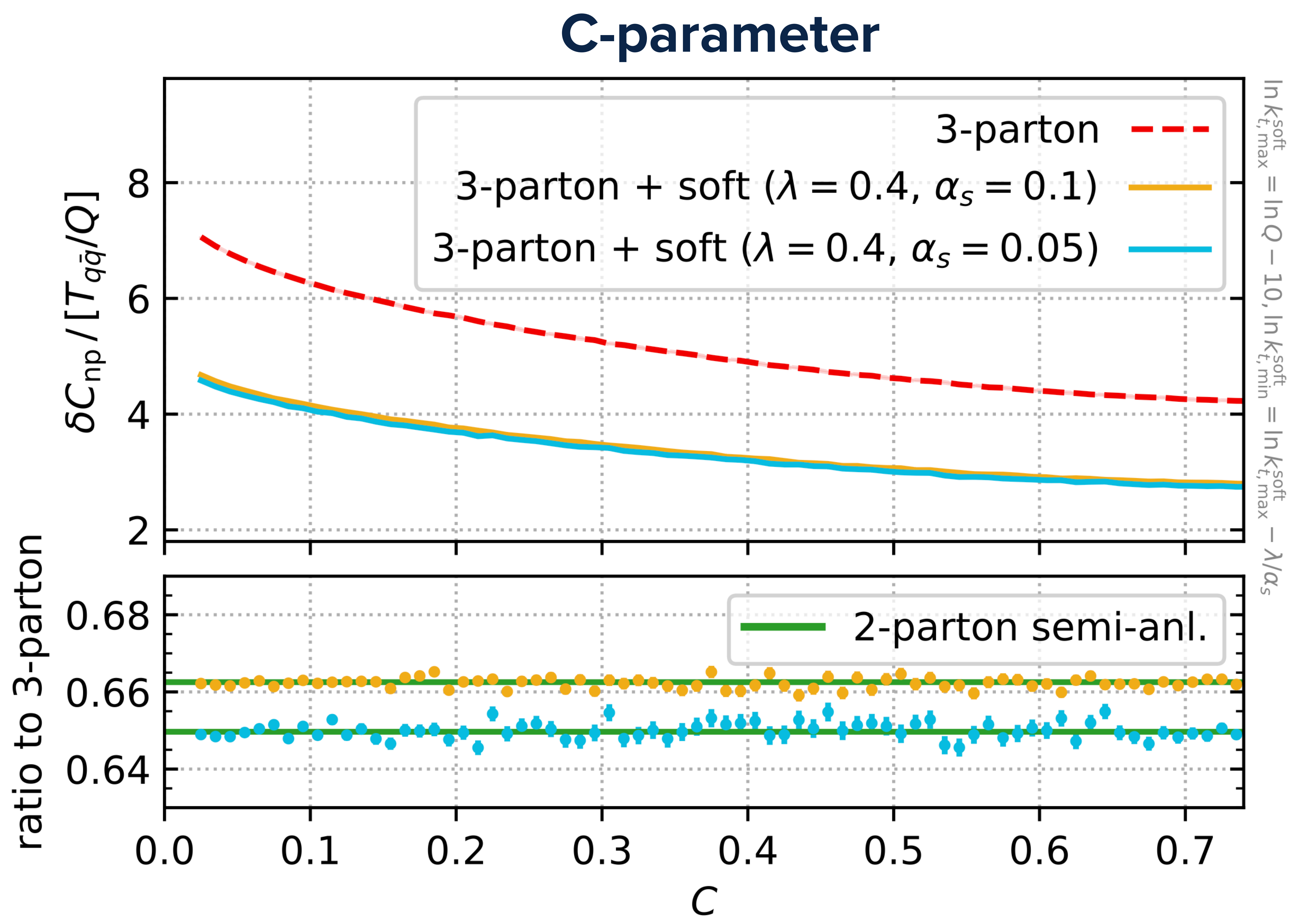


UNIVERSALITY ACROSS THE SPECTRUM

The linearity property of an observable holds beyond the simple 2-jet limit: we expect our result to hold also in the 3-jet limit.

1. Generate a hard $q\bar{q}g$ event with the correct tree level ME
2. Add a shower of soft gluons
3. Insert in the full event a soft gluer

The expected value is the 2-jet result at the corresponding λ and (fixed) α_s.



THE SIMPLICITY OF OUR RESULT HOLDS EVENT BEYOND THE 2-JET LIMIT

TAKE AWAY MESSAGES

- **We can calculate analytically the anomalous dimension $\mathcal{S}_1$** for linear event shapes studying the non-perturbative transverse momentum per unit rapidity induced by a gluer insertion.
- **We can resum analytically the anomalous dimension $\mathcal{S}_1$.** Even though it's a non-global resummation, the result turns out to be impressively simple, given by the trivial exponentiation of the first order.
- **The all order anomalous dimension has been checked numerically** through different independent tests within the PanScales framework, which confirmed our analytic result.
- **The anomalous dimension is universal in the entire spectrum,** holding also in the 3-jet limit.

The power of these results lies in the possibility of gaining new analytic insights into the structure of hadronisation

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A natural question arises: do these effects reflect the behaviour of hadronisation models?

MASSIVE VS MASSLESS SCHEME

[Salam, Wicke hep-ph/0102343]

Perturbative calculations assume massless partons, while experimental measurements deal with massive hadrons. In general, the definition of an event shape is different in the two cases.

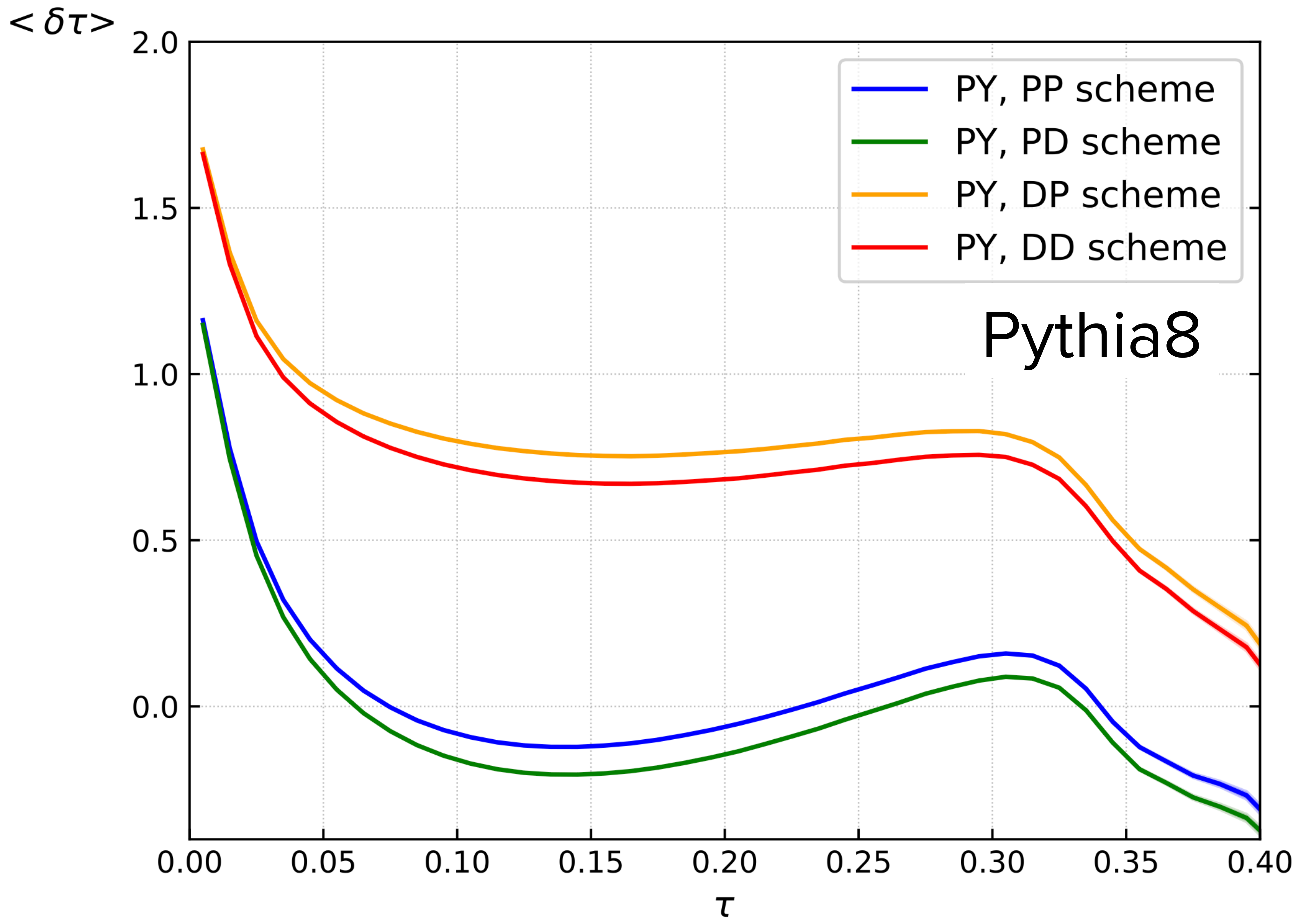
We can convert a massive event into a massless one using different schemes:

P scheme: $\vec{p}_i \to \vec{p}_i\,, \quad E_i \to |\vec{p}_i|$

E scheme: $E_i \to E_i\,, \quad \vec{p}_i \to E_i \frac{\vec{p}_i}{|p_i|}$ + boost in com frame

D scheme: massive particles are decayed isotropically into 2 massless partons

THRUST



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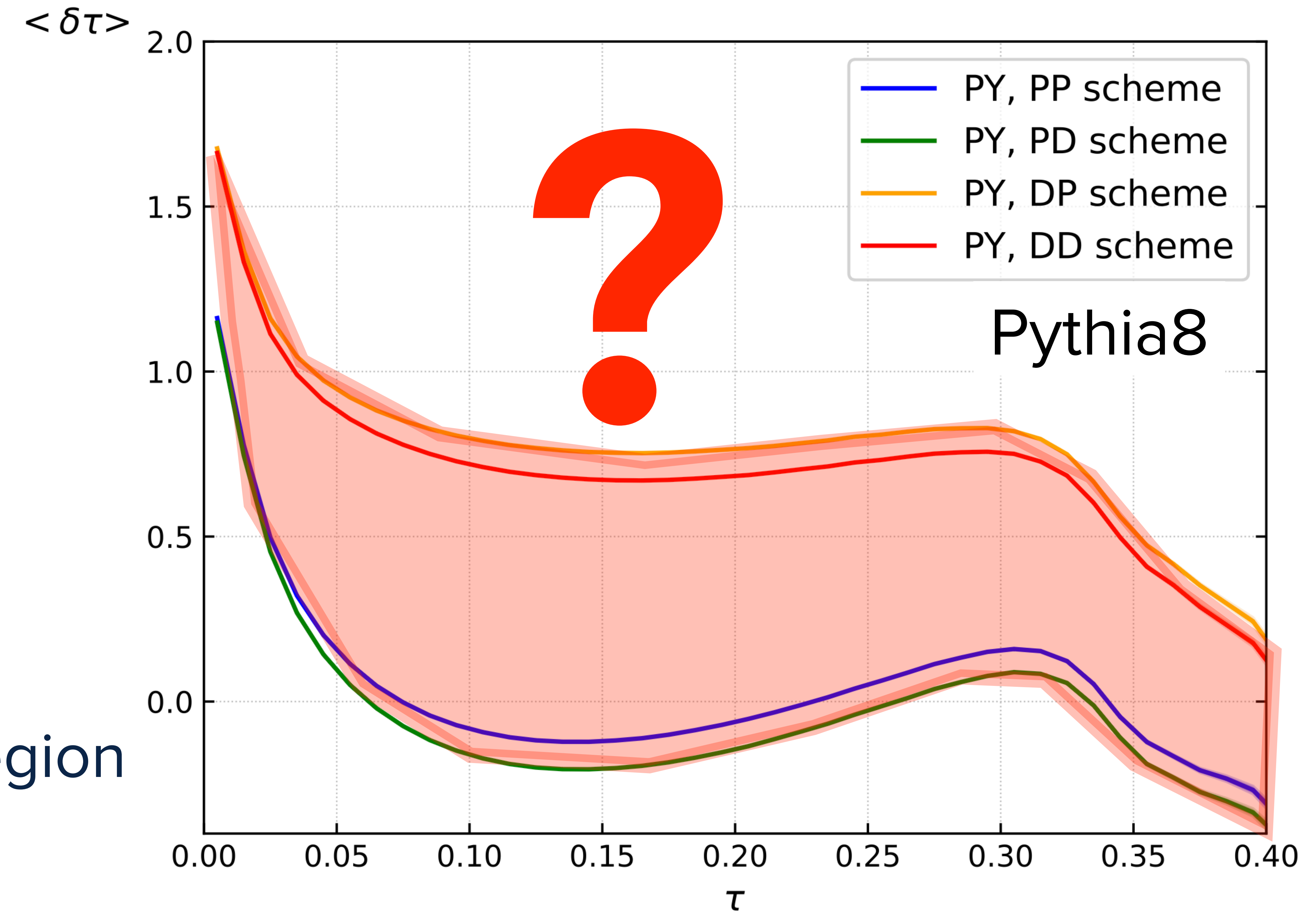
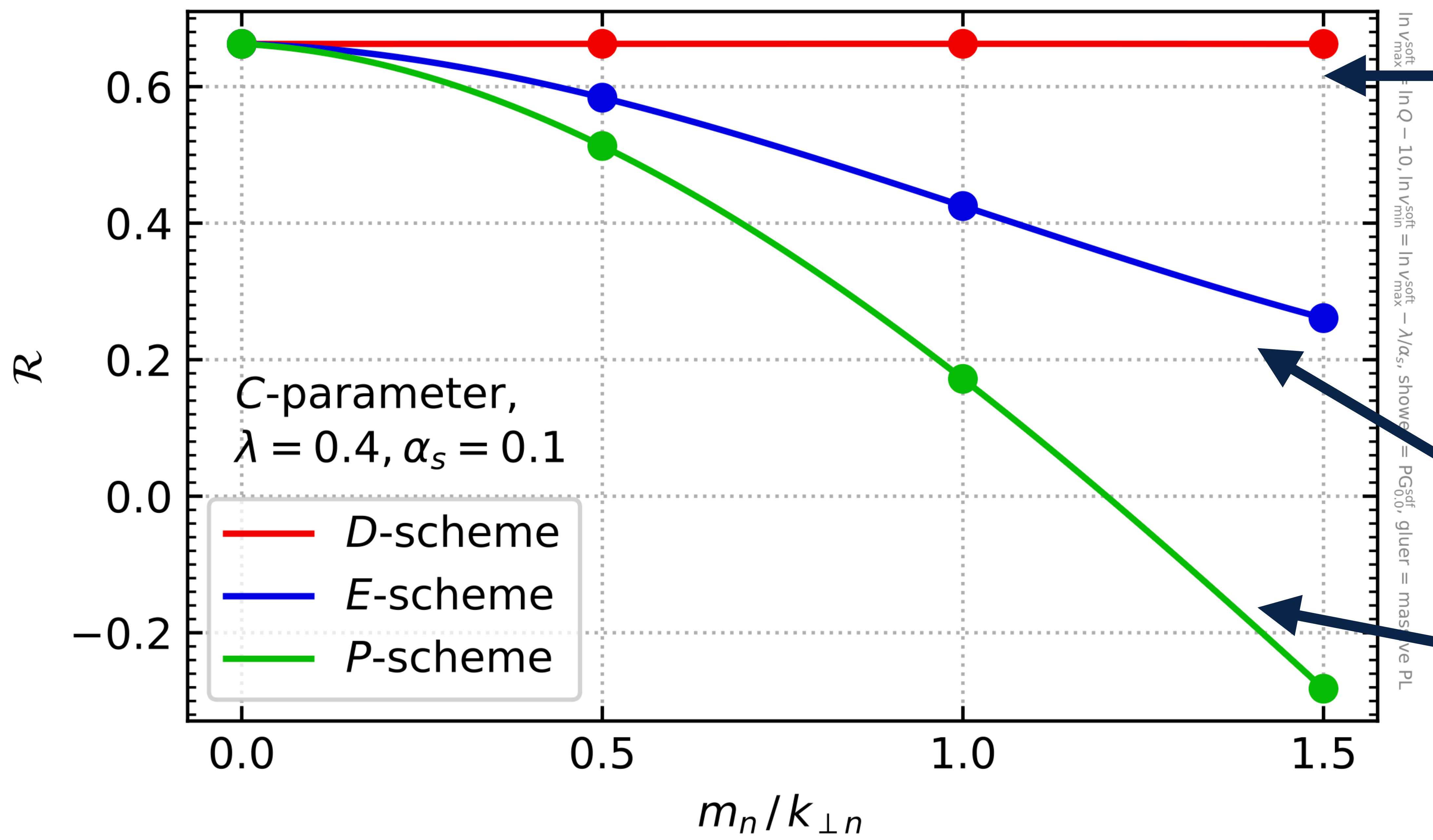


The choice of the scheme is ambiguous, because they all agree in the massless limit, but it induces large differences in the result!

This is the largest uncertainty (1.7%) in a fit of α_s in the 3-jet region [Nason, Zanderighi 2501.18173]

D-SCHEME

We can test the effect of a hadron-mass scheme inserting a **massive gluer**.



Flat in $m/k_\perp$: it does not depend in the non-perturbative mass distribution of the hadrons. The relation between transverse and +/− light-cone components along any given dipole's two directions is preserved.

Strong dependence on additional non-perturbative parameters, other than $T_{q\bar{q}}$.

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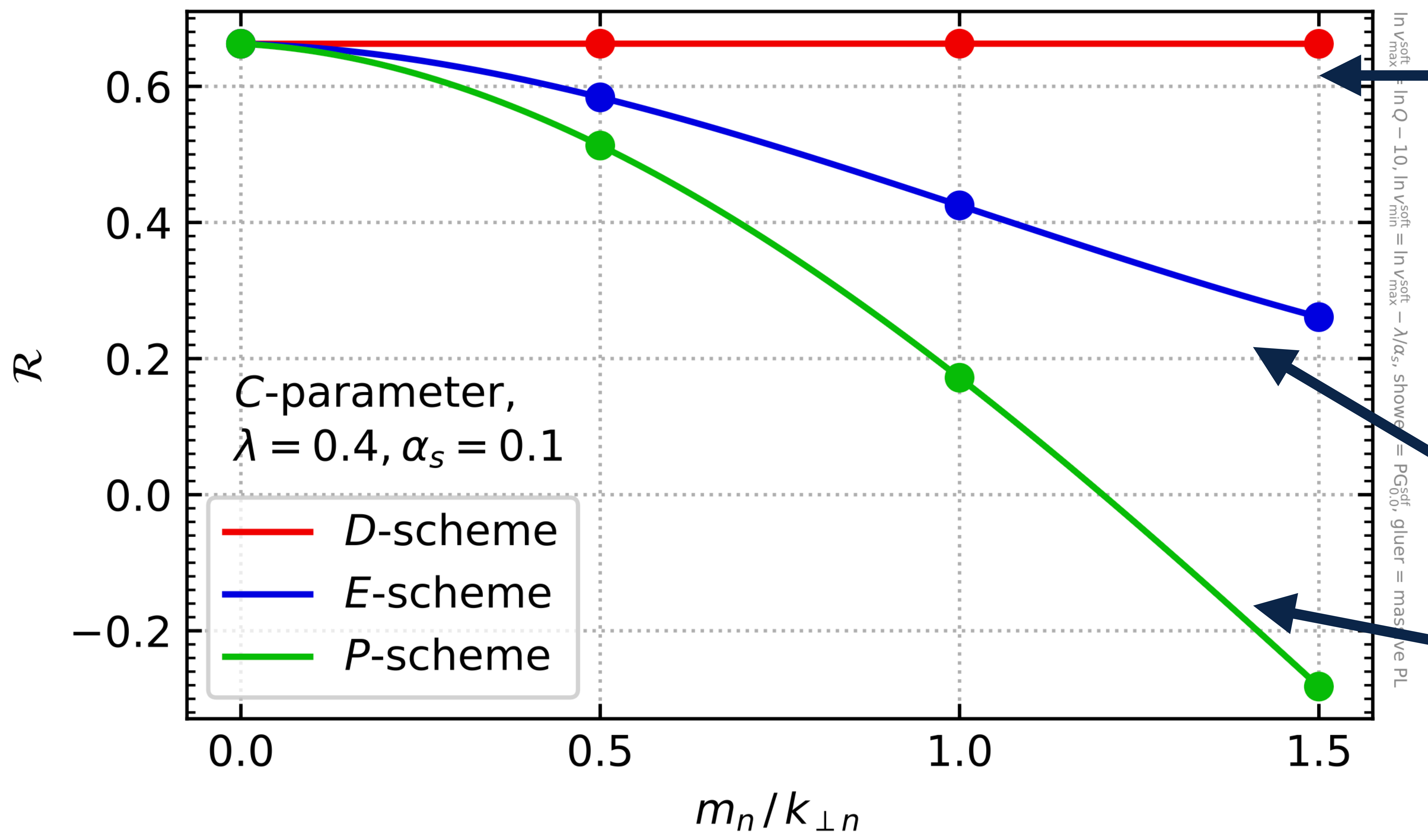

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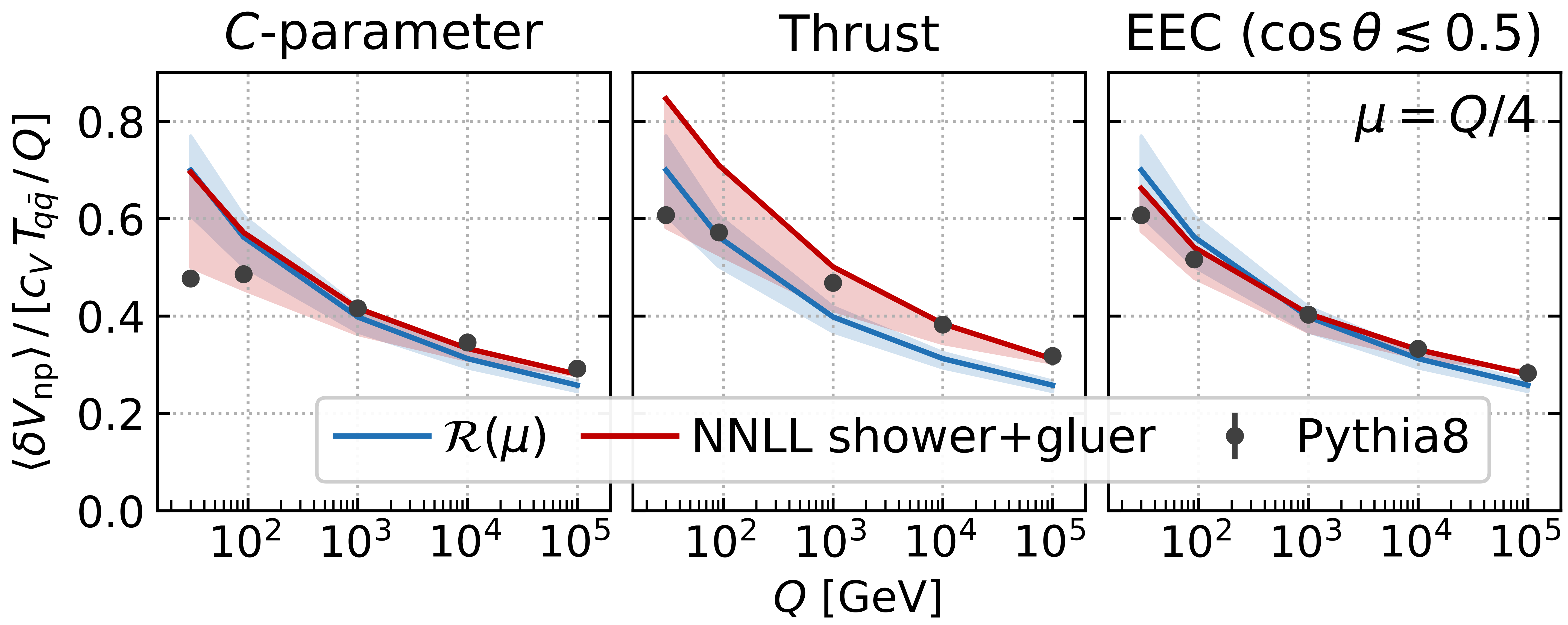
The D-scheme should be favoured!

COMPARISON WITH PYTHIA8

NON-PERTURBATIVE SHIFT IN EVENT-SHAPE MEAN VALUES

- $\mathscr{R}(\mu)$ **(analytic)**: error band obtained varying the scale $1/6 < \mu/Q < 1/2$; $\mu_{\rm np} = 2\,\mathrm{GeV}$.
- **NNLL shower + gluer insertion:** shower cutoff at $\mu_{\rm np}$; gluer insertion (PL) at $k_\perp = 0.0015\,\mathrm{GeV}$; error band obtained inserting the gluer (PG) at $k_\perp = 0.7\,\mathrm{GeV}$ (higher-power effect)
- **Pythia8** using the D-scheme: the non-perturbative parameter $T_{q\bar{q}}$ is obtained imposing agreement with NNLL shower + gluer for the average C-parameter at $Q = 1000\,\mathrm{GeV}$.



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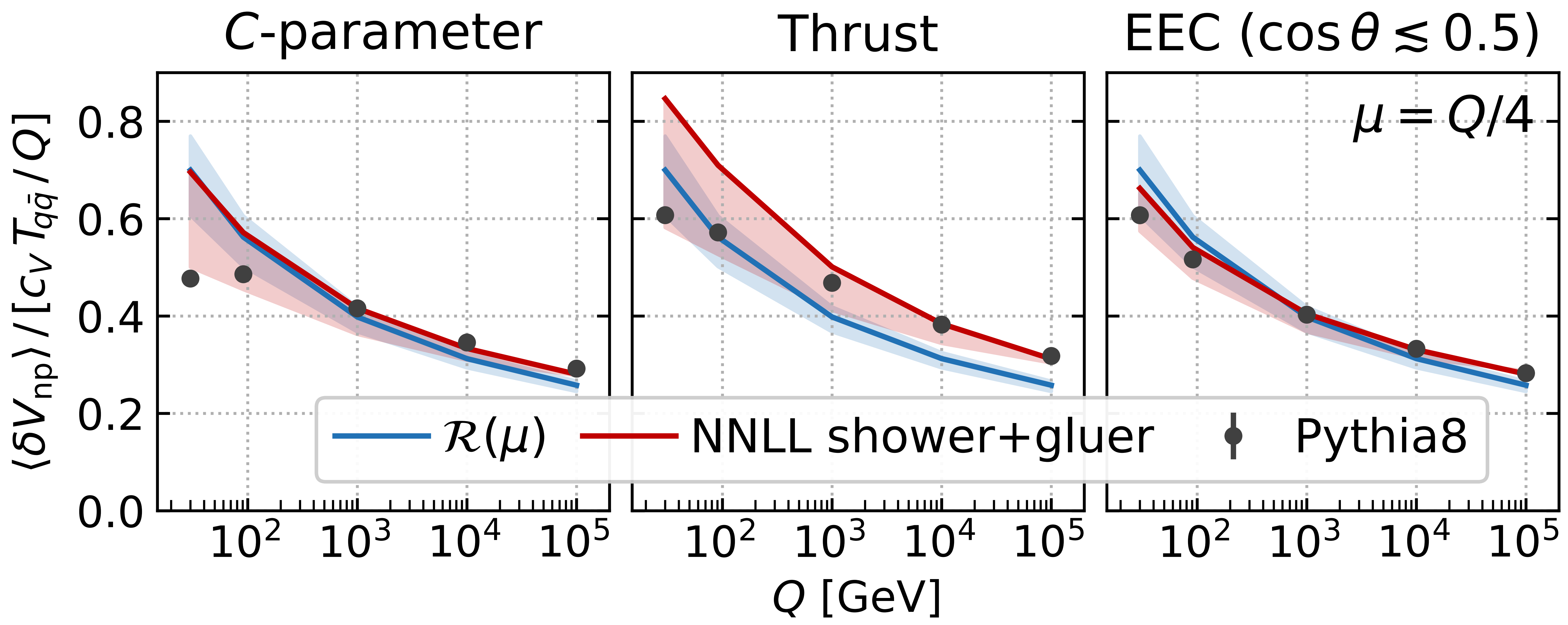
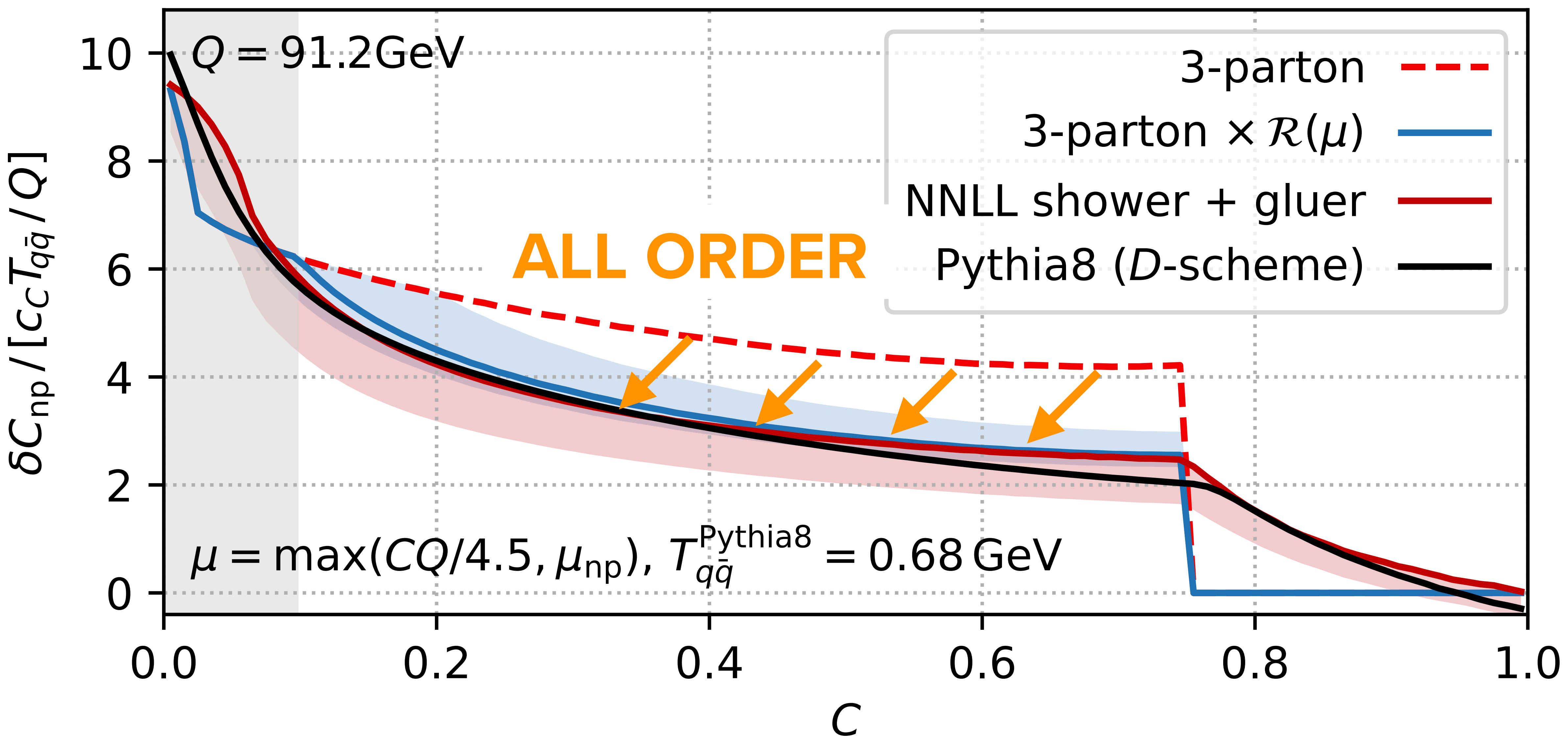


- Overall **good agreement in the entire spectrum** for all observables.
- The thrust shows large missing higher-order perturbative effects (blue vs red curves).
- Large higher-power effects up to $\sim 100\mathrm{GeV}$.

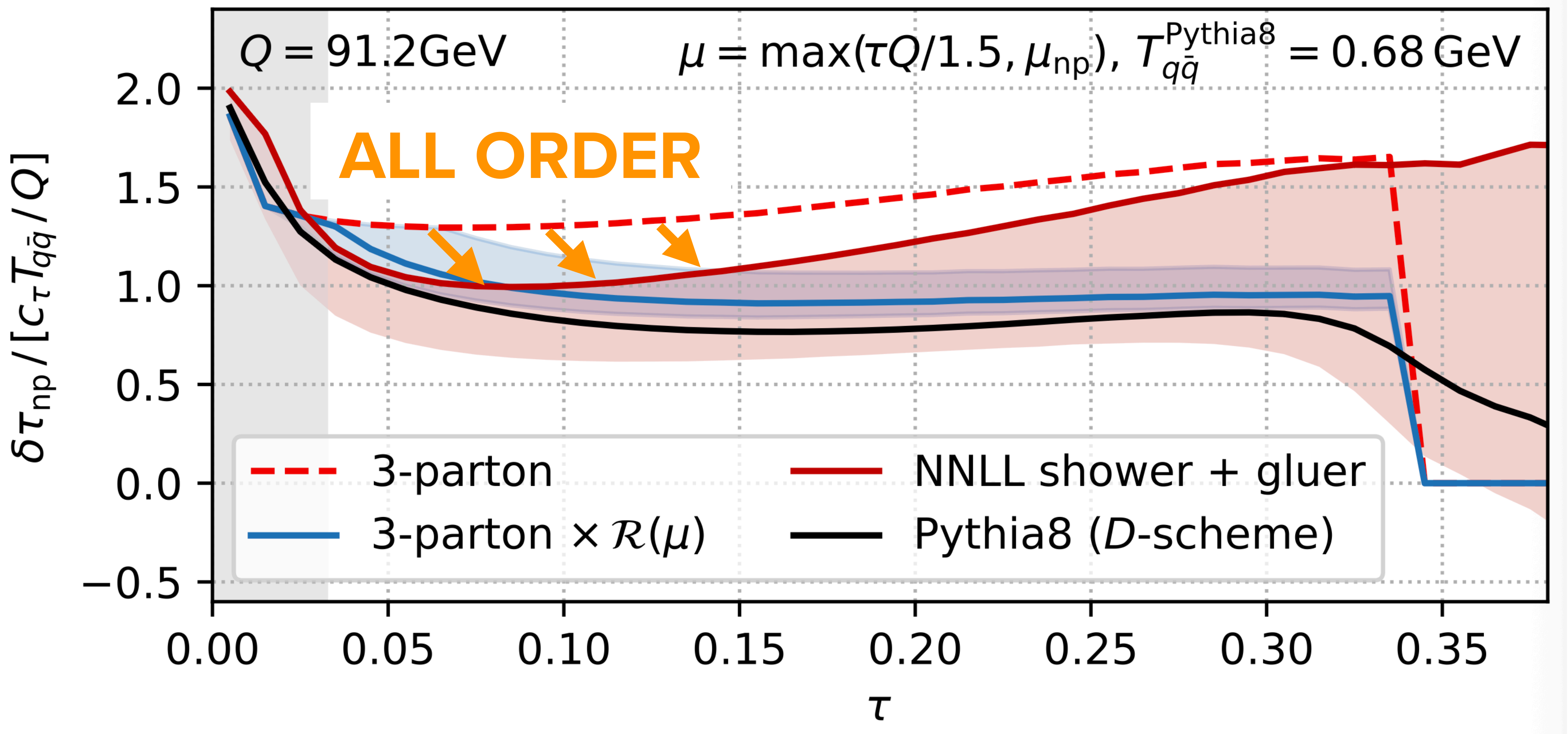
COMPARISON WITH PYTHIA8

NON-PERTURBATIVE SHIFT IN FULL SPECTRUM

C-parameter



Thrust

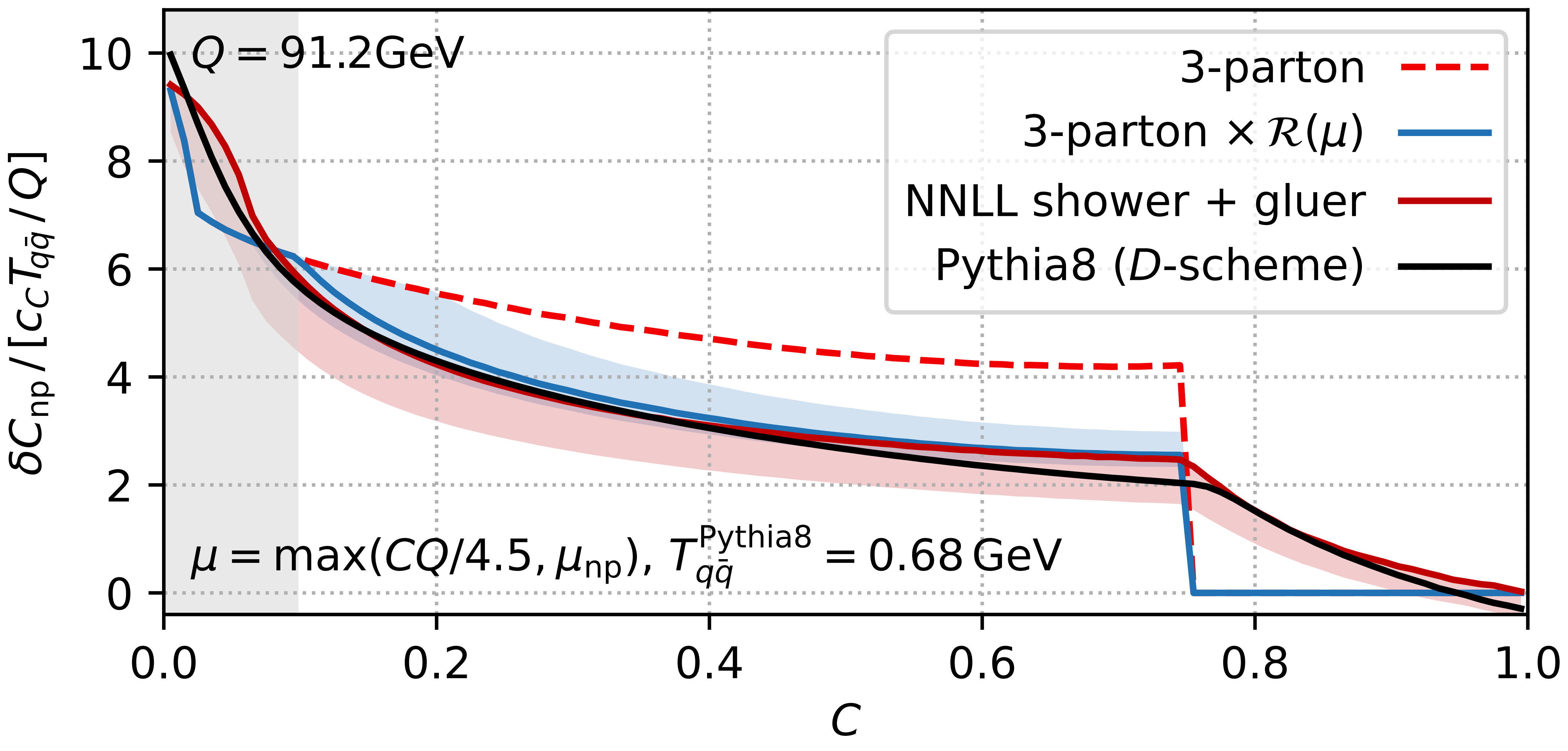


- **Significant effect when including resummation**, also in the shape.

COMPARISON WITH PYTHIA8

NON-PERTURBATIVE SHIFT IN FULL SPECTRUM

C-parameter



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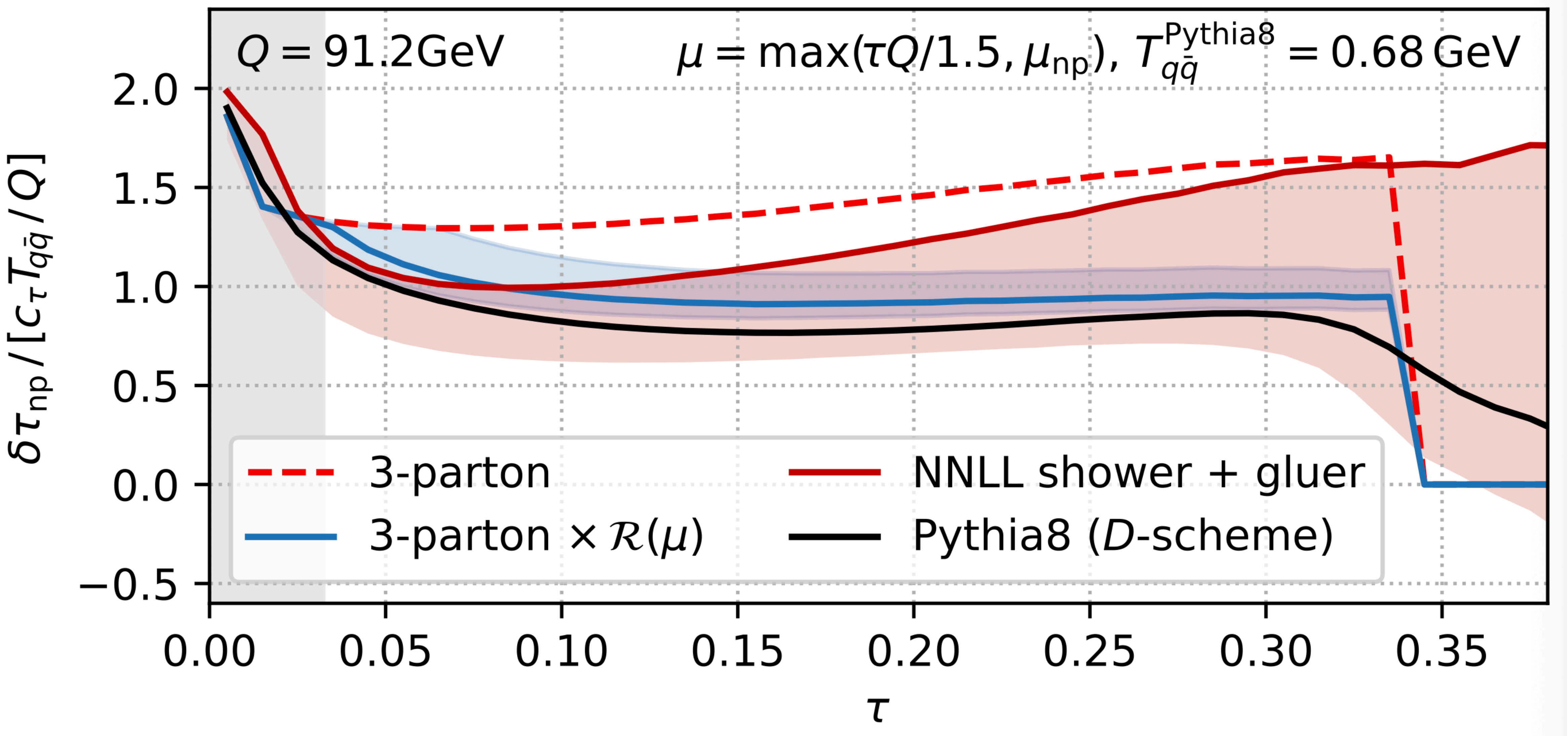
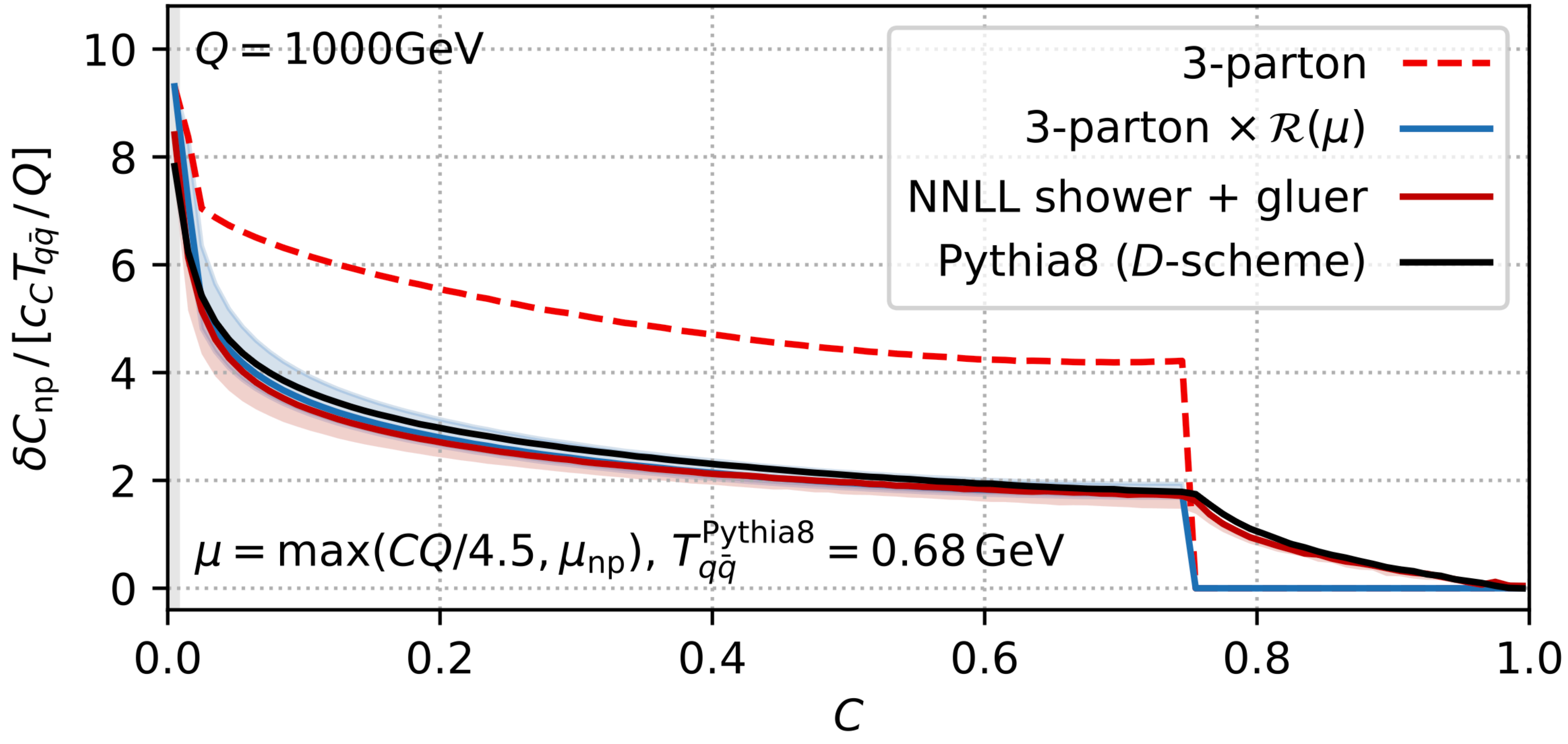


- **Significant effect when including resummation**, also in the shape.
- Striking agreement with PY8 for the C-parameter. The Thrust appears to be less stable (large higher-power and perturbative effects beyond accuracy).

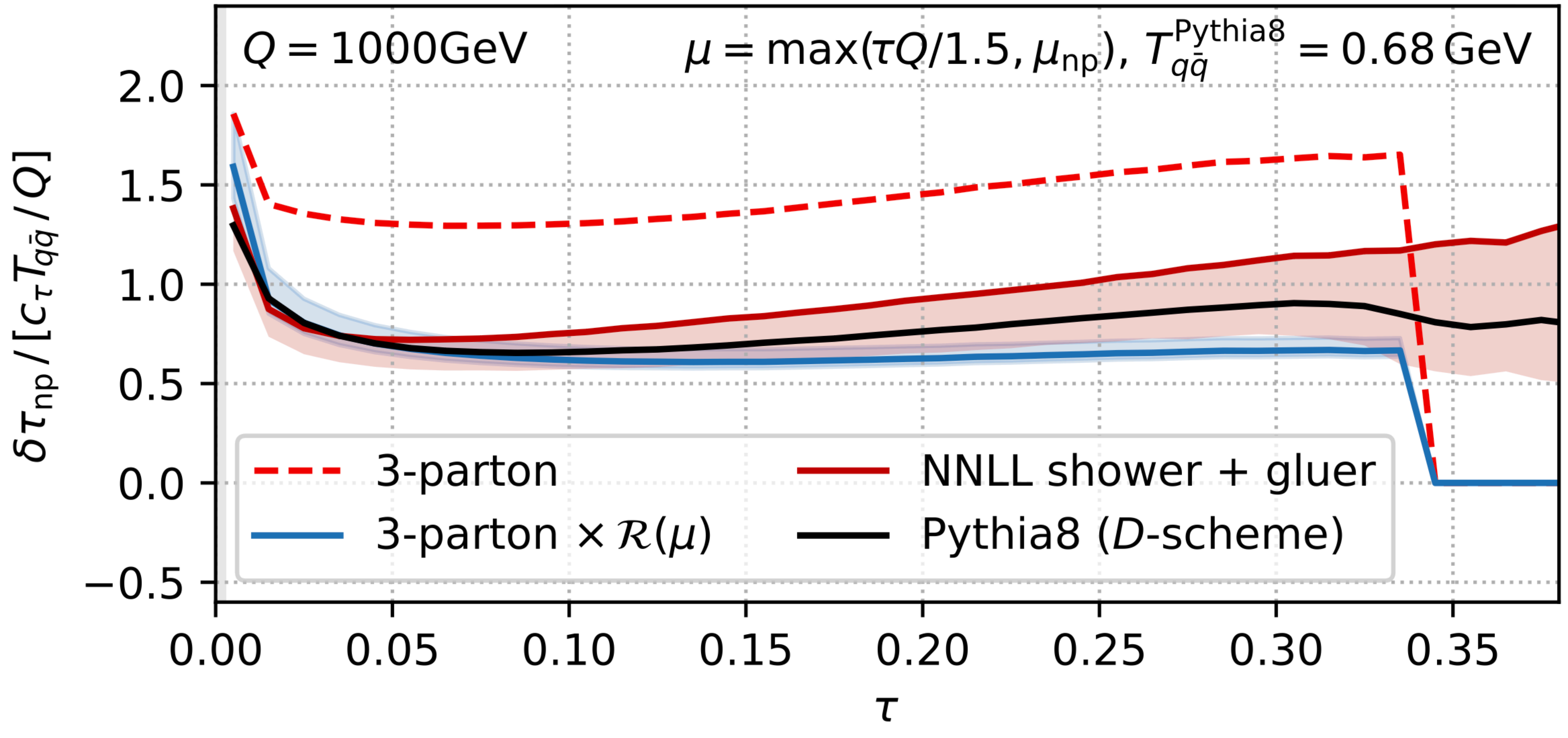
COMPARISON WITH PYTHIA8

NON-PERTURBATIVE SHIFT IN FULL SPECTRUM

C-parameter



Thrust



- **Significant effect when including resummation**, also in the shape.
- Striking agreement with PY8 for the C-parameter. The Thrust appears to be less stable (large higher-power and perturbative effects beyond accuracy).
- Same conclusions at higher scales.

CONCLUSION AND OUTLOOK

- **First analytic calculation of the all-order anomalous dimension** governing the energy scaling of non-perturbative power corrections for linear event shapes. The analytic results have been **confirmed numerically within the PanScales framework**.
- The simplicity of the result holds **beyond the simple 2-jet limit**.
- **The D-scheme should be favoured**, as it is the only one that does not depend on the non-perturbative distribution of m/p_t.
- Moreover, a **first comparison with Pythia8** (with the D-scheme) has been performed, showing a remarkable agreement for the analyzed observables.

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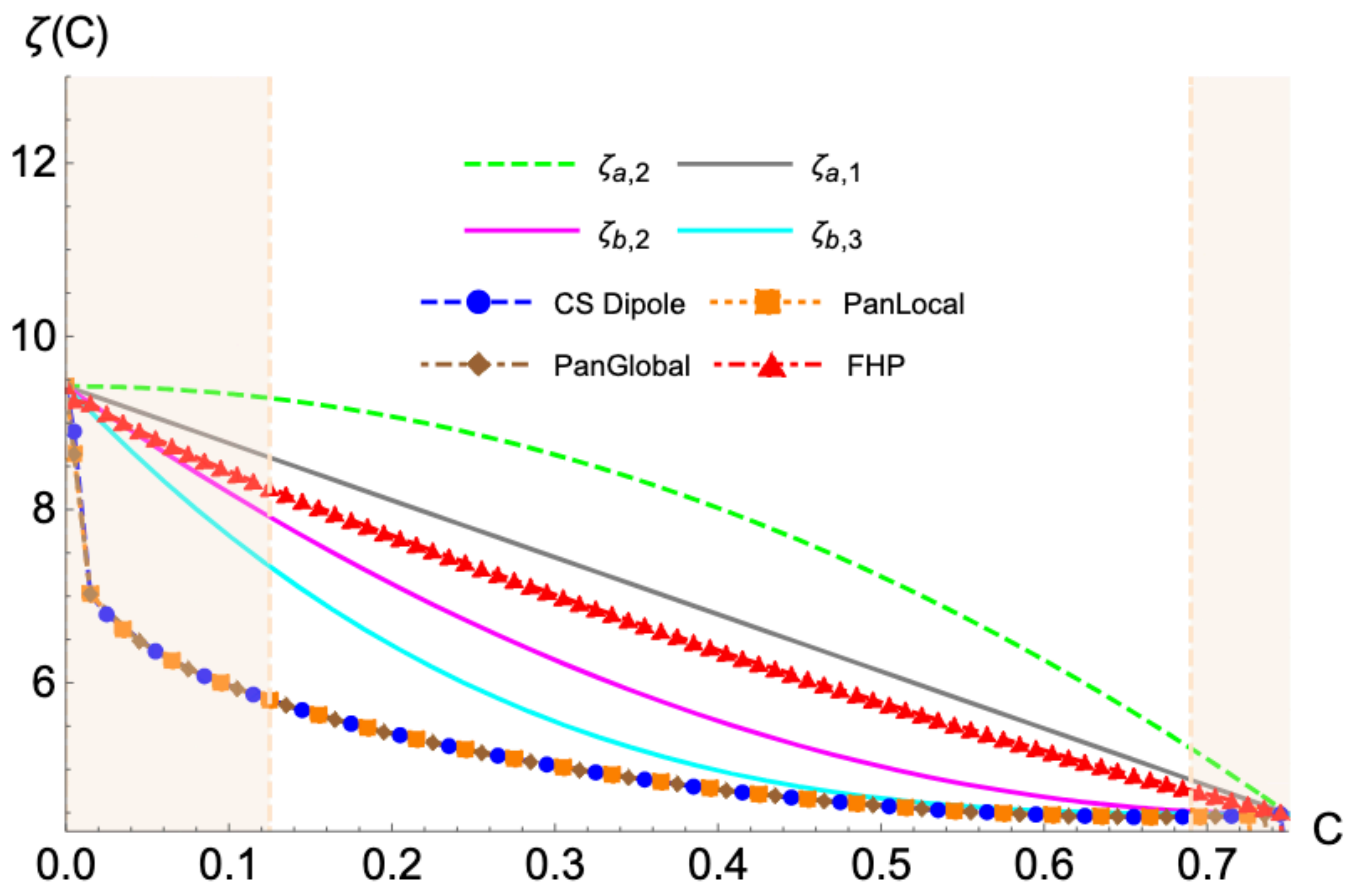
Many developments are possible:

- Can we extend this study to more complicated event shapes (e.g. hemisphere observables)?
- What's the impact of our findings in a fit of α_s?
- Can we obtain an analytic understanding of the comparison with Pythia8?
- Can we learn something more on the analytics behind hadronisation? Can we constrain hadronisation models?

BACKUP

IMPACT OF MAPPINGS

The calculation of the non perturbative shift associated to a gluer emission requires a recoil scheme to enforce energy-momentum conservation. What is the impact of this choice on our result?



[Luisoni, Monni, Salam 2012.00622]

→ **Same results for mappings in which the longitudinal recoil is kept local within the dipole**

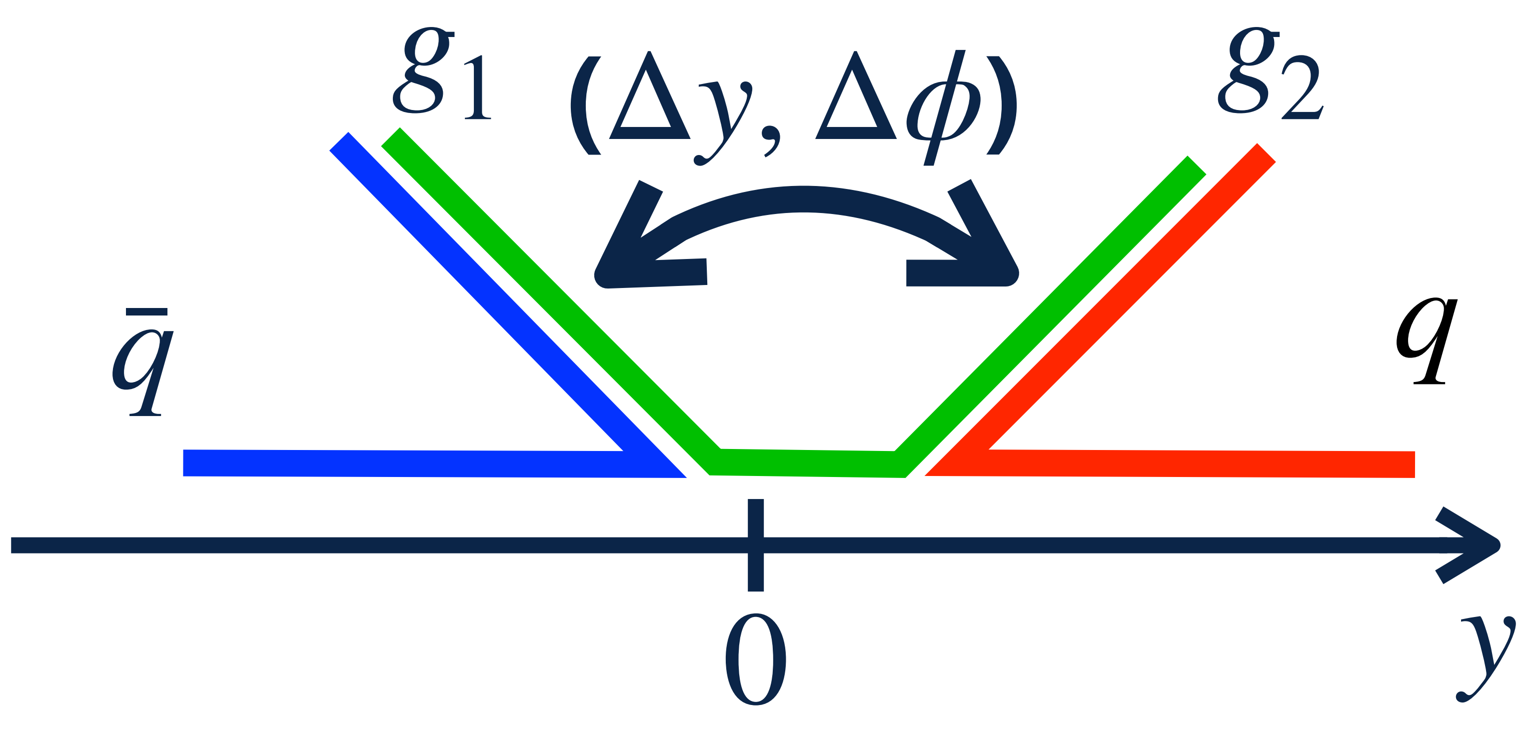
The mappings adopted in this work (PG and PL) satisfy the smoothness requirement in the soft limit that is needed in order for the recoil effects not to contribute to linear power corrections.

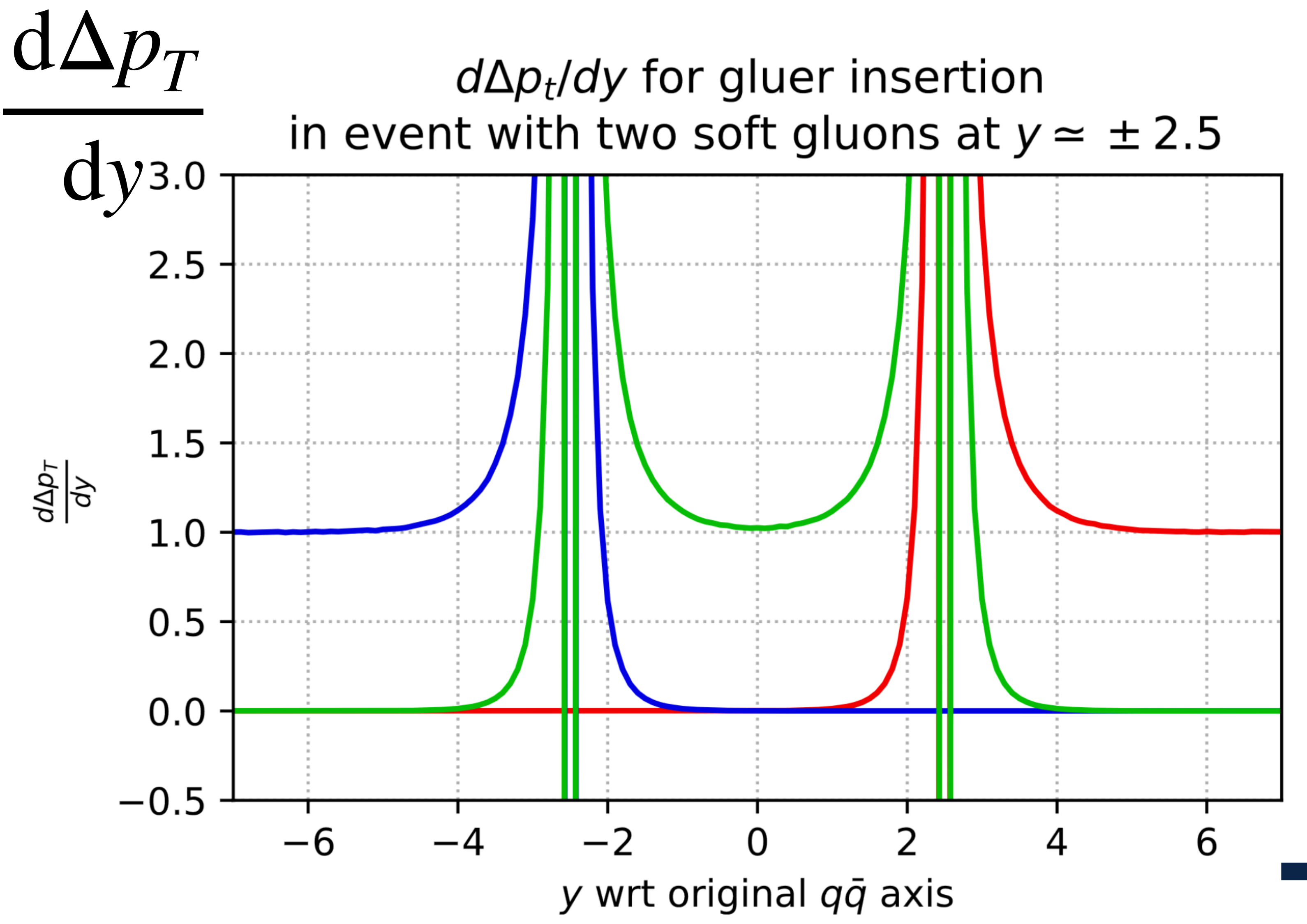
[Caola, Ferrario Ravasio, Limatola, Melnikov, Nason 2108.08897; +Ozcelik 2204.02247]

GENERAL DIPOLE CONFIGURATION

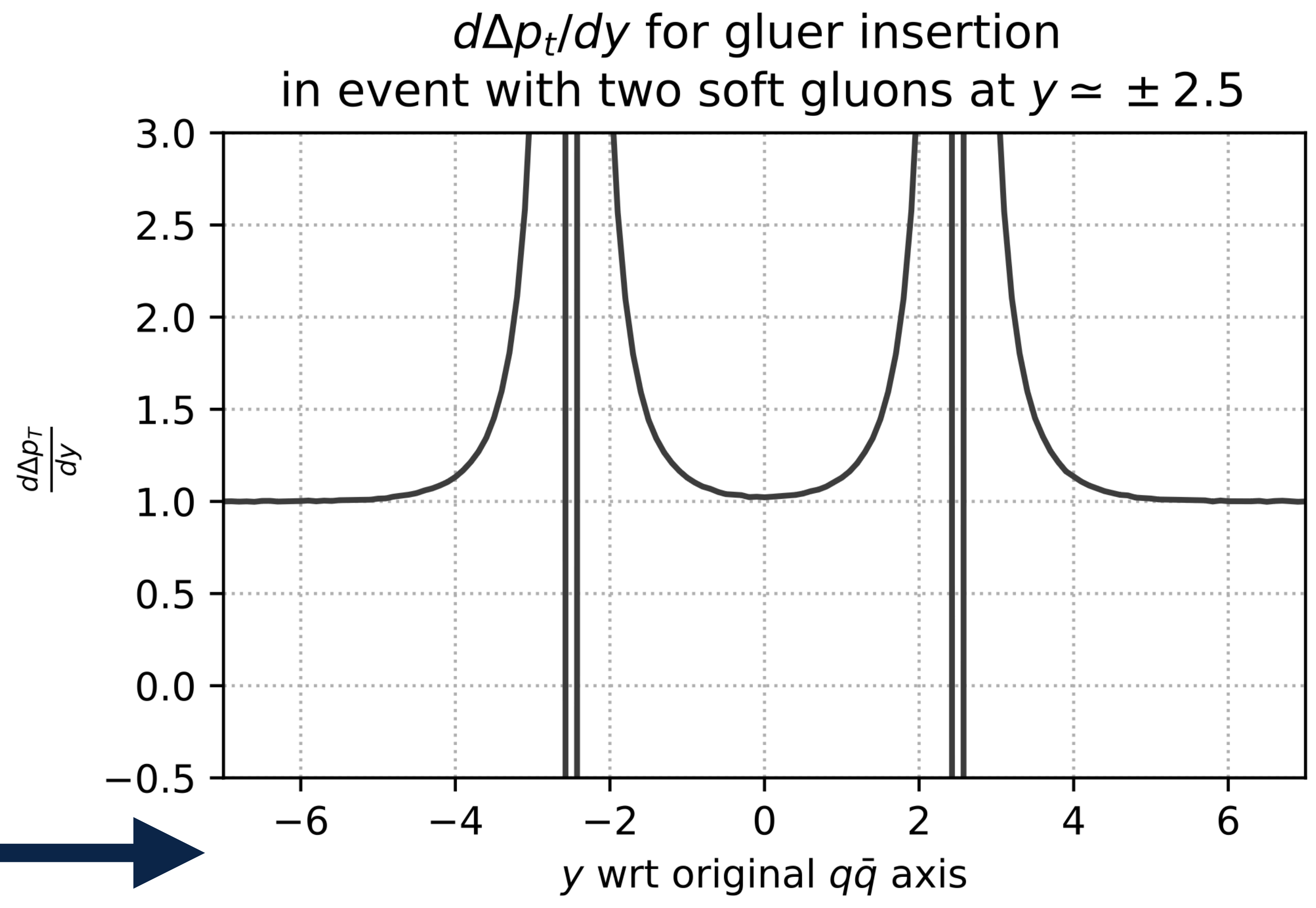
Can we extend this result to any possible dipole configuration?

Let's start from the case with two insertions:

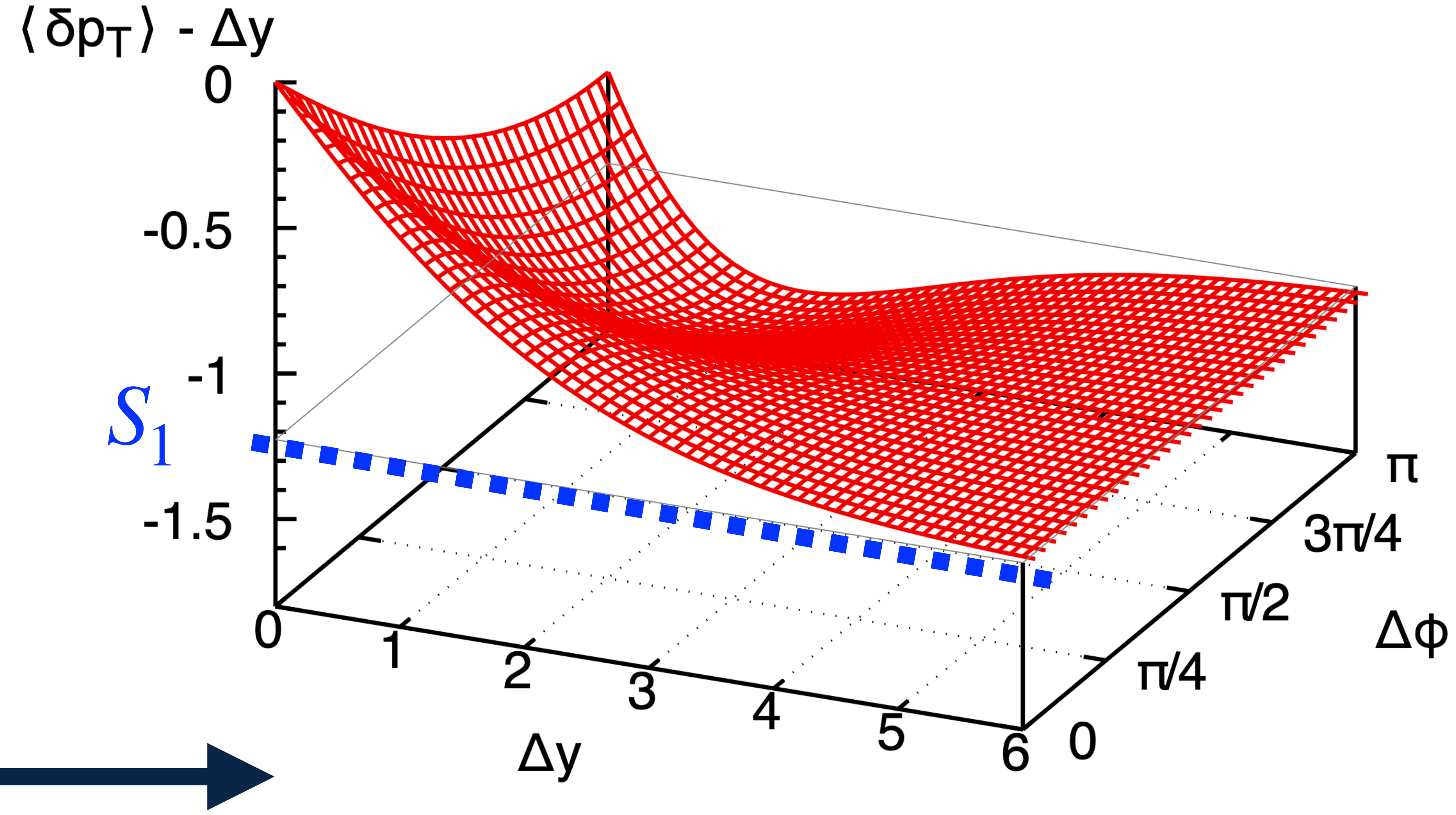




SUM OVER DIPOLES



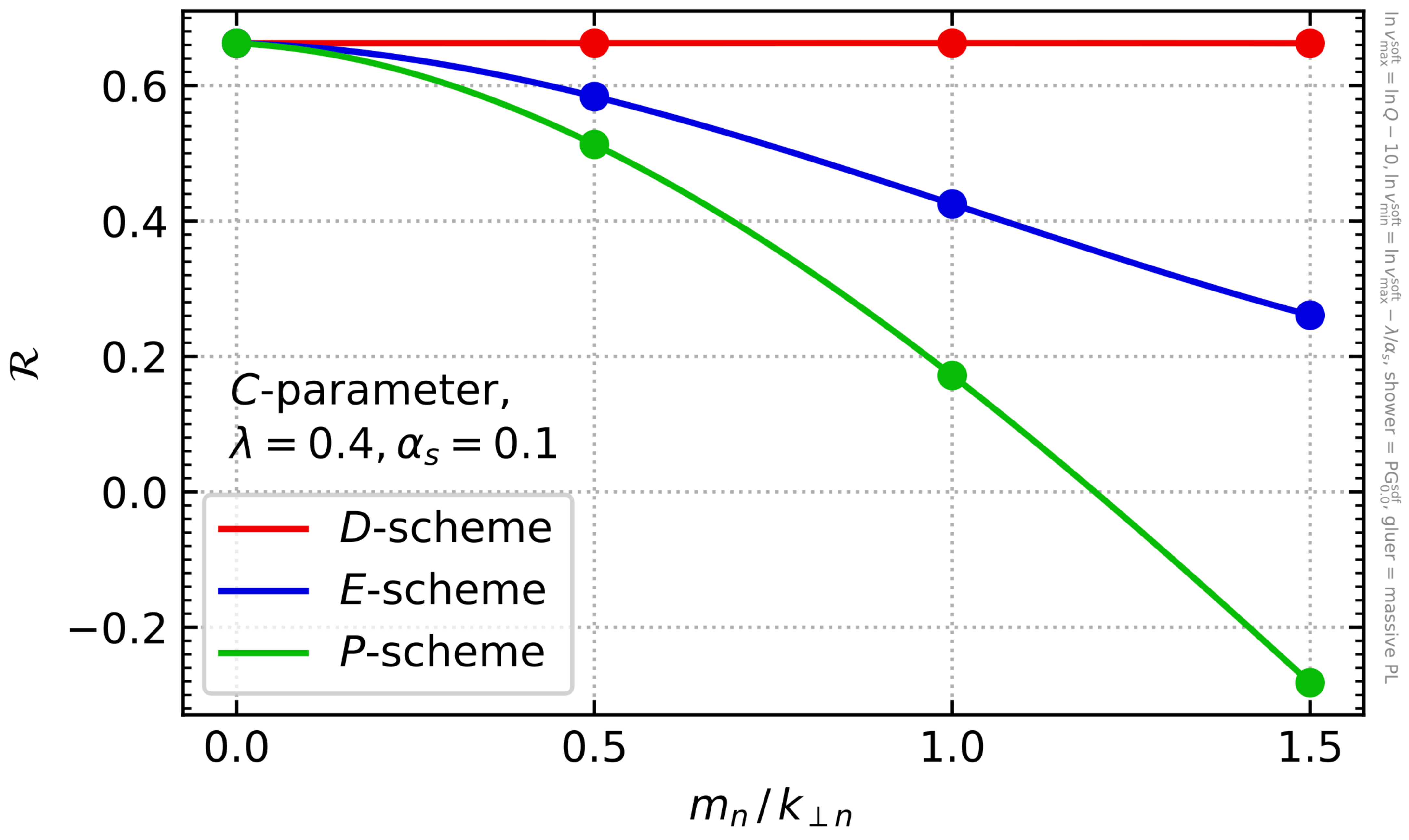
GRID in $(\Delta y, \Delta\phi)$

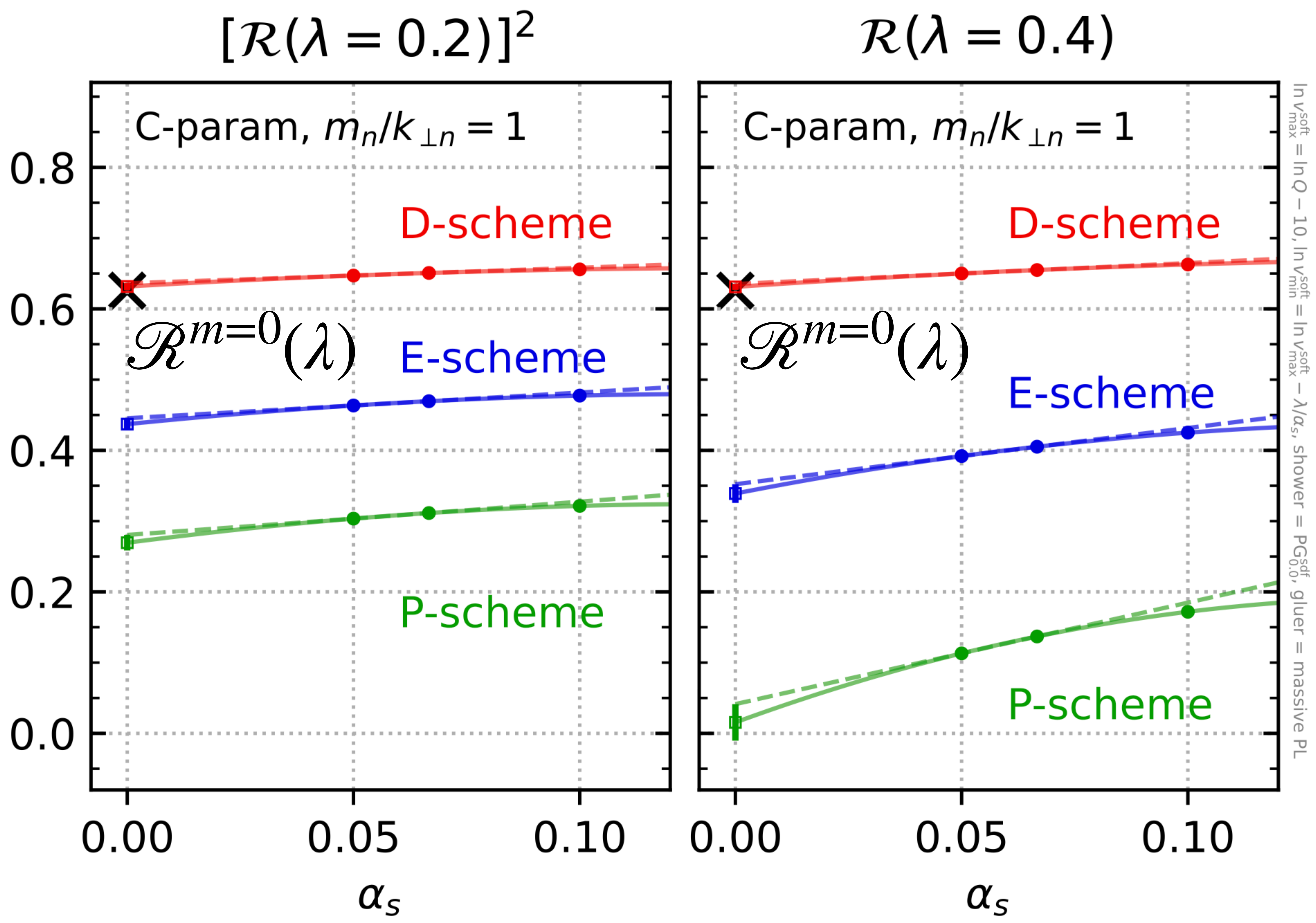


It can be used for a general dipole configuration!

D-SCHEME

We can test the effect of a hadron-mass scheme inserting a **massive gluer**.





No dependence on the non-perturbative distribution of $m/k_\perp$ only in the D-scheme.

The property of exponentiation holds only in the D-scheme.