# Decoding the Standard Model with Flavour Physics

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## Outline:

1. The Standard Model and the Flavour Problem
2. The $V_{cb}$ puzzle
3. Outlook and prospects on BSM

# Motivation

Despite the SM successes,
there are open problems:

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**Hierarchy problem**

**dark matter/dark energy**

**flavour hierarchies**

**neutrino masses**

**gravity**

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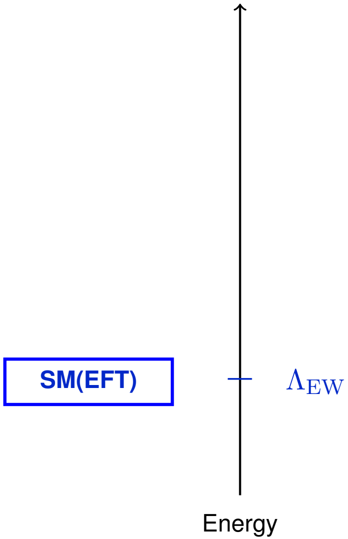

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flavour hierarchies

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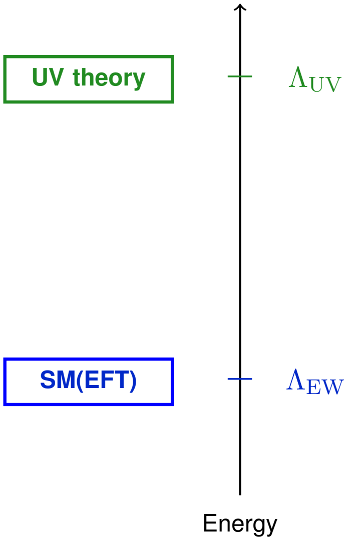

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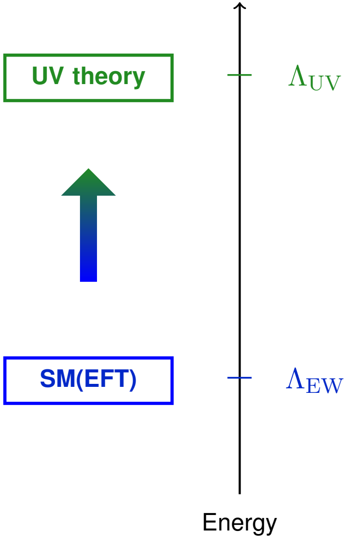
Despite the SM successes, there are open problems:

- Hierarchy problem
- dark matter/dark energy
- flavour hierarchies
- neutrino masses
- gravity



# The (two) flavour problems

1. **The SM flavour problem**: The measured Yukawa pattern doesn't seem accidental

   $\Rightarrow$ Is there any deeper reason for that?

2. **The NP flavour problem**: If we regard the SM as an EFT valid below a certain energy cutoff $\Lambda$, why don't we see any deviations in flavour changing processes?

   $\Rightarrow$ Which is the flavour structure of BSM physics?

# The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & 0.003 \\ & & 0.04 \\ & & 1 \end{pmatrix}$$

# The SM flavour problem

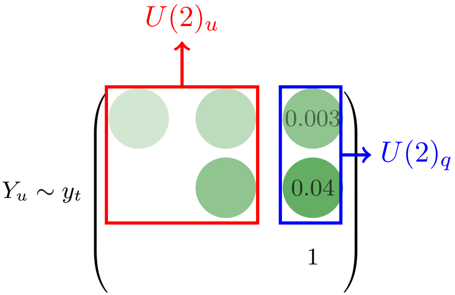
$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$

$$Y_u \sim y_t \begin{pmatrix} & & \\ & & \\ & & 1 \end{pmatrix}$$

Exact $U(2)^n$ limit

# The SM flavour problem

$$\mathcal{L}_{\text{Yukawa}} \supset Y_u^{ij} \bar{Q}_L^i H u_R^j$$




An approximate $U(2)^n$ is acting
on the light families!

# The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\text{gauge}}} + \boxed{\mathcal{L}_{\text{Higgs}}}$$

**Large Flavour symmetry**

Three replica of the same fermion fields

**$U(3)^5$ symmetry**

**Flavour degeneracy is broken**

The breaking is peculiar

- In the SM: accidental $U(3)^5 \to$ approx $U(2)^n$

# The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\rm gauge}} + \boxed{\mathcal{L}_{\rm Higgs}} + \boxed{\sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d}$$

Large Flavour symmetry

Three replica of the same fermion fields

$U(3)^5$ symmetry

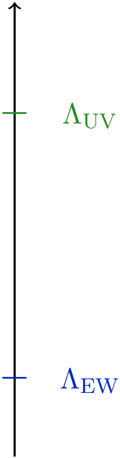
Flavour degeneracy is broken

The breaking is peculiar

- In the SM: accidental $U(3)^5 \to$ approx $U(2)^n$
- **What happens when we switch on NP?**

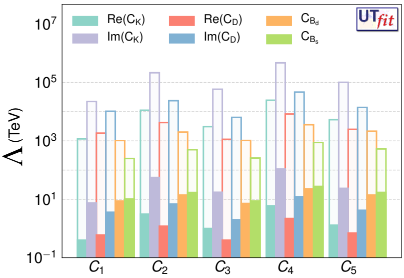
# The NP flavour problem

$$\mathcal{L} = \boxed{\mathcal{L}_{\text{gauge}}} + \boxed{\mathcal{L}_{\text{Higgs}}} + \boxed{\sum_{d,i} \frac{c_i^{(d)}}{\Lambda^{d-4}} \mathcal{O}_i^d}$$



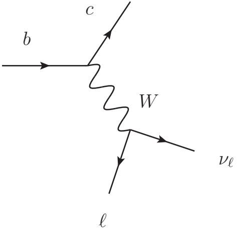

- What is the energy scale of NP?
- Why haven't observed any violation of accidental symmetries yet?

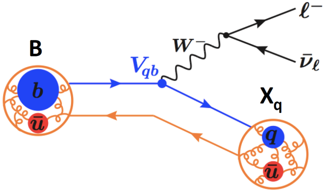
Pierini's EPS talk




**no breaking of the $U(2)^n$ flavour symmetry at low energies**

# Partonic vs Hadronic

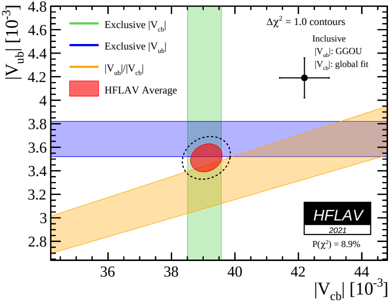


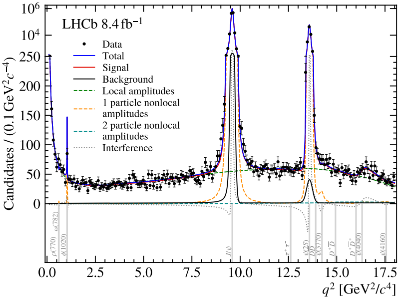



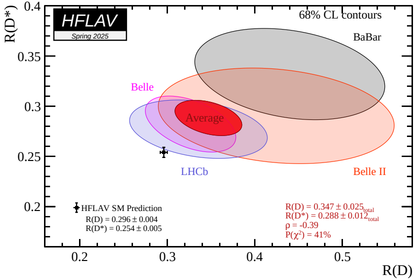

$$\mu_{\text{partonic}} = m_b \qquad \mu_{\text{hadronic}} = \Lambda_{\text{QCD}}$$

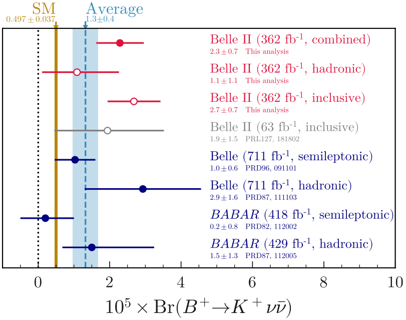
**Fundamental challenge to match partonic and hadronic descriptions**

# Old and new puzzles in flavour physics

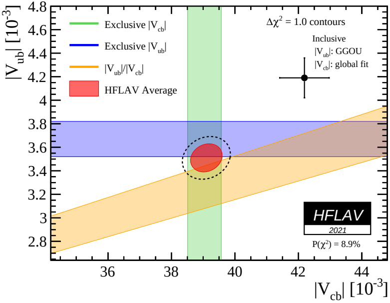


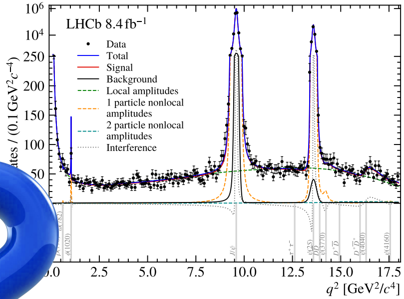



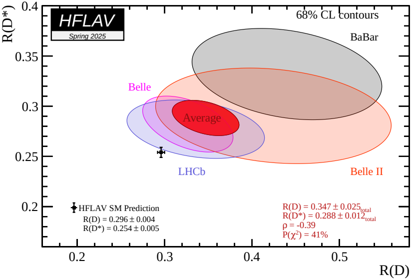



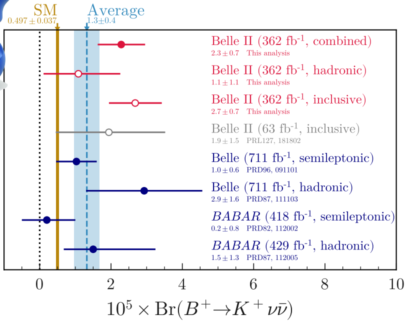



# Old and new puzzles in flavour physics



# The $V_{cb}$ puzzle

# The CKM matrix

## Interaction basis

⇒ gauge interactions are diagonal

⇒ mass terms are not diagonal

$$-\mathcal{L}_{\mathrm{Y}} = Y_d^{ij}\bar{Q}_L^i H d_R^j + Y_u^{ij}\bar{Q}_L^i \tilde{H} u_R^j + \mathrm{h.c.}$$

**Non-diagonal Yukawa**

## Mass basis

⇒ Yukawa couplings are diagonal

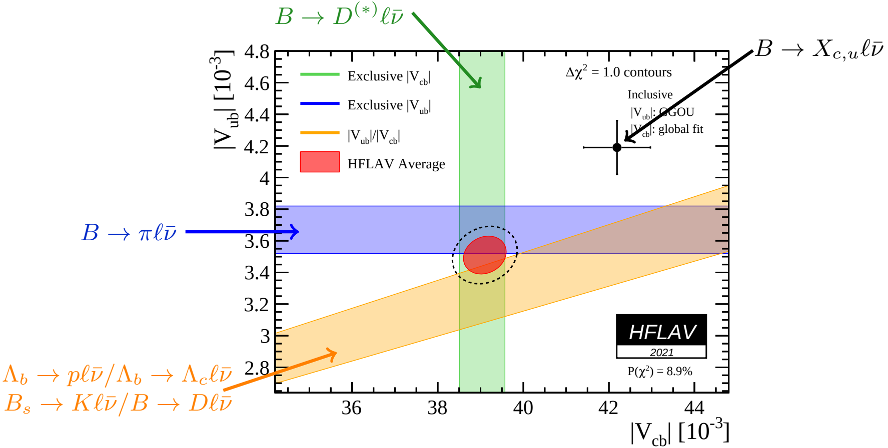
⇒ The CKM matrix is the remnant of the diagonalisation

$$\mathcal{L}_{cc} \propto \bar{u}_L^i \gamma^\mu d_L^j W_\mu^+ V_{ij}$$

**CKM matrix**

# The $V_{cb} - V_{ub}$ puzzle

- Large discrepancies between inclusive and exclusive determinations
- Recent work mostly on $B \to D^*$ due to new lattice QCD form factors determinations
- When precision increases, more puzzles arise



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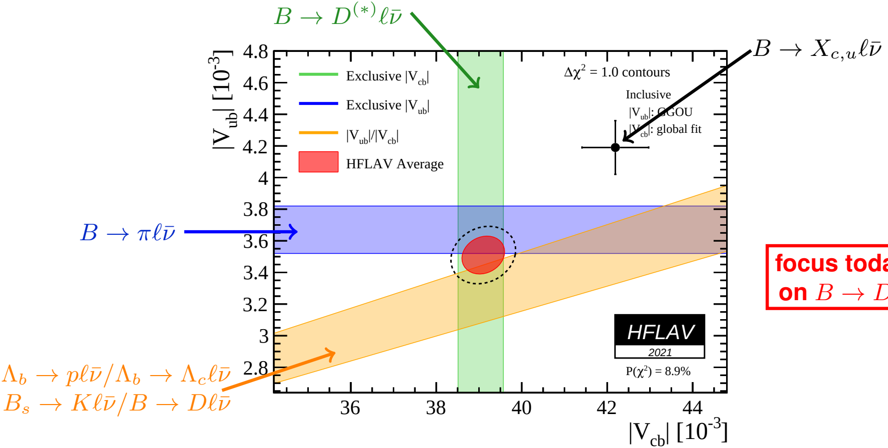
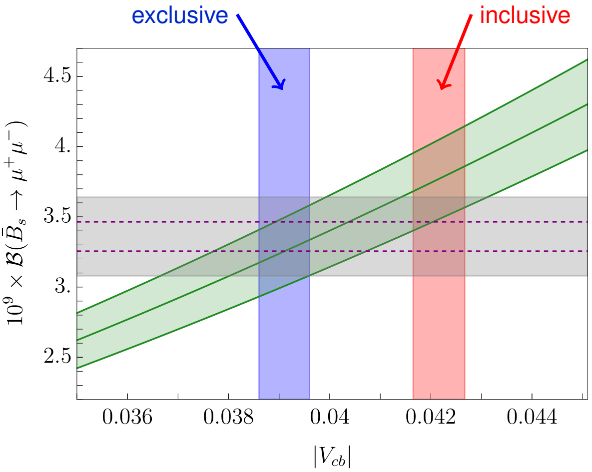



focus today on $B \to D^*$

# Why is $V_{cb}$ important?

$|V_{cb}|$ is a fundamental parameter to predict all flavour changing processes

$$\mathcal{B}(B_s \to \mu^+\mu^-) \sim |V_{cb}|^2$$



# Exclusive matrix elements

$$\langle H_c | J_\mu | H_b \rangle = \sum_i S_\mu^i \mathcal{F}_i$$

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$$\langle H_c|J_\mu|H_b\rangle = \sum_i S^i_\mu \mathcal{F}_i$$

form factor

scale $\Lambda_{\text{QCD}}$

independent

Lorentz structures

# Exclusive matrix elements

$$\underbrace{\langle H_c | J_\mu | H_b \rangle}_{\text{scale } \Lambda_{\text{QCD}}} = \sum_i S_\mu^i \mathcal{F}_i$$

form factor

scale $\Lambda_{\text{QCD}}$

independent

Lorentz structures

Form factors determinations

- Lattice QCD
- QCD SR, LCSR

only points at specific kinematic points

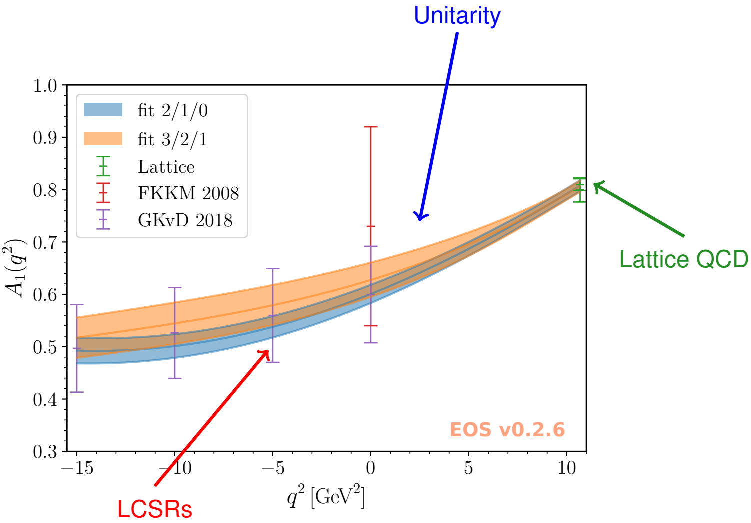
Form factors parametrisations

- HQET (CLN + improvements) $\Rightarrow$ reduce independent degrees of freedom
- Analytic properties $\rightarrow$ BGL

data points needed to fix the coefficients of the expansion

# $B \to D^*$ before 2021

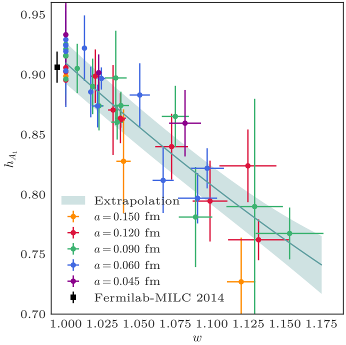
[MB, Gubernari, Jung, van Dyk, '19]



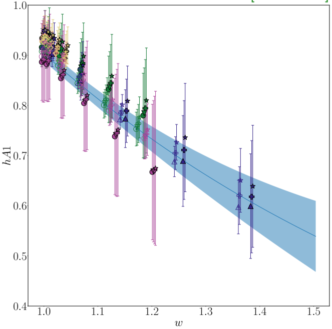

Other references: F. Bernlochner, Z. Ligeti, M. Papucci, M. Prim, D. Robinson, '22
P. Gambino, M. Jung, S. Schacht, '19

# $B \to D^*$ from lattice away from zero recoil

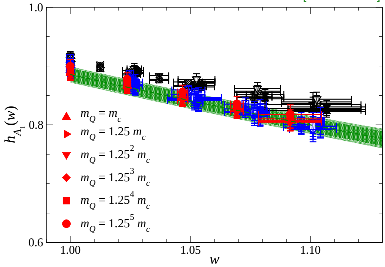
[2105.14019]




[2304.03137]

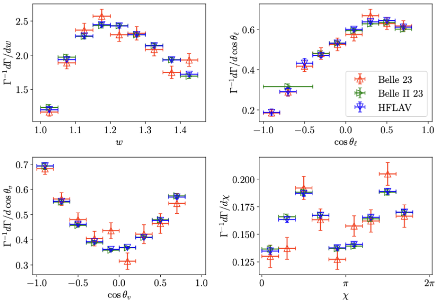


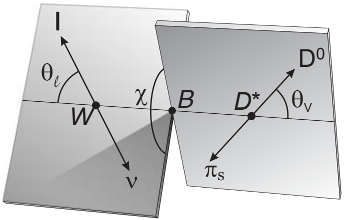

[2306.05657]




- Are these results compatible with each other?
- Are they compatible with experimental data?

# New $B \to D^* \ell \bar{\nu}$ Belle and Belle II data




$$
\begin{aligned}
\frac{d\Gamma}{dw\,d\cos(\theta_\ell)\,d\cos(\theta_v)\,d\chi} = & \frac{3G_F^2}{1024\pi^4}|V_{cb}|^2\eta_{EW}^2 M_B r^2\sqrt{w^2-1}\,q^2 \\
& \times \{(1-\cos(\theta_\ell))^2\sin^2(\theta_v)H_+^2(w) + (1+\cos(\theta_\ell))^2\sin^2(\theta_v)H_-^2(w) \\
& + 4\sin^2(\theta_\ell)\cos^2(\theta_v)H_0^2(w) - 2\sin^2(\theta_\ell)\sin^2(\theta_v)\cos(2\chi)H_+(w)H_-(w) \\
& - 4\sin(\theta_\ell)(1-\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi)H_+(w)H_0(w) \\
& + 4\sin(\theta_\ell)(1+\cos(\theta_\ell))\sin(\theta_v)\cos(\theta_v)\cos(\chi)H_-(w)H_0(w)\}
\end{aligned}
$$

- Between 7 to 10 bins per kinematic variable
- Available on HEPData with correlations
- Angular observables analysis are available, data just newly released

# Analysis strategies

## Setup

- BGL parametrisation
- Bayesian inference to apply unitarity

Flynn, Jüttner, Tsang, '23

## Questions

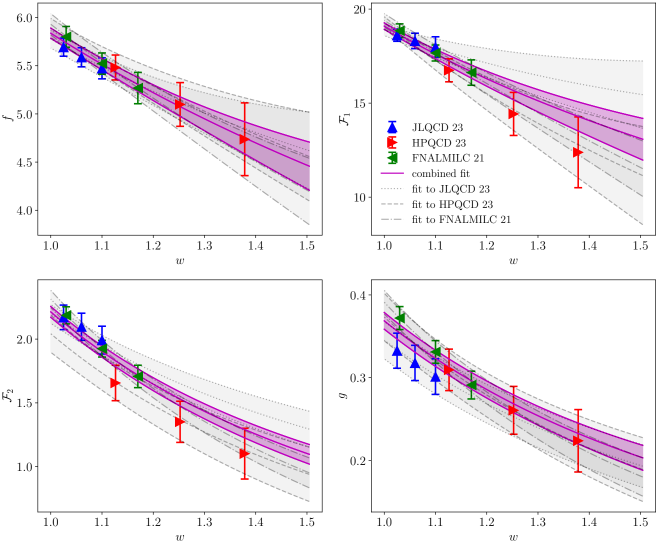
- Combine the three LQCD datasets

  $\Rightarrow$ Is the combination acceptable?

- Combine with experimental data
- What are the consequences for phenomenology?

# Lattice only

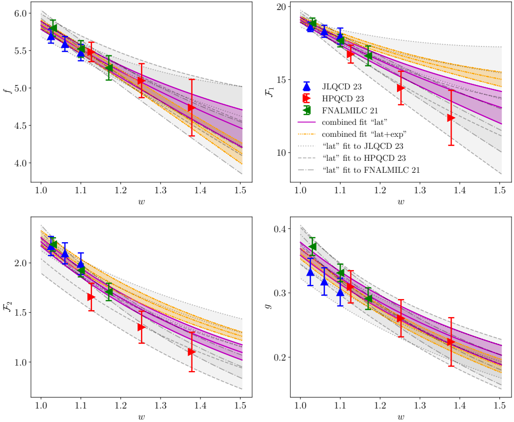
[MB, A. Jüttner, '24]




see also G. Martinelli, S. Simula, L. Vittorio, '23,'24

# Lattice + experimental data

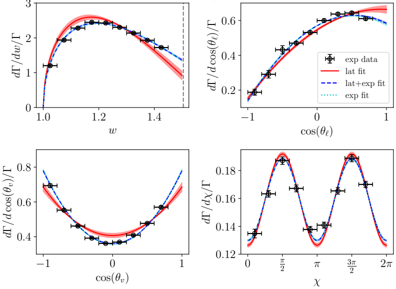
[MB, A. Jüttner, '24]



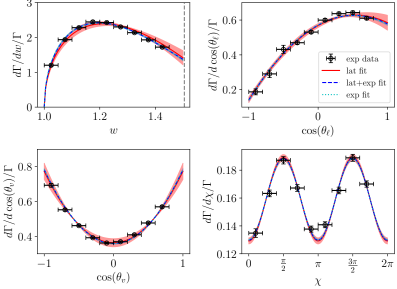

- Good fit quality ($p$-value $\sim 18\%$)
- Adding experimental data reduces the uncertainties, especially at large w
- Especially for $\mathcal{F}_1$ and $\mathcal{F}_2$, the shape changes when including experimental data

## FNAL/MILC + HPQCD +HFLAV

[MB, A. Jüttner, '24]



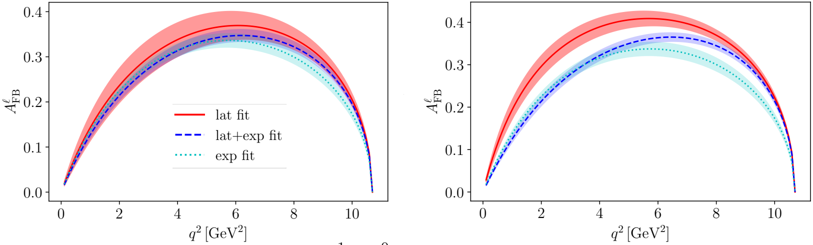

## JLQCD +HFLAV




- Fit to HPQCD and FNAL/MILC misses experimental points
- BGL fit to experimental and lattice data has $p-$value $\sim 18\%$
- BGL coefficients shift of a few $\sigma$ when including experimental data

# Differential observables

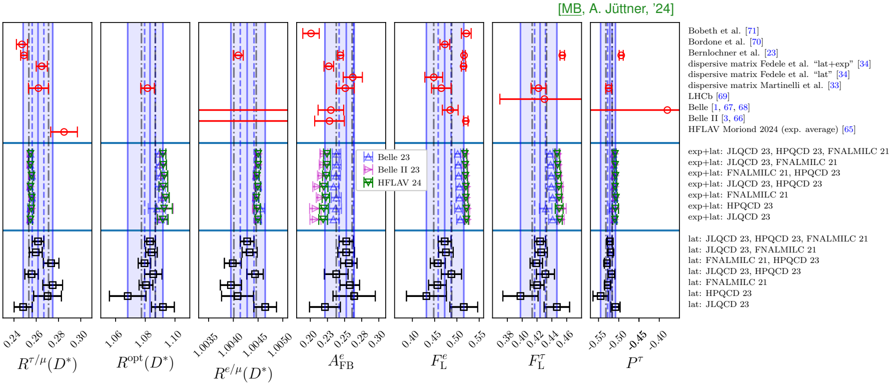
[MB, A. Jüttner, '24]




$$A^\ell_{\rm FB} = \frac{\int_0^1 - \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}{\int_0^1 + \int_{-1}^0 d\cos\theta_\ell d\Gamma/d\cos\theta_\ell}$$

- The combined lattice + experimental precision makes it possible to study the differences in the shape
- It is clear that there is a distinct difference between JLQCD and FNAL/MILC+HPQCD
- Difficult to understand what is going on, JLQCD errors are also a bit larger

# Integrated observables



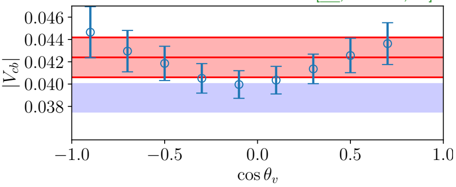

- Significant scatter between various combinations of lattice results
  - We apply a systematic error to account for the spread
- Consistent scatter of the experimental results independently of the lattice information

$\Rightarrow$ see also: Fedele et al, '23

# $|V_{cb}|$ extraction

$$|V_{cb}|_{\alpha,i} = \left(\Gamma_{\rm exp}\left[\frac{1}{\Gamma}\frac{d\Gamma}{d\alpha}\right]^{(i)}_{\rm exp} \Big/ \left[\frac{d\Gamma_0}{d\alpha}(\mathbf{a})\right]^{(i)}_{\rm lat}\right)^{1/2}$$

[MB, A. Jüttner, '24]




Blue band

- Frequentist fit $p-$value $\sim 0\%$
- Affected by d'Agostini Bias

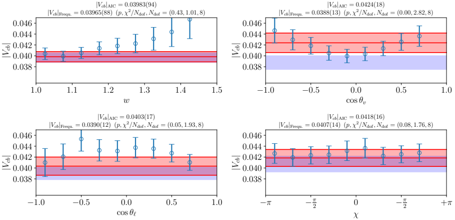
Red band

- Frequentist fit $p-$value $\sim 0\%$
- Akaike-Information-Criterion analysis: average over all possible fits with at least two data points and then weighted average

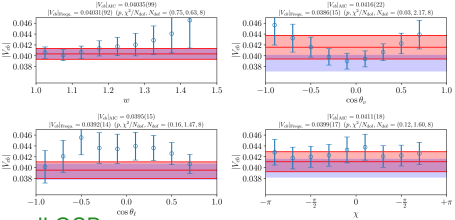
$$w_{\{\alpha,i\}} = \mathcal{N}^{-1}\exp\left(-\frac{1}{2}(\chi^2_{\{\alpha,i\}} - 2N_{{\rm dof},\{\alpha,i\}})\right) \qquad \text{where} \quad \mathcal{N} = \sum_{\text{sets }\{\alpha,i\}} w_{\rm set}$$

$$|V_{cb}| = \langle|V_{cb}|\rangle \equiv \sum_{\text{sets }\{\alpha,i\}} w_{\rm set}|V_{cb}|_{\rm set}$$

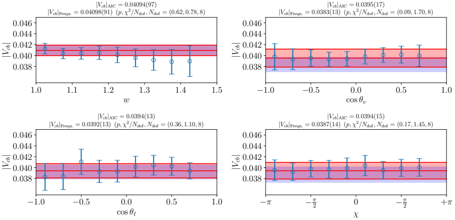
## FNAL/MILC




## HPQCD




## JLQCD

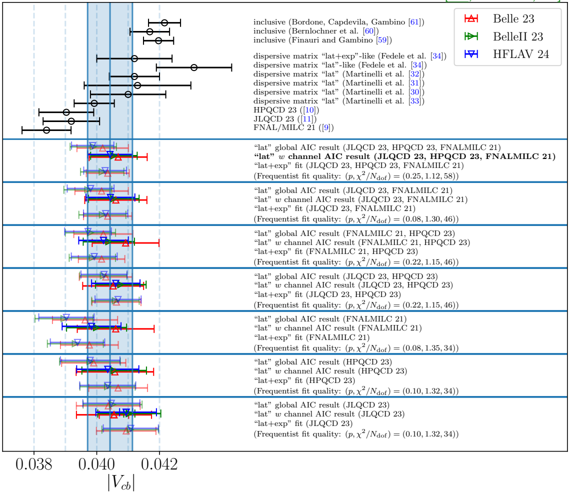



- The AIC nicely reduces the d'Agostini bias
- Some lattice data behave strangely
- Would it be safer to discard the angular distributions?
- Combining the three lattice datasets doesn't help, shape driven by FNAL/MILC and HPQCD

see also G. Martinelli, S. Simula, L. Vittorio, '23,'24

# $|V_{cb}|$ - Summary

[MB, A. Jüttner, '24]



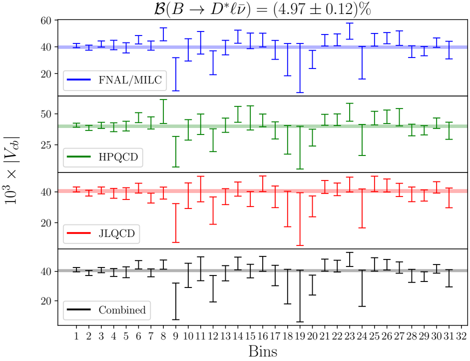

- Residual $2\sigma$ difference with inclusive
- The AIC produces slightly larger uncertainties, overall all results are quite consistent

# What about other datasets?

[MB, O. Heald, A. Jüttner, in preparation]

Belle angular observables

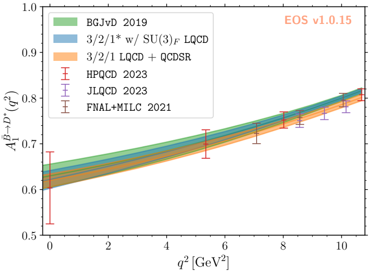
$$\frac{\mathrm{d}\Gamma(\bar{B}\to D^*\ell\bar{\nu}_\ell)}{\mathrm{d}w\,\mathrm{d}\cos\theta_\ell\,\mathrm{d}\cos\theta_\mathrm{V}\,\mathrm{d}\chi} = \frac{2G_\mathrm{F}^2\eta_\mathrm{EW}^2|V_\mathrm{cb}|^2 m_B^4 m_{\mathrm{D}^*}}{2\pi^4}\times\Big(J_{1s}\sin^2\theta_\mathrm{V}+J_{1c}\cos^2\theta_\mathrm{V}$$
$$+(J_{2s}\sin^2\theta_\mathrm{V}+J_{2c}\cos^2\theta_\mathrm{V})\cos 2\theta_\ell + J_3\sin^2\theta_\mathrm{V}\sin^2\theta_\ell\cos 2\chi$$
$$+J_4\sin 2\theta_V\sin 2\theta_\ell\cos\chi + J_5\sin 2\theta_V\sin\theta_\ell\cos\chi + (J_{6s}\sin^2\theta_\mathrm{V}+J_{6c}\cos^2\theta_\mathrm{V})\cos\theta_\ell$$
$$+J_7\sin 2\theta_V\sin\theta_\ell\sin\chi + J_8\sin 2\theta_V\sin 2\theta_\ell\sin\chi + J_9\sin^2\theta_\mathrm{V}\sin^2\theta_\ell\sin 2\chi\Big)\,.$$

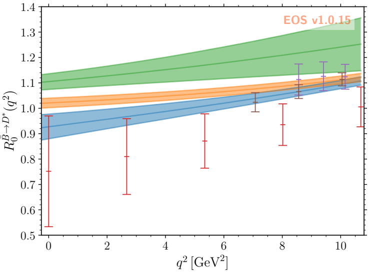


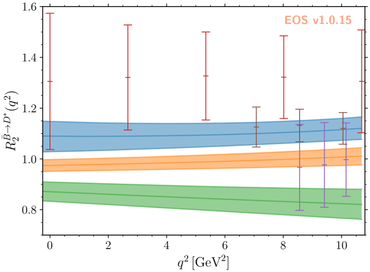

**Results are consistent with our previous analysis**

# Can a different parametrisation help?

[MB, N. Gubernari, M. Jung D. van Dyk, '25]



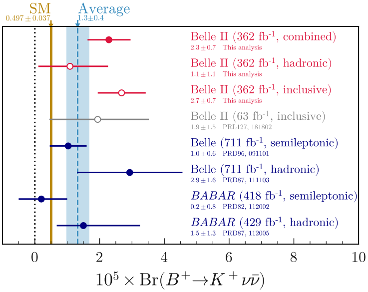



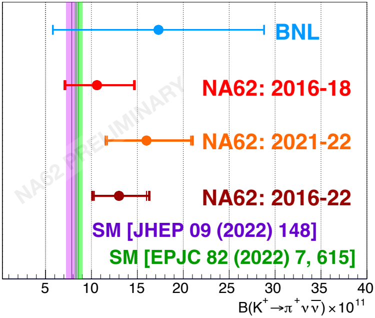



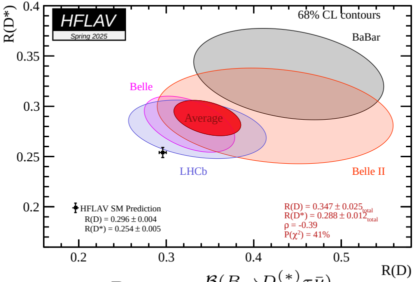

**same pattern of deviations observed**

# Outlook and prospects on BSM

# What about BSM?

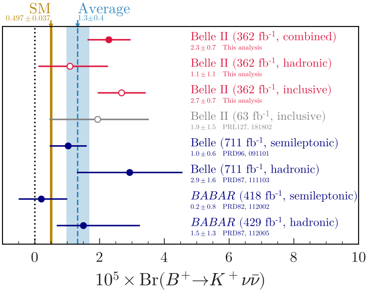


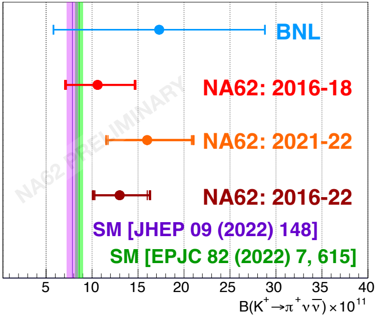



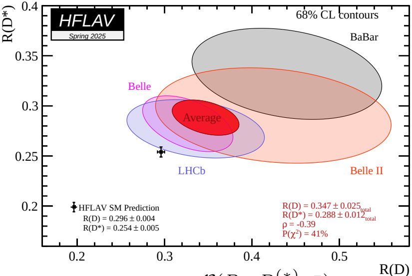




$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}$$

# What about BSM?




$$R_{D^{(*)}} = \frac{\mathcal{B}(B \to D^{(*)} \tau \bar{\nu})}{\mathcal{B}(B \to D^{(*)} \ell \bar{\nu})}$$

**Can we accommodate all these deviations together?**

# The EFT approach

- Since we haven't observed any clear sign of NP yet at low energies, we can work in an EFT context
  - $\Rightarrow$ Agnostic of the nature of new physics, describe more than one UV model with the same operators
  - $\Rightarrow$ Try to derive model-independent bounds
- We use the SMEFT
  - $\Rightarrow$ Build all possible operators with SM fields and respecting SM symmetries
- The remnant of high-energy new physics is contained in the Wilson Coefficients
  - $\Rightarrow$ With flavour, we have a lot of free degrees of freedom
  - $\Rightarrow$ We need a criterium to infer their magnitude

# The $U(2)^n$ symmetry for BSM

$$q_{3L} \sim (\mathbf{1}, \mathbf{1}) \qquad \ell_{3L} \sim (\mathbf{1}, \mathbf{1})$$

$$Q_L = (Q_L^1, Q_L^2) \sim (\bar{\mathbf{2}}, \mathbf{1}) \qquad L_L = (\ell_L^1, \ell_L^2) \sim (\mathbf{1}, \bar{\mathbf{2}})$$

Unbroken $U(2)^5$

$$Y_u = y_t \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Longrightarrow \begin{array}{l} \bar{q}_{3L}\Gamma q_{3L} \ \checkmark \\ \bar{q}_{3L}\Gamma Q \ \text{✗} \end{array}$$

# The $U(2)^n$ symmetry for BSM

$$\begin{aligned} q_{3L} &\sim (\mathbf{1},\mathbf{1}) \\ Q_L = (Q_L^1, Q_L^2) &\sim (\bar{\mathbf{2}},\mathbf{1}) \\ V_q &\sim (\mathbf{2},\mathbf{1}) \end{aligned} \qquad \begin{aligned} \ell_{3L} &\sim (\mathbf{1},\mathbf{1}) \\ L_L = (\ell_L^1, \ell_L^2) &\sim (\mathbf{1},\bar{\mathbf{2}}) \\ V_\ell &\sim (\mathbf{1},\mathbf{2}) \end{aligned}$$

Unbroken $U(2)^5$

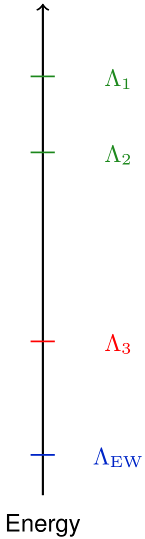
$$Y_u = y_t \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Longrightarrow \quad \bar{q}_{3L}\Gamma q_{3L} \ \checkmark \qquad \bar{q}_{3L}\Gamma Q \ \text{✗}$$

Soft symmetry breaking

$$Y_u = y_t \begin{pmatrix} \Delta & V_q \\ 0 & 1 \end{pmatrix} \Longrightarrow \quad \bar{q}_{3L}\Gamma q_{3L} \ \checkmark \qquad \bar{q}_{3L}\Gamma (V_q Q) \ \checkmark$$

# Flavour Non-Universal New Physics

Dvali, Shifman, '00
Panico, Pomarol, '16
MB, Cornella, Fuentes-Martin, Isidori '17
Allwicher, Isidori, Thomsen '20
Barbieri, Cornella, Isidori, '21
Davighi, Isidori '21




**Basic idea:**

- 1st and 2nd have small masses and small couplings to NP because they are generated by dynamics at a heavier scale
- 3rd generation is linked to dynamics at lower scales and has stronger couplings

**Flavour deconstruction**:
fermion families interact with different gauge groups and flavour hierarchies emerge as accidental symmetries

# Flavour Non-Universal New Physics

Dvali, Shifman, '00
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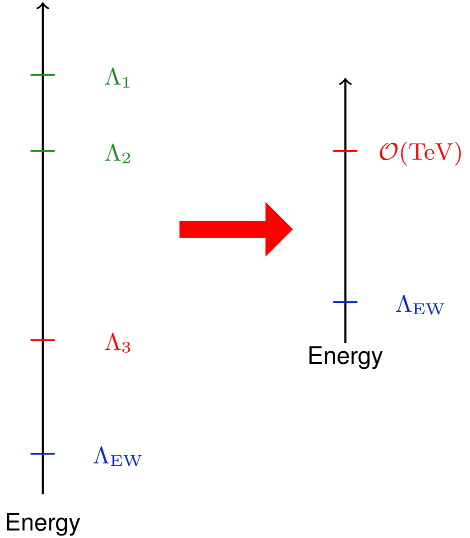
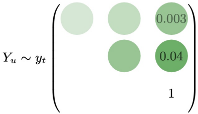



$\mathrm{G}^{12} \times \mathrm{G}^3$

$U(2)^n$ limit

NP coupled to 3rd gen only

broken $U(2)^n$



# Which operators?

$$Q_{\ell q}^{\pm} = (\bar{q}_L^3 \gamma^\mu q_L^3)(\bar{\ell}_L^3 \gamma_\mu \ell_L^3) \pm (\bar{q}_L^3 \gamma^\mu \sigma^a q_L^3)(\bar{\ell}_L^3 \gamma_\mu \sigma^a \ell_L^3) \quad Q_S = (\bar{\ell}_L^3 \tau_R)(\bar{b}_R q_L^3)$$

# Which operators?

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$SU(2)$ singlet

$SU(2)$ triplet

scalar

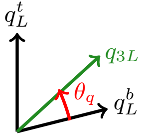
# Which operators?

$$Q^{\pm}_{\ell q} = (\bar{q}^3_L \gamma^\mu q^3_L)(\bar{\ell}^3_L \gamma_\mu \ell^3_L) \pm (\bar{q}^3_L \gamma^\mu \sigma^a q^3_L)(\bar{\ell}^3_L \gamma_\mu \sigma^a \ell^3_L) \qquad Q_S = (\bar{\ell}^3_L \tau_R)(\bar{b}_R q^3_L)$$

$SU(2)$ singlet — $SU(2)$ triplet — scalar

- Only left-handed neutrinos
- $q_{3L} \equiv q^b_L + \hat{V} \cdot Q_L$

$$q^b_L = \begin{pmatrix} V^*_{j3} u^j_L \\ b_L \end{pmatrix} \qquad Q^i_L = \begin{pmatrix} V^*_{ji} u^j_L \\ d^i_L \end{pmatrix} \qquad \hat{V}_q \equiv -\epsilon V_{ts} \begin{pmatrix} \kappa V_{td}/V_{ts} \\ 1 \end{pmatrix}$$



$$
\begin{tikzcd}
 & (\bar{q}_{3L}\gamma_\mu q_{3L})(\bar{\ell}_{3L}\gamma^\mu \ell_{3L}) \arrow[dl, "\text{CKM or } \hat{V}"'] \arrow[dr, "\hat{V}"] & \\
(\bar{c}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \tau_L) & & (\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau) \arrow[d, "\text{two insertion of } \hat{V}"] \\
 & & (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau)
\end{tikzcd}
$$

$$\begin{tikzcd} & (\bar{q}_{3L}\gamma_\mu q_{3L})(\bar{\ell}_{3L}\gamma^\mu \ell_{3L}) \arrow[dl, "\text{CKM or } \hat{V}"'] \arrow[dr, "\hat{V}"] & \\ (\bar{c}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \tau_L) & & (\bar{s}_L\gamma_\mu b_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau) \arrow[d, "\text{two insertion of } \hat{V}"] \\ & & (\bar{s}_L\gamma_\mu d_L)(\bar{\nu}_\tau\gamma^\mu \nu_\tau) \end{tikzcd}$$

**Correlations among all these modes is essential to prove NP scenarios**

# What do we expect in the SMEFT?

$$\mathcal{L}_{\rm EFT} \supset \boxed{\frac{C_{bc\tau\tau}}{\Lambda^2}}(\bar{b}_L\gamma_\nu c_L)(\bar{\nu}_\tau\gamma^\mu\tau_L)$$

From $U(2)^n \Rightarrow C_{bc\tau\tau} \sim V_{cb}\mathcal{O}(1)$

From $R_{D^{(*)}} \Rightarrow \Lambda \sim \mathcal{O}(\mathrm{TeV})$

Using $SU(2)_L$ invariance, we have

$$\mathcal{L}_{\rm EFT} \supset \frac{C_{ij\tau\tau}}{\Lambda^2}(\bar{d}^i_L\gamma_\nu d^j_L)(\bar{\nu}_\tau\gamma^\mu\nu_\tau)$$

$B^+ \to K^+\nu\bar{\nu}$

From $U(2)^n \Rightarrow C_{bs\tau\tau} \sim V_{cb}\mathcal{O}(1)$

$K^+ \to \pi^+\nu\bar{\nu}$

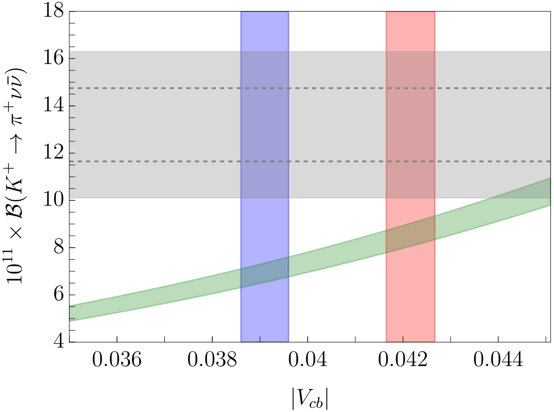
From $U(2)^n \Rightarrow C_{sd\tau\tau} \sim 10^{-1}V_{cb}\mathcal{O}(1)$

# On the $V_{cb}$ puzzle (again)

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}|^2 \left[ (\bar{\rho} - 1) \left( 1 - \frac{\lambda^2}{2} \right) + i\bar{\eta} \left( 1 + \frac{\lambda^2}{2} \right) \right] + \mathcal{O}(\lambda^4)$$

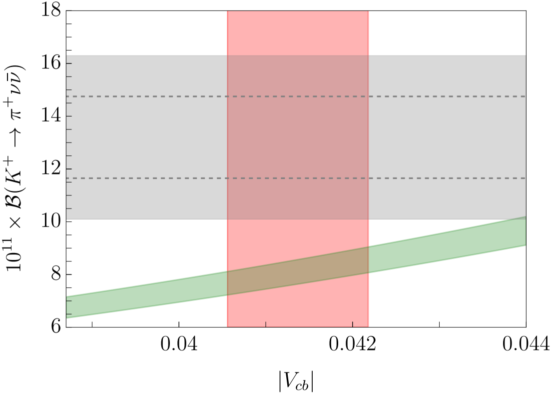
# On the $V_{cb}$ puzzle (again)

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}|^2 \left[ (\bar{\rho} - 1) \left( 1 - \frac{\lambda^2}{2} \right) + i\bar{\eta} \left( 1 + \frac{\lambda^2}{2} \right) \right] + \mathcal{O}(\lambda^4)$$



# On the $V_{cb}$ puzzle (again)

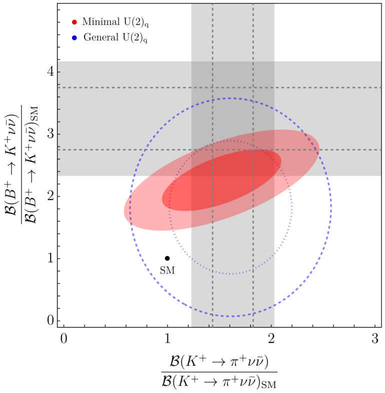
$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}) \propto |\lambda_{ts}|^2 \quad \lambda_{ts} \equiv \lambda |V_{cb}|^2 \left[ (\bar{\rho} - 1)\left(1 - \frac{\lambda^2}{2}\right) + i\bar{\eta}\left(1 + \frac{\lambda^2}{2}\right)\right] + \mathcal{O}(\lambda^4)$$

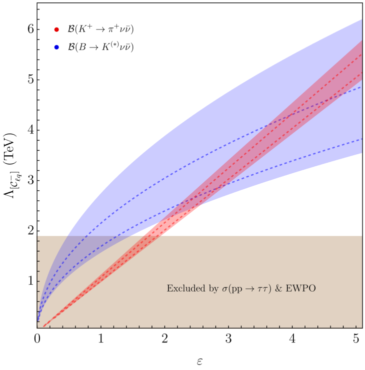



$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu})^{\rm SM} = (8.09 \pm 0.63) \times 10^{-11}$$

$$\mathcal{B}(K_L \to \pi^0 \nu\bar{\nu})^{\rm SM} = (2.58 \pm 0.30) \times 10^{-11}$$

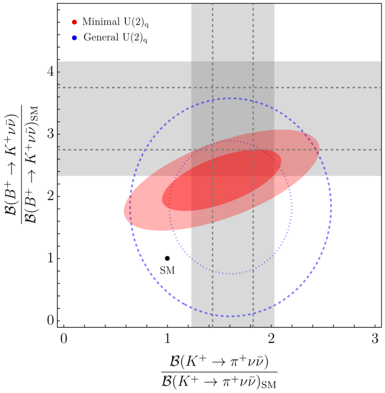
[L. Allwicher, MB, G. Isidori, G. Piazza, A. Stanzione, '24]

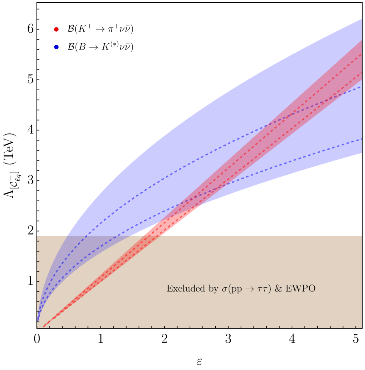



- The $U(2)^n$ symmetry creates a natural link between all this observables
- The complementarity between low- and high-energy data is useful to probe the parameter space

[L. Allwicher, MB, G. Isidori, G. Piazza, A. Stanzione, '24]

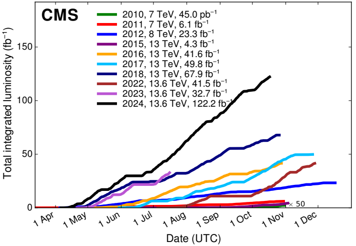


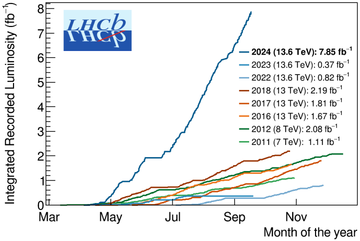




- The $U(2)^n$ symmetry creates a natural link between all this observables
- The complementarity between low- and high-energy data is useful to probe the parameter space

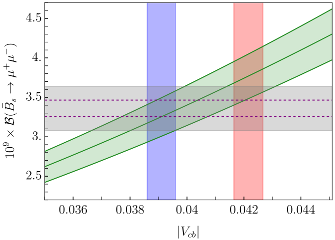
**Further data is essential!**

# Experimental prospects




- Experimental facilities are delivering unprecedented datasets
- The experimental reach supported by new analysis techniques already superseded the expectations
- Theoretical advancements are crucial for achieving greater precision in understanding flavor processes and evaluating potential signs of new physics

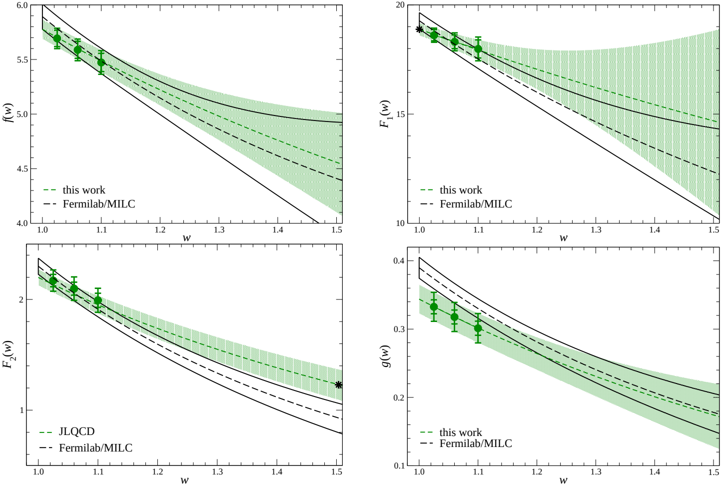


# Summary

- Flavour physics has the potential to test for possible hints of extensions of the SM
- The main showstopper is the theoretical precision
- A lot of progress has been made, but a few pivotal puzzles persist
- There are hints for possible BSM directions, but more efforts and more data are needed to shed light on their nature

# Appendix

# Compatiblity of lattice data




- Similar results with HPQCD
- There are some differences in the slopes
- How good is the compatibility?
- Do the differences yield significant pheno consequences?

# Strategy A

[MB, A. Jüttner, '24]

## Frequentist fit

| $K_f$ | $K_{\mathcal{F}_1}$ | $K_{\mathcal{F}_2}$ | $K_g$ | $a_{g,0}$ | $a_{g,1}$ | $a_{g,2}$ | $a_{g,3}$ | $p$ | $\chi^2/N_{\rm dof}$ | $N_{\rm dof}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 0.03138(87) | -0.059(24) | - | - | 0.95 | 0.62 | 30 |
| 3 | 3 | 3 | 3 | 0.03131(87) | -0.046(36) | -1.2(1.8) | - | 0.90 | 0.67 | 26 |
| 4 | 4 | 4 | 4 | 0.03126(87) | -0.017(48) | -3.7(3.3) | 49.9(53.6) | 0.79 | 0.75 | 22 |

- good fit quality
- lattice data are compatible
- no unitarity

## Bayesian Fit

| $K_f$ | $K_{\mathcal{F}_1}$ | $K_{\mathcal{F}_2}$ | $K_g$ | $a_{g,0}$ | $a_{g,1}$ | $a_{g,2}$ | $a_{g,3}$ |
|---|---|---|---|---|---|---|---|
| 2 | 2 | 2 | 2 | 0.03018(76) | -0.101(21) | - | - |
| 3 | 3 | 3 | 3 | 0.03034(78) | -0.087(24) | -0.34(45) | - |
| 4 | 4 | 4 | 4 | 0.03035(77) | -0.089(23) | -0.27(41) | -0.04(45) |

- unitarity regulates higher orders
- truncation dependent

# What's the problem for BSM?

$B$-physics

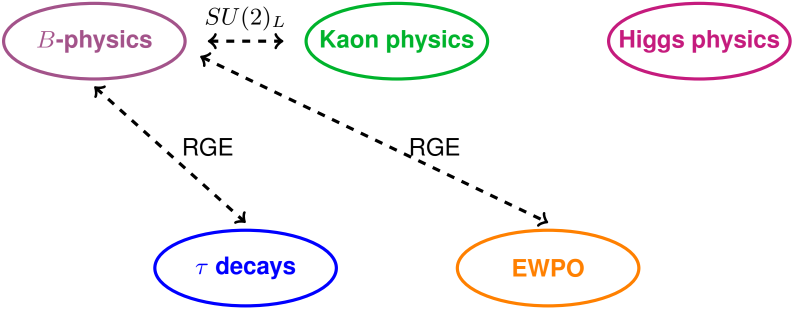
Kaon physics

Higgs physics

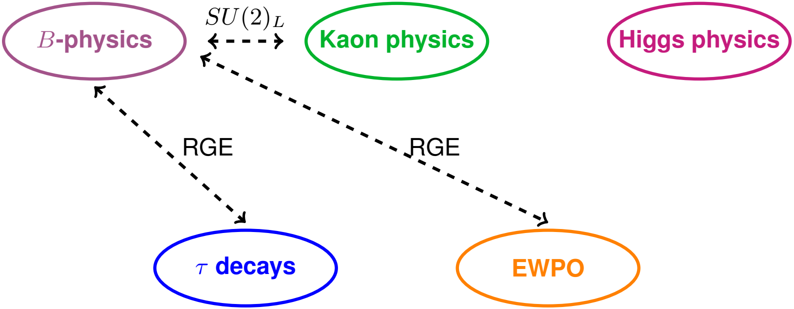
$\tau$ decays

EWPO

# What's the problem for BSM?

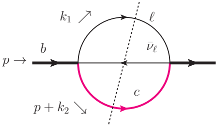


# What's the problem for BSM?



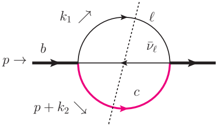

**How to satisfy all the constraints at the same time?**

# Theory framework




$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T \left\{ \mathcal{H}^\dagger_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} |B(p)\rangle$$

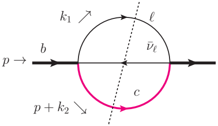
# Theory framework




$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p) | T \left\{ \mathcal{H}^\dagger_{\mathrm{eff}}(x) \mathcal{H}_{\mathrm{eff}}(0) \right\} | B(p) \rangle$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

# Theory framework

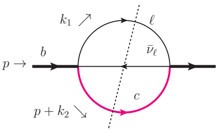



$$\Gamma = \frac{1}{m_B} \mathrm{Im} \int d^4x \langle B(p)| T\left\{\mathcal{H}^\dagger_{\rm eff}(x)\mathcal{H}_{\rm eff}(0)\right\}|B(p)\rangle$$

$$\sum_{n,i} \frac{1}{m_b^n} \mathcal{C}_{n,i} \mathcal{O}_{n+3,i}$$

- The Wilson coefficients are calculated perturbatively
- The matrix elements $\langle B(p)|\mathcal{O}_{n+3,i}|B(p)\rangle$ are non perturbative
  - $\Rightarrow$ They need to be determined with non-perturbative methods, e.g. Lattice QCD
  - $\Rightarrow$ They can be extracted from data
  - $\Rightarrow$ With large $n$, large number of operators

# Theory framework

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**loss of predictivity**

# Theory framework for $B \to X_c \ell \bar{\nu}$

Double expansion in $1/m$ and $\alpha_s$

$$\Gamma_{sl} = \Gamma_0 f(\rho) \Big[ 1 + a_1 \left(\frac{\alpha_s}{\pi}\right) + a_2 \left(\frac{\alpha_s}{\pi}\right)^2 + a_3 \left(\frac{\alpha_s}{\pi}\right)^3 - \left(\frac{1}{2} - p_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_\pi^2}{m_b^2}$$
$$+ \left(g_0 + g_1 \left(\frac{\alpha_s}{\pi}\right)\right) \frac{\mu_G^2(m_b)}{m_b^2} + d_0 \frac{\rho_D^3}{m_b^3} - g_0 \frac{\rho_{LS}^3}{m_b^3} + \dots \Big]$$

- The coefficients are known
- $\mu_\pi^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v (i\vec{D})^2 b_v | B \rangle_\mu$ $\qquad$ $\mu_G^2(\mu) = \frac{1}{2m_B} \langle B | \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v | B \rangle_\mu$

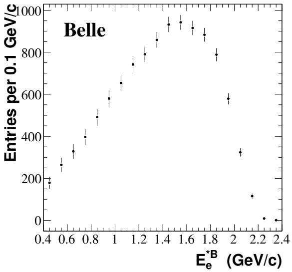
  $\Rightarrow$ No Lattice QCD determinations are available yet

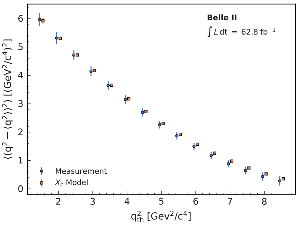
- Use for the first time of $\alpha_s^3$ corrections [Fael, Schönwald, Steinhauser, '20]
- Ellipses stands for higher orders

  $\Rightarrow$ proliferation of terms and loss of predictivity

# How do we constrain the hadronic parameters?

We need information from kinematic distributions



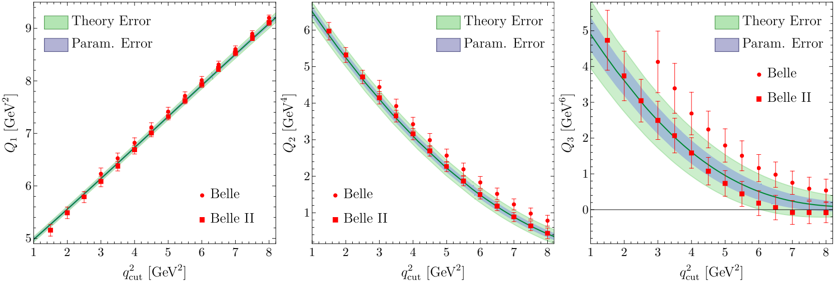




- Traditional method: Extract the hadronic parameters from moments of kinematic distributions in $E_l$ and $M_X$
- New idea: Use $q^2$ moments to exploit the reduction of free parameters due to RPI [Fael, Mannel, Vos, '18, Bernlochner et al, '22]
- Measurements of branching fractions are needed and are at the moment quite old
- Can we do it on the lattice? [Gambino, Hashimoto, '20, '23, Hashimoto, Jüttner, et al, '23]

# Global fit

[MB, Capdevila, Gambino, '21, Finauri, Gambino, '23]

| | $m_b^{\rm kin}$ | $\overline{m}_c$ | $\mu_\pi^2$ | $\mu_G^2$ | $\rho_D^3$ | $\rho_{LS}^3$ | $10^2{\rm BR}_{c\ell\nu}$ | $10^3\lvert V_{cb}\rvert$ | $\chi^2_{\rm min}(/{\rm dof})$ |
|---|---|---|---|---|---|---|---|---|---|
| without | 4.573 | 1.092 | 0.477 | 0.306 | 0.185 | −0.130 | 10.66 | 42.16 | 22.3 |
| $q^2$-moments | 0.012 | 0.008 | 0.056 | 0.050 | 0.031 | 0.092 | 0.15 | 0.51 | 0.474 |
| Belle II | 4.573 | 1.092 | 0.460 | 0.303 | 0.175 | −0.118 | 10.65 | 42.08 | 26.4 |
| | 0.012 | 0.008 | 0.044 | 0.049 | 0.020 | 0.090 | 0.15 | 0.48 | 0.425 |
| Belle | 4.572 | 1.092 | 0.434 | 0.302 | 0.157 | −0.100 | 10.64 | 41.96 | 28.1 |
| | 0.012 | 0.008 | 0.043 | 0.048 | 0.020 | 0.089 | 0.15 | 0.48 | 0.476 |
| Belle & | 4.572 | 1.092 | 0.449 | 0.301 | 0.167 | −0.109 | 10.65 | 42.02 | 41.3 |
| Belle II | 0.012 | 0.008 | 0.042 | 0.048 | 0.018 | 0.089 | 0.15 | 0.48 | 0.559 |



# Two calculation approaches

## 1. Splitting Functions

$$\left(\frac{d\Gamma}{dy}\right)^{(1)} = \frac{\alpha}{2\pi}\,\bar{L}_{b/e}\int_y^{1-\rho}\frac{dx}{x}\,P_{ee}^{(0)}\left(\frac{y}{x}\right)\left(\frac{d\Gamma}{dx}\right)^{(0)}$$

$\log(m_b^2/m_e^2)$ plus distribution

- Correction vanishes for the inclusive branching fraction
- Suitable for evaluating $\mathcal{O}(\alpha^2)$ and $\mathcal{O}(\alpha/m_b^n)$ corrections

## 2. Full $\mathcal{O}(\alpha)$ corrections

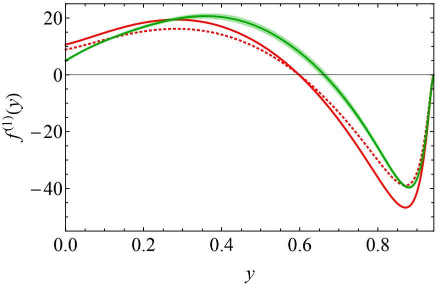
- Access all corrections, not only the one that factorise
- Real corrections are computationally expensive
  - $\Rightarrow$ Cuba library employed to carry out the 4-body integration
  - $\Rightarrow$ Phase space splitting used to reduce the size of the integrands

# Lepton Energy spectrum

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- We compute bins in the lepton energy using the full $\mathcal{O}(\alpha)$ calculation
- We compare them to the results given by the splitting functions
- The difference the two calculations for the lepton energy spectrum and obtain a full analytic formula for the radiative corrections

$\Rightarrow$ Relatively small, easy-to-use formula to obtain branching fractions, lepton energy moments w/o cuts




$$f^{(1)}(y) = \frac{\bar{L}_{b/e}}{2}\, f^{(1)}_{\mathrm{LL}}(y) + \Delta f^{(1)}(y)$$

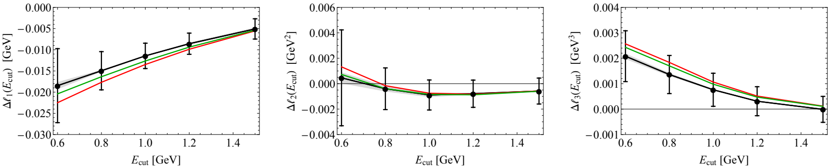
# Comparison with data

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- Babar provides data with and without applying PHOTOS to subtract QED effects

  ⇒ Perfect ground to test our calculations

  ⇒ Not the same for Belle at the moment, could be possible for future analysis

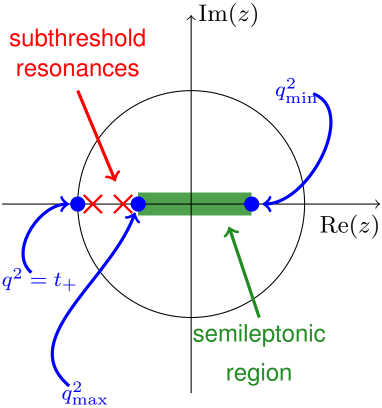



- The moments, since they are normalised, are not affected by the large threshold corrections
- The agreement with BaBar is very good

$$\langle E_\ell^n \rangle = \frac{\int_{E_\ell > E_{\ell,\rm cut}} dE_\ell E_\ell^n \frac{d\Gamma}{dE_\ell}}{\Gamma_{E_\ell > E_{\ell,\rm cut}}}$$

# The $z$-expansion and unitarity

[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]




- in the complex plane form factors are real analytic functions
- $q^2$ is mapped onto the conformal complex variable $z$

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

- $q^2$ is mapped onto a disk in the complex $z$ plane, where $|z(q^2, t_0)| < 1$

$$F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k$$

$$\sum_{k=0}^{n_i} |a_k^i|^2 < 1$$

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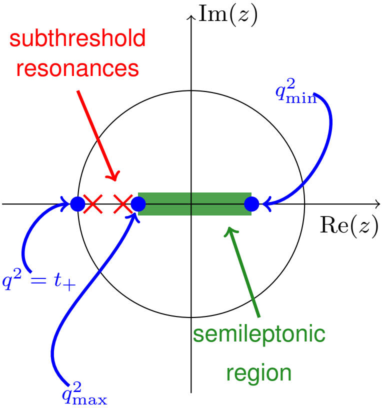
[Boyd, Grinstein, Lebed, '95, Caprini, Lellouch, Neubert, '98]

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$$\boxed{F_i = \frac{1}{P_i(z)\phi_i(z)} \sum_{k=0}^{n_i} a_k^i z^k \qquad \sum_{k=0}^{n_i} |a_k^i|^2 < 1}$$

**BGL**

# How to apply unitarity

- Penalty function in the $\chi^2$ or likelihood [P. Gambino, M. Jung, S. Schacht, '19]

$$\chi^2 \to \chi^2(a_k^i, a_k^i|_{\mathrm{data}}) + w_i \theta\left(\sum_{k=0}^{n_i} |a_k^i|^2 - 1\right)$$

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- Dispersive Matrix Method [M. Di Carlo, G. Martinelli, M. Naviglio, F. Sanfilippo, S. Simula, L. Vittorio, '21] [G. Martinelli, S. Simula, L. Vittorio, '21,'23]

$$\mathbf{M} = \begin{pmatrix} \chi & \phi f & \phi_1 f_1 & \phi_2 f_2 & \dots & \phi_N f_N \\ \phi f & \frac{1}{1-z^2} & \frac{1}{1-z z_1} & \frac{1}{1-z z_2} & \dots & \frac{1}{1-z z_N} \\ \phi_1 f_1 & \frac{1}{1-z_1 z} & \frac{1}{1-z_1^2} & \frac{1}{1-z_1 z_2} & \dots & \frac{1}{1-z_1 z_N} \\ \phi_2 f_2 & \frac{1}{1-z_2 z} & \frac{1}{1-z_2 z_1} & \frac{1}{1-z_2^2} & \dots & \frac{1}{1-z_2 z_N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \phi_N f_N & \frac{1}{1-z_N z} & \frac{1}{1-z_N z_1} & \frac{1}{1-z_N z_2} & \dots & \frac{1}{1-z_N^2} \end{pmatrix}$$

$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \le f_0 \le \beta + \sqrt{\gamma}$$

# How to apply unitarity

- Penalty function in the $\chi^2$ or likelihood [P. Gambino, M. Jung, S. Schacht, '19]

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$$\det \mathbf{M} > 0 \Rightarrow \beta - \sqrt{\gamma} \le f_0 \le \beta + \sqrt{\gamma}$$

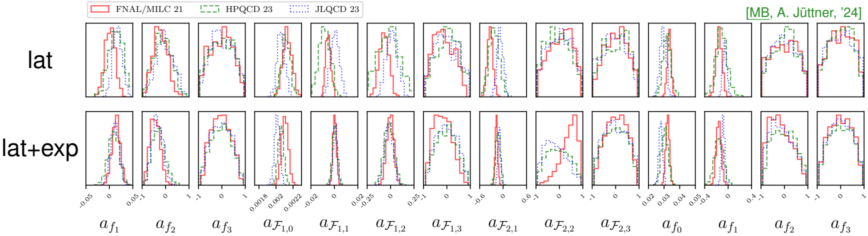
- Bayesian inference [J. Flynn, A. Jüttner, T. Tsang, '23]

$$\theta(1 - |\mathbf{a}|^2)$$

$$\langle g(\mathbf{a}) \rangle = \mathcal{N} \int d\mathbf{a}\, g(\mathbf{a})\, \pi(\mathbf{a}|\mathbf{f}, C_{\mathbf{f}}) \pi_{\mathbf{a}}$$

contains the lattice $\chi^2$

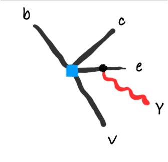
# Posterior distribution




- Small shifts between lattice only and lattice + data
- Higher order coefficients well constrained by unitarity
- $a_{\mathcal{F}_{2,2}}$ has a strange behaviour, maybe kinematic constraints?

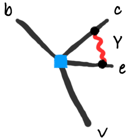
# QED effects for inclusive $V_{cb}$

**1. Collinear logs**: captured by splitting functions



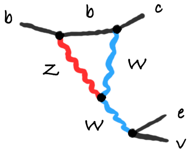

$$\sim \frac{\alpha_e}{\pi} \log^2\left(\frac{m_b^2}{m_e^2}\right)$$

**2. Threshold effects** or Coulomb terms




$$\sim \frac{2\pi\alpha_e}{3}$$

**3. Wilson Coefficient**




$$\sim \frac{\alpha_e}{\pi}\left[\log\left(\frac{M_Z^2}{\mu^2}\right) - \frac{11}{6}\right]$$

# Branching ratio

[Bigi, MB, Gambino, Haisch, Piccione, '23]

- The total branching ratio is not affected by large logs due to KLN theorem
- The large corrections are from the Wilson Coefficient and the threshold effects

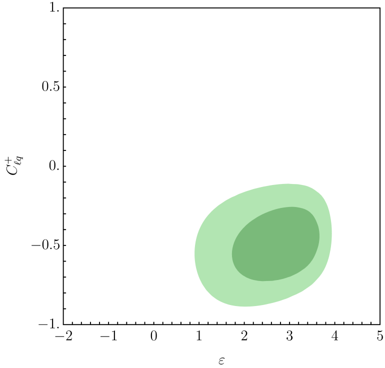
$$\frac{\Gamma}{\Gamma^{(0)} g(\rho)} = 1 + \frac{\alpha}{\pi}\left[\ln\left(\frac{M_Z^2}{m_b^2}\right) - \frac{11}{6} + 5.516(14)\right]$$

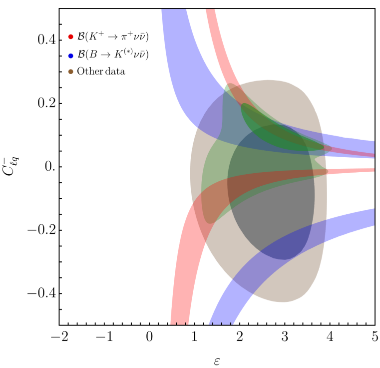
$$= 1 + 1.43\% - 0.44\% + 1.32\% = 1 + 2.31\%$$

Wilson Coefficient

Threshold effects

- Large shift of the branching ratio of the same order of the current error on $V_{cb}$
- How do we incorporate in the current datasets?
  - $\Rightarrow$ Possible only on BaBar data
  - $\Rightarrow$ A systematic approach is needed and foreseen for future experimental analysis
  - $\Rightarrow$ How to evaluate structure-dependent terms is an open task




- EWPO and direct searches
- $R_{D^{(*)}}$
- $B \to K^{(*)}\mu^+\mu^-$

- $B \to K^{(*)}\nu\bar{\nu}$
- $K^+ \to \pi^+\nu\bar{\nu}$