Advanced perturbative predictions for $t\bar{t}H$ and $t\bar{t}W$

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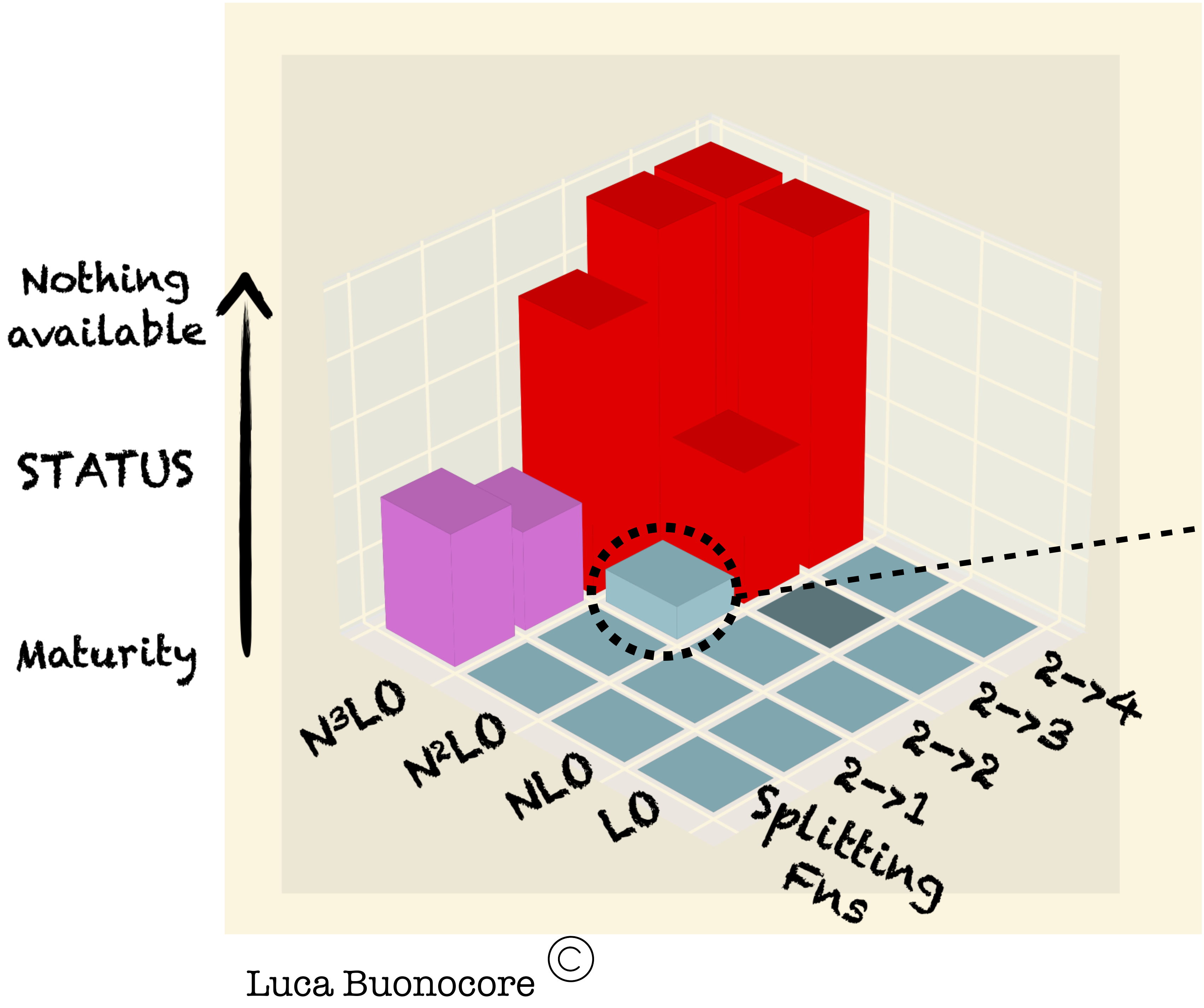
based on Phys.Rev.Lett. 130 (2023), Phys.Rev.Lett. 131 (2023) and arXiv 2411.15340

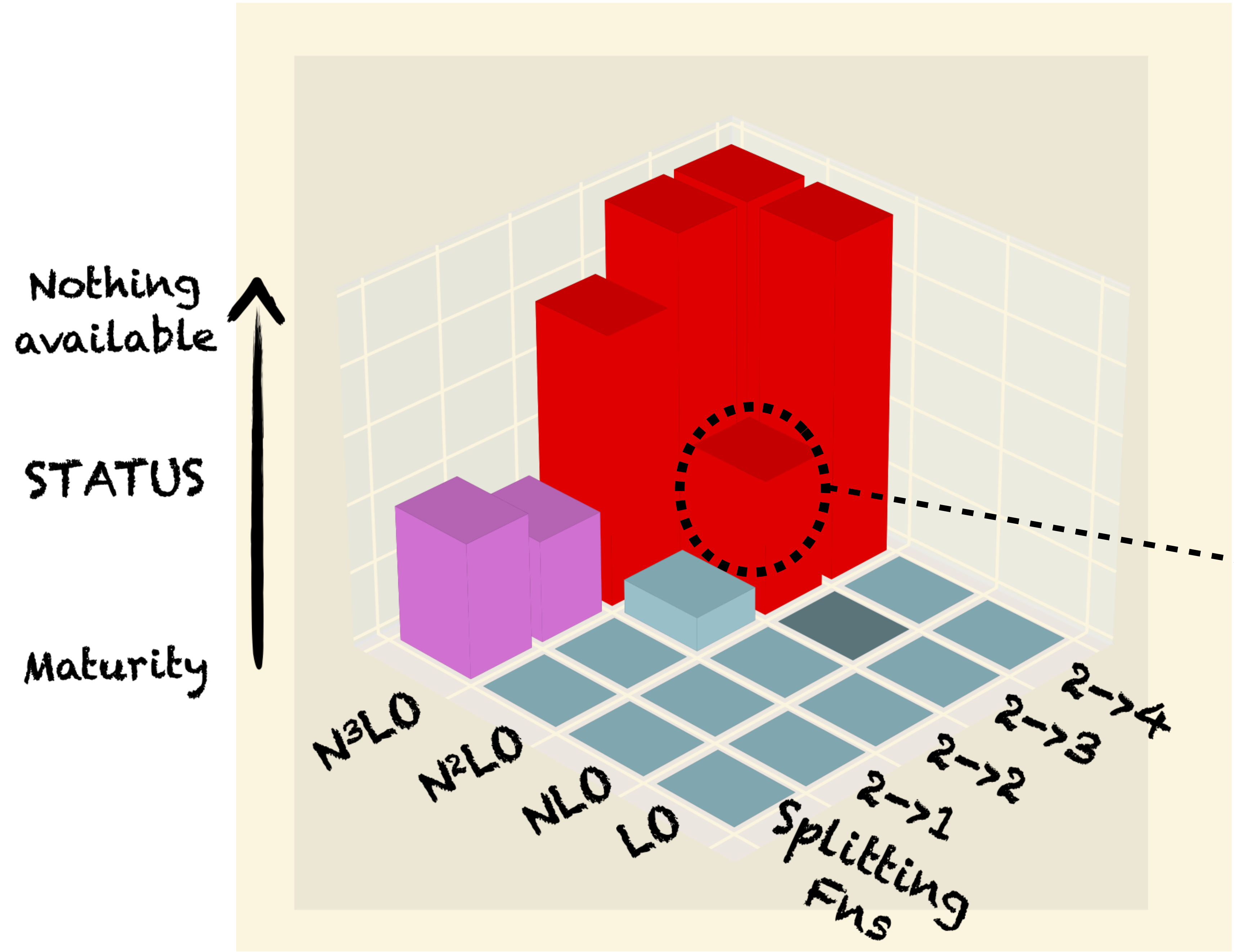
TUM-MPP Collider Phenomenology Seminar — December 17^{th} 2024

The NNLO frontier



- <u>tremendous progress</u> in the past ∼10 years!
- … but still far from reaching the same level of automation, efficiency and generalisation as at NLO
- $2 \to 2$ processes are under control (independent calculations)

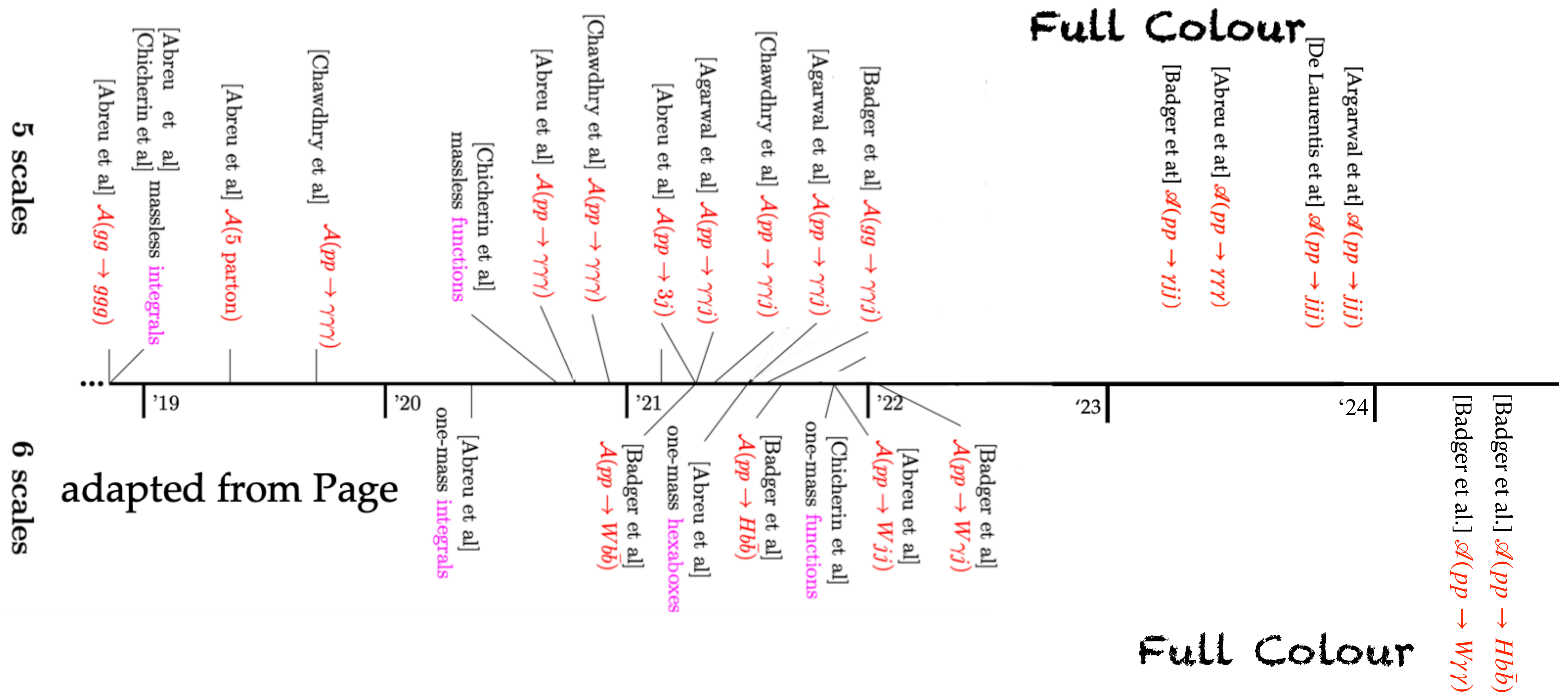
The NNLO frontier



Luca Buonocore ©

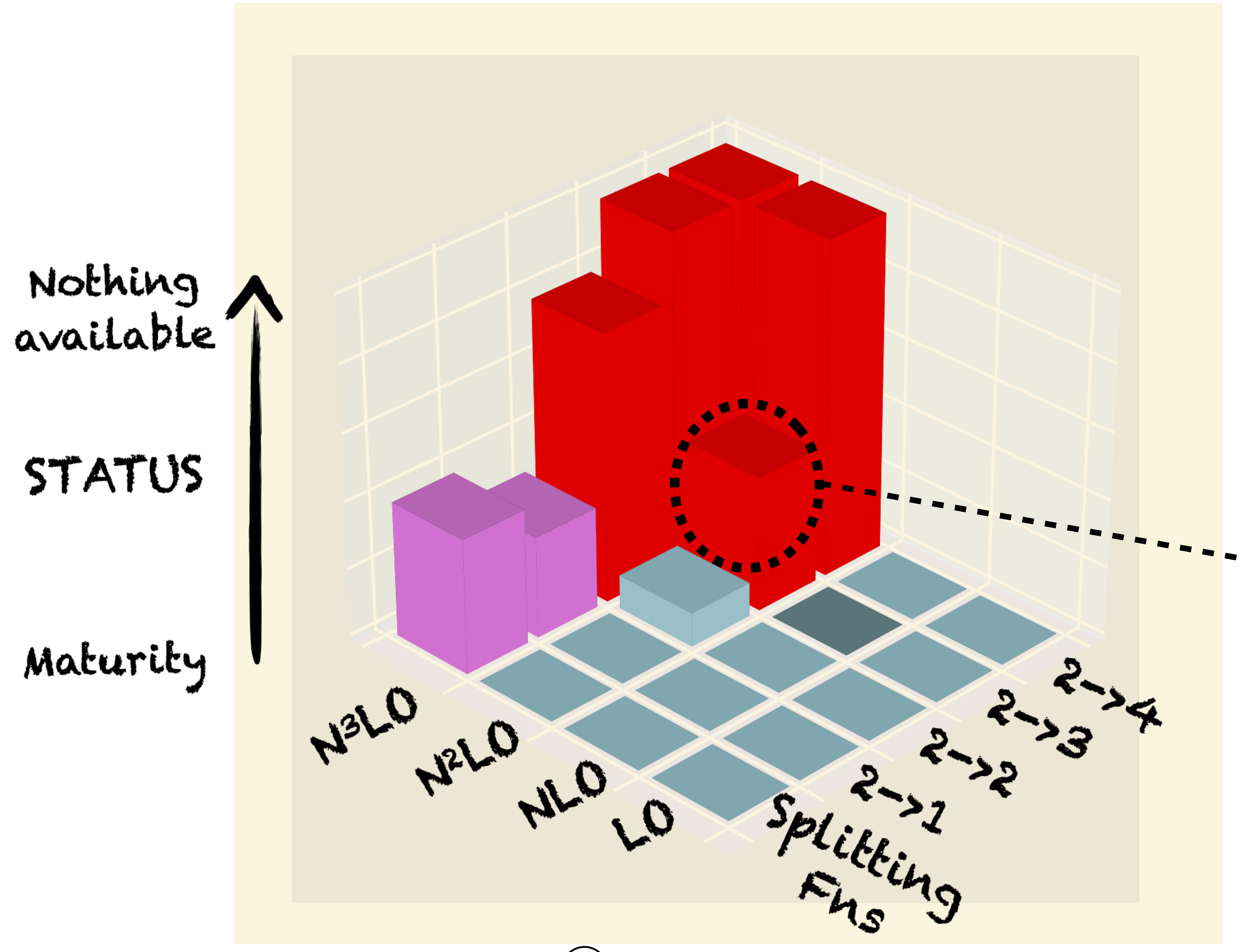
- tremendous progress in the past ∼10 years!
- … but still far from reaching the same level of automation, efficiency and generalisation as at NLO
- great development in two-loop amplitude computations

- 2 → 2 processes are under control (independent calculations)
- **2 → 3** processes represent the **current frontier**
  - massless computations (up to one massive leg) basically done!



adapted from Page

The NNLO frontier



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$pp \to \gamma\gamma\gamma$
[Chawdhry et al. (2019)]
[Kallweit et al. (2020)]

$pp \to \gamma\gamma j$
[Chawdhry et al. (2021)]

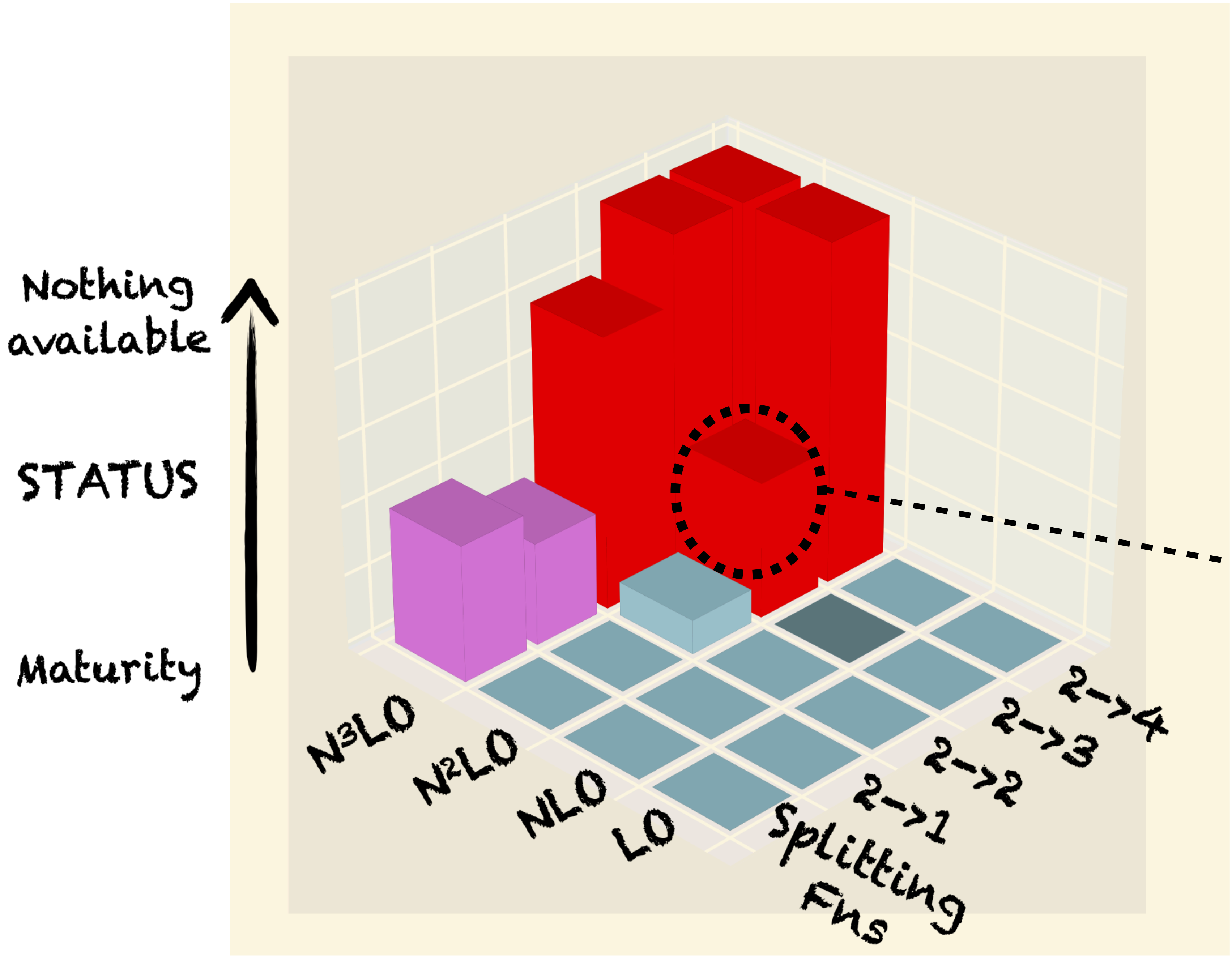
$pp \to jjj$
[Czakon et al. (2021)]

$pp \to Wb\bar{b}$ (5FS)
[Hartanto et al. (2022)]

$pp \to \gamma jj$
[Badger et al. (2023)]

immediate phenomenological applications at the cross section level

The NNLO frontier



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- $2 \to 2$ processes are under control (independent calculations)
- $\mathbf{2 \to 3}$ processes represent the **current frontier**
  - the complexity considerably grows when more external massive legs are present

$pp \to Wb\bar{b}$ (4FS) [Buonocore et al. (2022)]

$pp \to t\bar{t}W$ [Buonocore et al. (2023)]

$pp \to t\bar{t}H$ [Catani et al. (2022)] [Devoto et al. (2024)]

fixed order

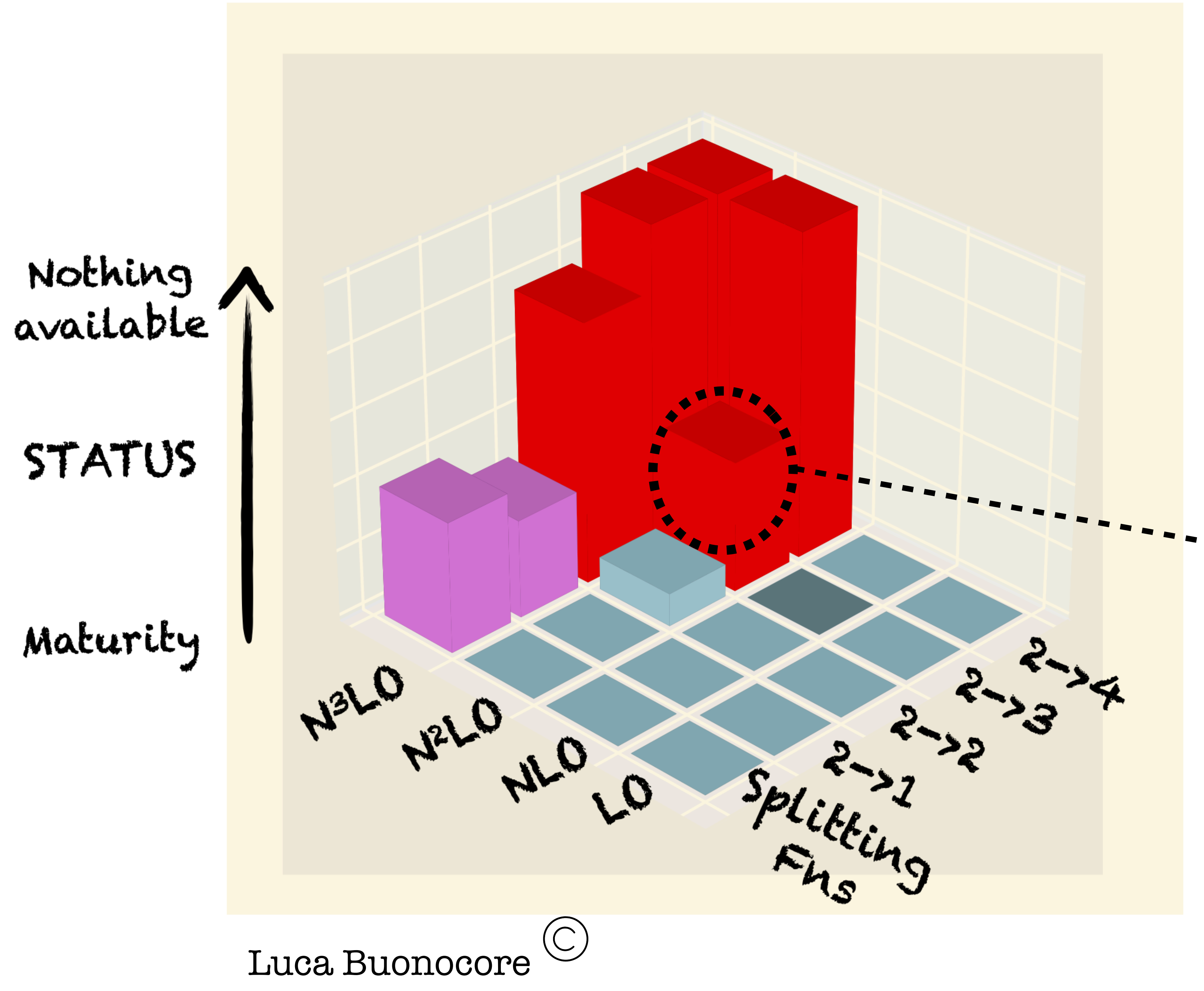
few computations with approximate double-virtual contribution

$pp \to Zb\bar{b}$ (4FS) [Mazzitelli et al. (2024)]

$pp \to Hb\bar{b}$ [Biello et al. (2024)]

fixed order matched with parton shower

The NNLO frontier



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to complete an NNLO computation: crucial to construct an NNLO subtraction/slicing scheme and have all scattering amplitudes available

FOCUS OF THIS TALK!

Our subtraction framework: q_T-slicing

[Catani, Grazzini (2007)]

- cross section for the production of a triggered $Q\bar{Q}V$ final state at N^{k}LO

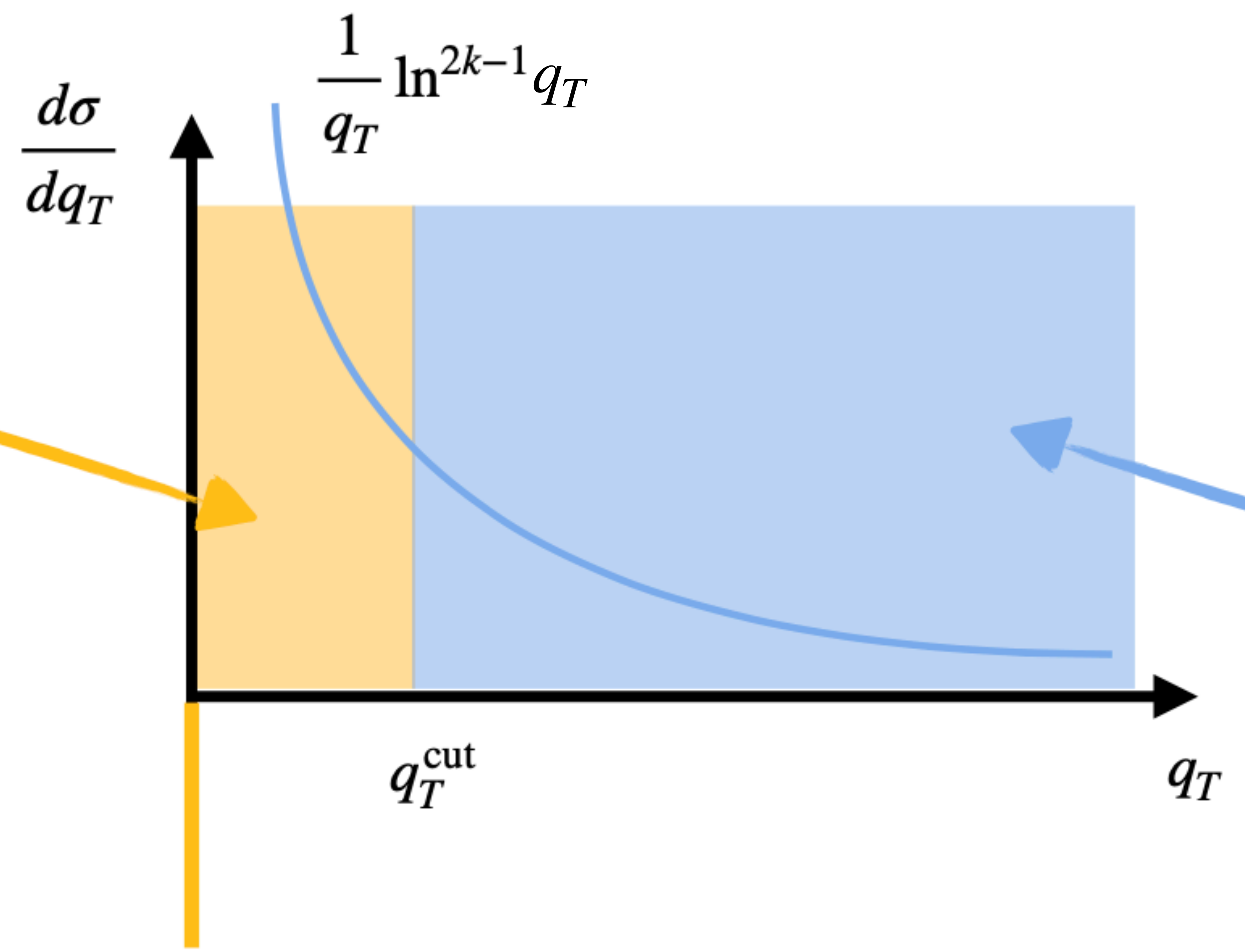
crucial to keep the mass of the heavy quark m_Q

$$\sigma = \int_{<q_T^{\rm cut}} dq_T \frac{d\sigma}{dq_T} + \int_{>q_T^{\rm cut}} dq_T \frac{d\sigma}{dq_T}$$

all emissions are unresolved

we can exploit the QCD factorisation of the matrix elements in the singular soft and/or collinear limits

ingredients from q_T - resummation



1 emission is always resolved

the complexity of the calculation is reduced by 1 order

logarithmic IR sensitivity to the cut

q_T is the transverse momentum of the $Q\bar{Q}V$ system

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + [d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}]_{q_T > q_T^{\rm cut}} + \mathcal{O}((q_T^{\rm cut})^p)$$

missing power corrections

Our subtraction framework: q_T-slicing [Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\mathrm{cut}}} + \mathcal{O}((q_T^{\mathrm{cut}})^p)$$

- ☑ all required **tree-level** and **one-loop** matrix elements are known and can be evaluated with **automated tools** like OpenLoops2 [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]

- ☑ the remaining NLO-type singularities can be removed by applying a **local subtraction** method
[Catani, Seymour (1998)] [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

- ☑ **automatised numerical implementation** in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]

Our subtraction framework: q_T-slicing

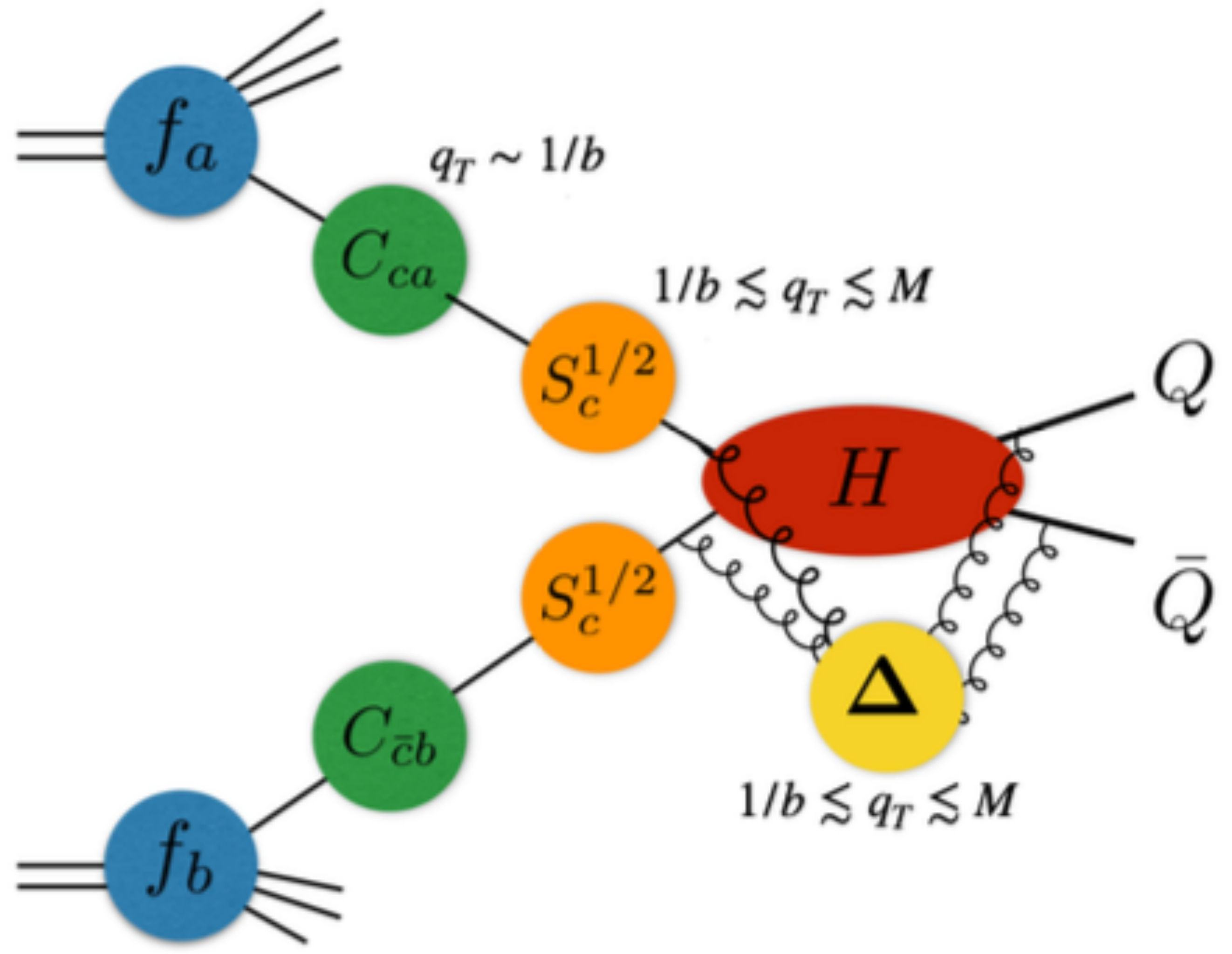
[Catani, Grazzini (2007)]

▹ master formula at NNLO

$$d\sigma_{NNLO} = \mathcal{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\rm cut}} + \mathcal{O}((q_T^{\rm cut})^p)$$

☑ non trivial ingredient: **two-loop soft function** for an **arbitrary kinematics** of the heavy quarks

[Catani, Devoto, Grazzini, Mazzitelli (2023)] [Devoto, Mazzitelli (in preparation)]



- the resummation formula shows a **richer structure** due to the additional soft singularities
- the factor Δ (operator in colour space) is specific for heavy-quark production and it encodes the **soft wide-angle radiation** from the $Q\bar{Q}$ pair and from initial-state final-state interference
- the log-enhanced contributions are controlled by the **transverse momentum anomalous dimension** Γ_t
- also the hard coefficient gets a non-trivial colour structure and azimuthal correlations

Our subtraction framework: q_T-slicing

[Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \boxed{\mathscr{H}_{NNLO}} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\rm cut}} + \mathcal{O}((q_T^{\rm cut})^p)$$

☑ the hard-collinear coefficient receives contributions also from the **two-loop virtual amplitudes**

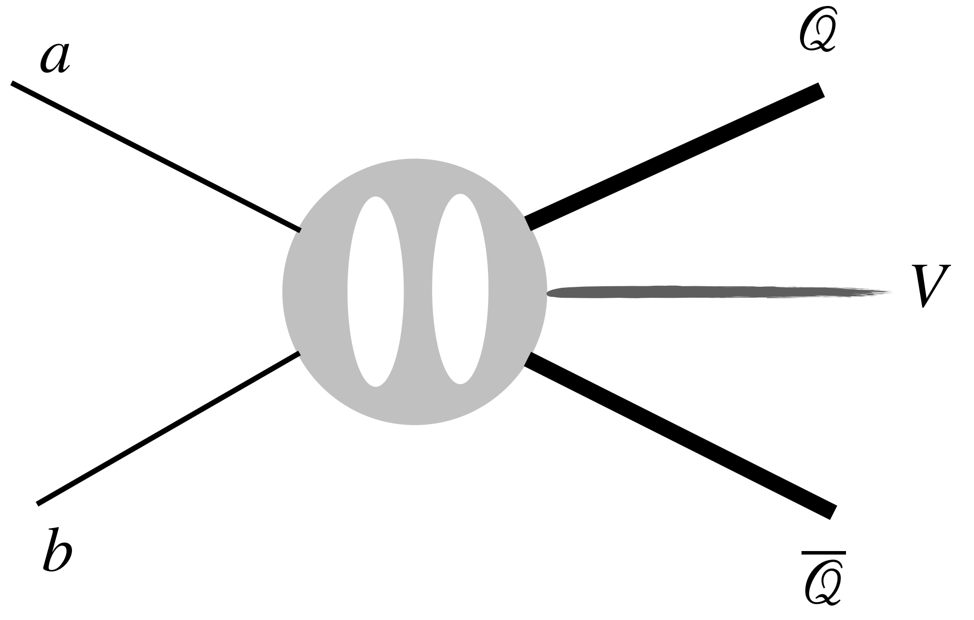
$$\mathscr{H}_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}^{(2)}(z_1, z_2)$$

where $$H^{(2)} = \left. \frac{2\Re(\mathscr{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathscr{M}^{(0)*})}{|\mathscr{M}^{(0)}|^2} \right|_{\mu_R=\mu_{IR}=Q}$$

UV renormalised and IR subtracted amplitude at scale μ_{IR}

[Ferroglia, Neubert, Pecjac, Yang (2009)]

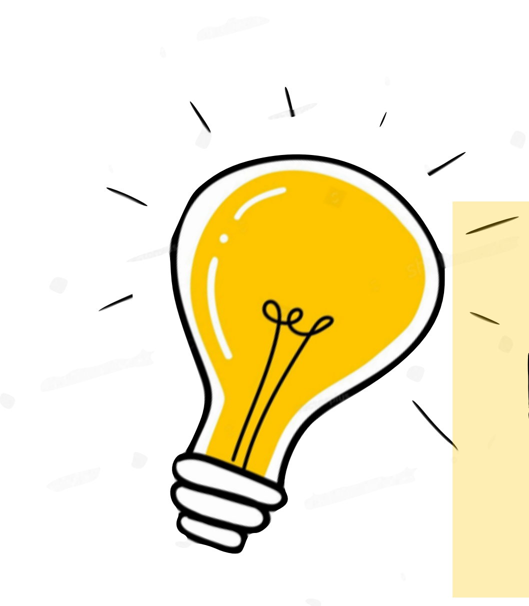
Q is the invariant mass of the $Q\bar{Q}V$ system



conceptual and technical challenges:

1. appearance of new mathematical functions
2. current analytic and numerical methods may not be enough
3. possible to find an amplitude representation that allows us for a numerically stable evaluation?

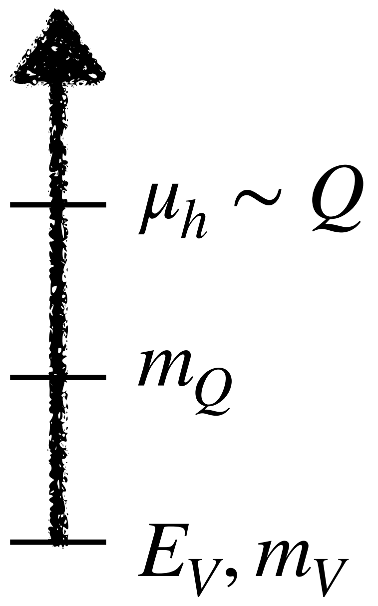
main bottleneck:
$2 \to 3$ and higher multiplicity two-loop amplitudes involving heavy loops and (many) external massive legs are currently out of reach. They require major breakthroughs



viable strategy: exploit the factorisation properties of QCD matrix elements in two different and rather complementary kinematic regimes

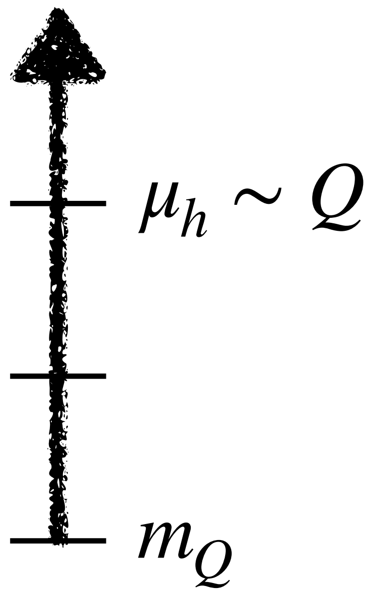
SOFT-BOSON approximation

1. soft limit for the external boson
$(E_V \to 0, m_V \to 0)$



MASSIFICATION

2. high-energy limit (ultra-relativistic quarks)
$(m_Q \ll \mu_h)$



disclaimer:
for $t\bar{t}H$ and $t\bar{t}W$, none of the two approximations is (a priori) justified in the bulk of the events. The quality of the approximation must be carefully assessed

Soft W-boson approximation

- It is well-known that when a soft photon, with momentum q^μ, is emitted in a high-energy process, the corresponding amplitude obeys the **factorisation formula**

$$\mathcal{M}(\{p_i\}, q) \simeq \mathcal{J}^\mu(q)\epsilon_\mu(q)\mathcal{M}(\{p_i\}) \qquad \text{with} \qquad \mathcal{J}^\mu(q) = e\sum_i \sigma_i Q_i \frac{p_i^\mu}{p_i \cdot q}$$

valid at **leading power** (LP) in the energy of the soft boson

$$\sigma_i = \begin{cases} +1 & \text{if } i \text{ (incoming) outgoing (anti-)fermion} \\ -1 & \text{otherwise} \end{cases}$$

eikonal current:

1. gauge-invariant

2. universal, process-independent: the soft boson cannot resolve the details of the hard process but only the charge and direction of the external charged particles

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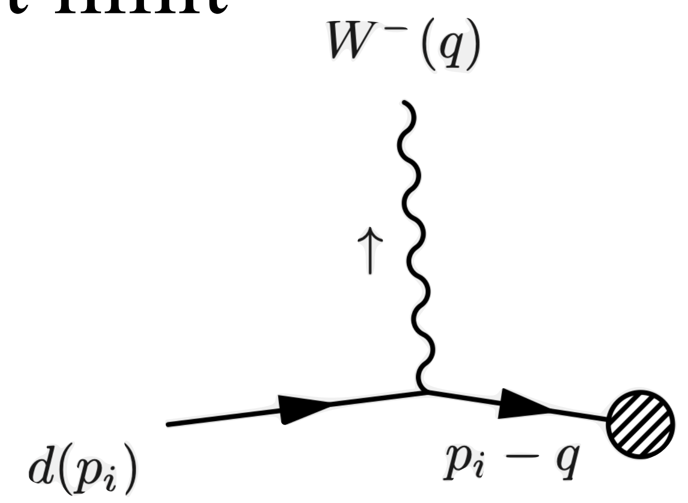
- since a W boson behaves as a photon under QCD corrections, an analogous factorisation holds in the soft limit

only the case of massless emitters is considered

$$\mathcal{J}^\mu(q) \to \mathcal{J}^\mu_W(q) = \frac{g_W}{\sqrt{2}}\left(\sum_i \sigma_i \frac{p_i^\mu}{p_i\cdot q}\right)\boxed{\frac{1-\gamma_5}{2}}$$

it selects the left-handed polarisation state of the emitter

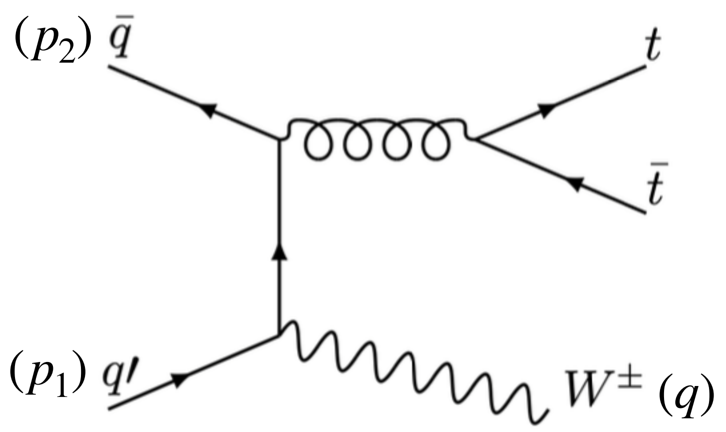


- the factorisation holds true at all orders in α_s since conserved currents do not renormalise

- in the specific case of **$t\bar{t}W$ production**:

reduced **polarised** $t\bar{t}$ amplitude

$$\lim_{q\to 0}\mathcal{M}_{t\bar{t}W}(\{p_i\},q) = \frac{g_W}{\sqrt{2}}\left(\frac{p_2\cdot\epsilon(q)}{p_2\cdot q} - \frac{p_1\cdot\epsilon(q)}{p_1\cdot q}\right)\boxed{\mathcal{M}^L_{t\bar{t}}(\{p_i\})}$$



[Bärnreuther, Czakon, Fiedler (2013)]
[Chen, Czakon, Poncelet (2017)]

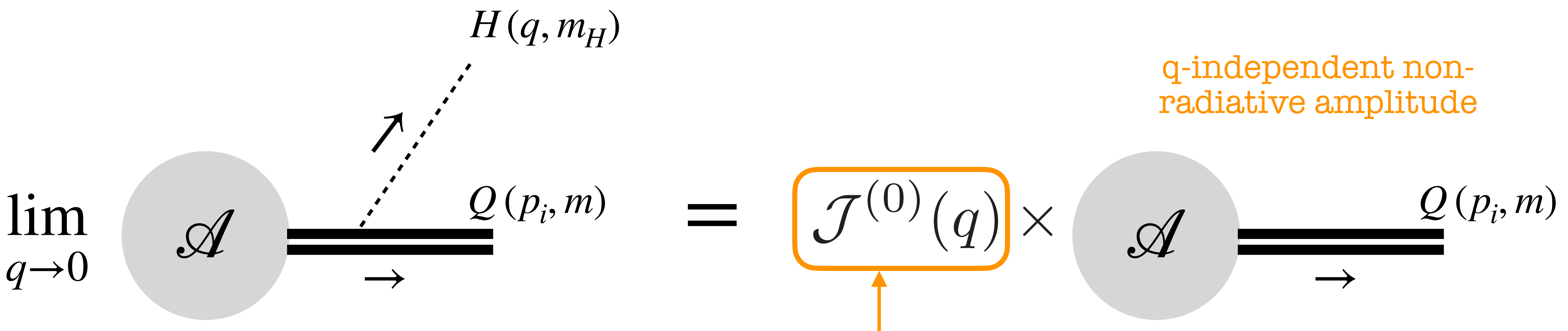
Soft Higgs-boson approximation

- Analogously to the soft W-boson limit, we want to study the soft Higgs-boson limit for the amplitude associated with

$$a_1(p_1) + a_2(p_2) \to \mathcal{Q}(p_3, m)\overline{\mathcal{Q}}(p_4, m)....\mathcal{Q}(p_{N+1}, m)\overline{\mathcal{Q}}(p_{N+2}, m) + H(q, m_H)$$

one or more heavy-quark pairs with the same mass

- at tree-level, it is straightforward to show that the LP factorisation reads



$$\mathcal{J}^{(0)}(q) = \frac{m}{v}\sum_i \frac{m}{p_i \cdot q}$$

scalar eikonal current

- at bare level, the naïve factorisation formula holds true at all orders in α_s, due to the **abelian nature** of the Higgs boson

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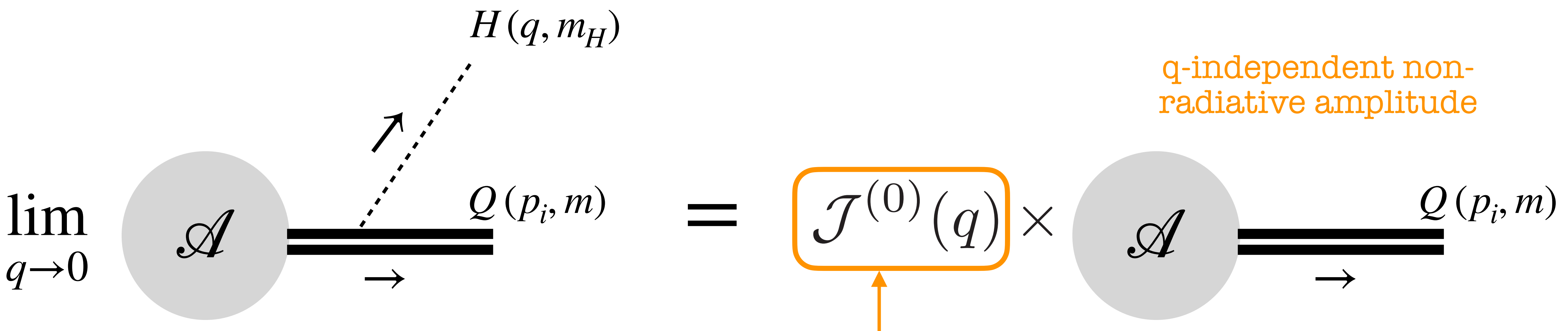

$$\mathcal{J}^{(0)}(q) = \frac{m}{v}\sum_i \frac{m}{p_i \cdot q}$$ scalar eikonal current

- at <u>bare level</u>, the naïve factorisation formula holds true at all orders in α_s, due to the **abelian nature** of the Higgs boson
- … but the renormalisation of the heavy-quark mass and wave function changes the overall normalisation by

up to two-loop order

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F\left(\alpha_s^{(n_l)}(\mu_R^2), \frac{\mu_R}{m}\right) = 1 + \frac{\alpha_s^{(n_l)}(\mu_R^2)}{2\pi}(-3C_F) + \left(\frac{\alpha_s^{(n_l)}(\mu_R^2)}{2\pi}\right)^2\left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_l + n_h) - 3C_F\beta_0^{(n_l)}\ln\frac{\mu_R^2}{m^2}\right) + \mathcal{O}({\alpha_s^{(n_l)}}^3)$$

Soft Higgs-boson approximation

how did we derive F ?

- To extract the explicit form of F up to three-loop order, we rely on the well-known **Higgs low-energy theorems** (LETs)

[Shifman, Vainshtein, Voloshin, Zakharov (1979)]
[Kniehl, Spira (1995)]

- they provide a connection between amplitudes of two processes which differ by the insertion of an external Higgs-boson line carrying zero momentum
- in our specific case:

$$\lim_{q\to 0} \mathcal{M}^{bare}_{Q\to QH}(p,q) = \frac{m_0}{v}\frac{\partial}{\partial m_0} \boxed{\mathcal{M}^{bare}_{Q\to Q}(p)}\Big|_{p^2=m^2}$$

$$\mathcal{M}^{bare}_{Q\to Q}(p) = \overline{Q}_0 \left\{ m_0(-1+\Sigma_S(p^2,m_0)) + \not{p}\,\Sigma_V(p^2,m_0) \right\} Q_0$$

unrenormalised heavy-quark self-energy

[Broadhurst, Grafe, Gray, Schilcher (1990)]
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[Melnikov, van Ritbergen (2000)]

The LETs can be derived by observing that:

1. the Higgs-boson interaction with a massive fermion emerges from the mass term by substituting:

$$m_0 \to m_0\left(1+\frac{H}{v}\right) \equiv m_0(H)$$

2. if the Higgs boson carries zero momentum, the corresponding field is constant

$$\frac{1}{\not{p}-m_0(H)} \simeq \frac{1}{\not{p}-m_0}\frac{m_0}{v}\frac{1}{\not{p}-m_0}H = \frac{m_0}{v}H\left(\frac{\partial}{\partial m_0}\frac{1}{\not{p}-m_0}\right)$$

- <u>next steps</u>:
  - renormalisation of the quark mass and wave function $m_0\overline{Q}_0Q_0 = m\overline{Q}QZ_mZ_2$
  - $\overline{\text{MS}}$ renormalisation of the strong coupling + decoupling of the n_h heavy quarks of mass m [Chetyrkin, Kniehl, Steinhauser (1997)]

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perfect agreement with the soft limit of the scalar heavy-quark form factor up to three-loop order [Fael et al. (2022)]

Soft Higgs-boson approximation

- **LP master formula** in the soft Higgs limit ($q \to 0,\, m_H \ll m$):

$$\mathcal{M}(p_1, p_2 \ldots p_N, q) \simeq F(\alpha_s(\mu_R); m/\mu_R)\, \frac{m}{v} \left(\sum_{i=1}^{N} \frac{m}{p_i \cdot q} \right) \mathcal{M}(p_1, p_2 \ldots p_N)$$

all-order UV renormalised amplitudes

- observations:
  - $F(\alpha_s(\mu_R); m/\mu_R)$ is perturbatively calculable, finite and gauge-independent
  - it can be derived by applying the so-called Higgs Low Energy theorems (LETs)
  - the IR singularity structure of the scattering amplitude is left changed
  - the non-radiative amplitude must be evaluated on a set of projected momenta (to preserve momentum conservation)

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  - the IR singularity structure of the scattering amplitude is left changed
  - the non-radiative amplitude must be evaluated on a set of projected momenta (to preserve momentum conservation)
  - for the specific case of **$t\bar{t}H$ production**, the non-radiative amplitude is known up to two-loop order

[Bärnreuther, Czakon, Fiedler (2013)]

the soft factorisation formulae could provide a powerful cross check of future exact amplitude calculations, in this specific kinematic limit

Mass factorisation or massification

ultra-relativistic quarks

[Penin (2006)]
[Becher, Melnikov (2007)]
[Moch, Mitov (2007)]

- massification relies on the **factorisation** properties of massless QCD amplitudes into a product of functions that organise the contributions of momentum regions relevant to the ϵ poles in the scattering amplitude [Sterman, Tejeda-Yeomans (2003)]

$$|\mathcal{M}\rangle = \boxed{\mathcal{J}_0\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)} \boxed{\mathscr{S}_0\left(\{p_i\}, \frac{Q'^2}{\mu^2}, \frac{Q'^2}{Q^2}, \alpha_s(\mu^2), \epsilon\right)} \boxed{|\mathcal{H}\rangle}$$

scheme-dependent

JET function: collinear contributions

SOFT function: soft wide-angle radiation

HARD function: short-distance dynamics

$$\mathcal{J}_0\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \prod_{i=1}^{n+2} \mathcal{J}_0^{[i]}\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

Sudakov-defined jet function

$$\mathcal{J}_0^{[i]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{J}_0^{[\bar{i}]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \left(\mathcal{F}_0^{[i\bar{i}\to F]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right)^{1/2}$$

singlet time-like form factor

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SOFT function: soft wide-angle radiation

HARD function: short-distance dynamics

$$\mathcal{J}_0\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \prod_{i=1}^{n+2} \mathcal{J}_0^{[i]}\left(\frac{Q'^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)$$

Sudakov-defined jet function

$$\mathcal{J}_0^{[i]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \mathcal{J}_0^{[\bar{i}]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) = \left(\mathcal{F}_0^{[i\bar{i}\to F]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right)\right)^{1/2}$$

singlet time-like form factor

- if one or more external partons acquire a non-vanishing mass, in the limit $m \ll \mu_h \sim Q$, a **LP factorisation** holds

$$|\mathcal{M}_m\rangle = \mathcal{J}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \mathscr{S}\left(\{p_i\}, \frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s(\mu^2), \epsilon\right) \boxed{|\mathcal{H}\rangle}$$

same as in the massless case

Mass factorisation or massification

original formulation

[Penin (2006)]
[Moch, Mitov (2007)]

- **idea**: reconstruct the massive amplitudes, in the ultra-relativistic quark limit $m \ll Q$, up to power corrections $\mathcal{O}(m^2/Q^2)$
- If contributions from **heavy-quark loops** are **neglected**, the master formula is

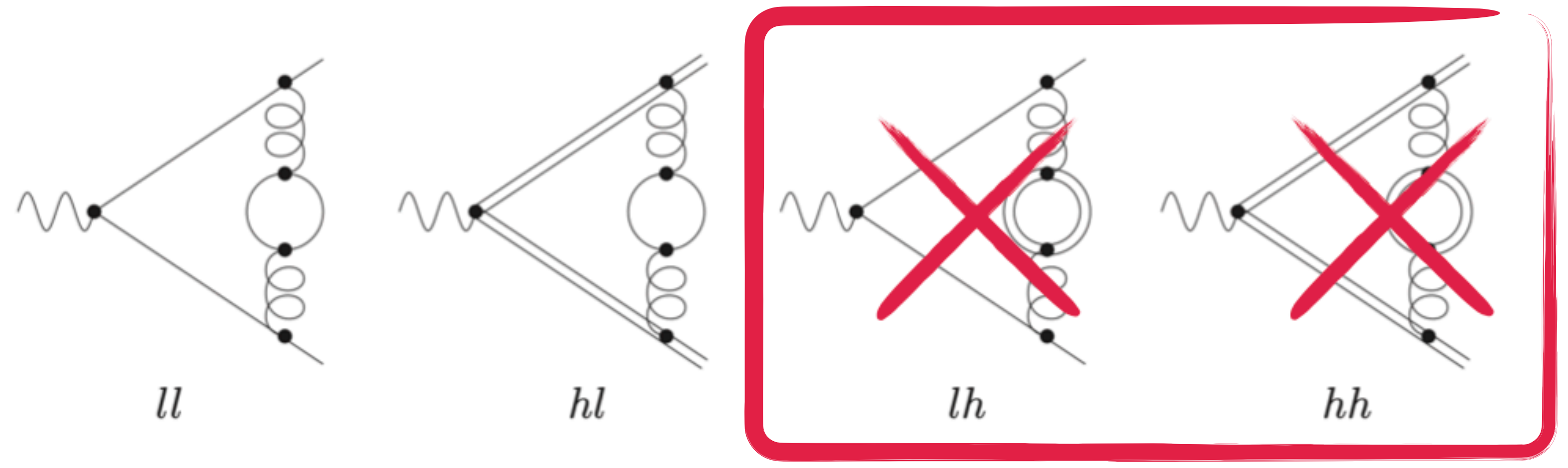
we are "dressing" n_Q external quarks with a mass m

$$|\mathcal{M}_m\rangle = \left(\boxed{Z_{[Q]}^{(m|0)} \left(\alpha_s^{(n_l)}, \frac{\mu^2}{m^2}, \epsilon \right)} \right)^{n_Q/2} |\mathcal{M}\rangle$$

all-order UV renormalised amplitudes in $\overline{\text{MS}}$ scheme with n_l running quarks

universal, perturbatively computable, ratio between massive and massless FFs

$$Z_{[Q]}^{(m|0)} \left(\alpha_s^{(n_l)}, \frac{\mu^2}{m^2}, \epsilon \right) = \mathcal{F}^{[Q\overline{Q} \to F]} \left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s^{(n_l)}(\mu^2), \epsilon \right) \left(\mathcal{F}_0^{[q\bar{q} \to F]} \left(\frac{Q^2}{\mu^2}, \alpha_s^{(n_l)}(\mu^2), \epsilon \right) \right)^{-1}$$



the mass "screens" collinear singularities

1. all ϵ poles, n_h-independent logarithms of the mass and finite terms of the massive amplitude are predicted
2. it can be viewed as a change in regularisation scheme

generalised formulation

Mass factorisation or massification

[Wang et al. (2023)]

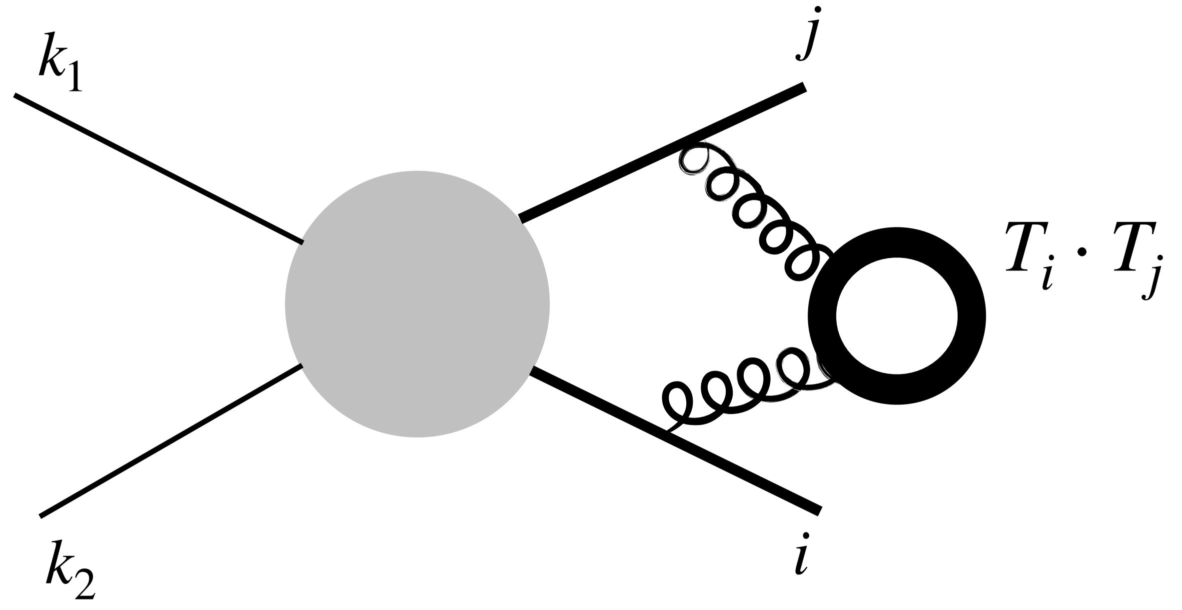
- If contributions from **heavy-quark loops** are **included**, a non-trivial soft function emerges starting from α_s^2
- the master formula gets modified as

[Becher, Melnikov (2007)]
[Engel et al. (2019)]

$$|\mathcal{M}_m\rangle = \prod_i \left(Z_{[i]}^{(m|0)} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon \right) \right)^{1/2} \boxed{\mathbf{S} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right)} |\mathcal{M}\rangle$$

all-order UV renormalised amplitudes in $\overline{\mathrm{MS}}$ scheme with $n_f = n_l + n_h$ running quarks

process-dependent **SOFT** function, operator in colour space, it starts contributing at two-loop order



$$\mathbf{S} \left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) = 1 + \left(\frac{\alpha_s^{(n_f)}(\mu^2)}{4\pi} \right)^2 n_h \sum_{i>j} (-\mathbf{T}_i \cdot \mathbf{T}_j)\, S^{(2)} \left(\frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) + \mathcal{O}(\alpha_s^3)$$

with $$S^{(2)} \left(\frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon \right) = T_R \left(\frac{\mu^2}{m^2} \right)^{2\epsilon} \left(-\frac{4}{3\epsilon^2} + \frac{20}{9\epsilon} - \frac{112}{27} - \frac{4\zeta_2}{3} \right) \ln \left(\frac{-s_{ij}}{m^2} \right)$$

Mass factorisation or massification

generalised formulation

[Wang et al. (2023)]

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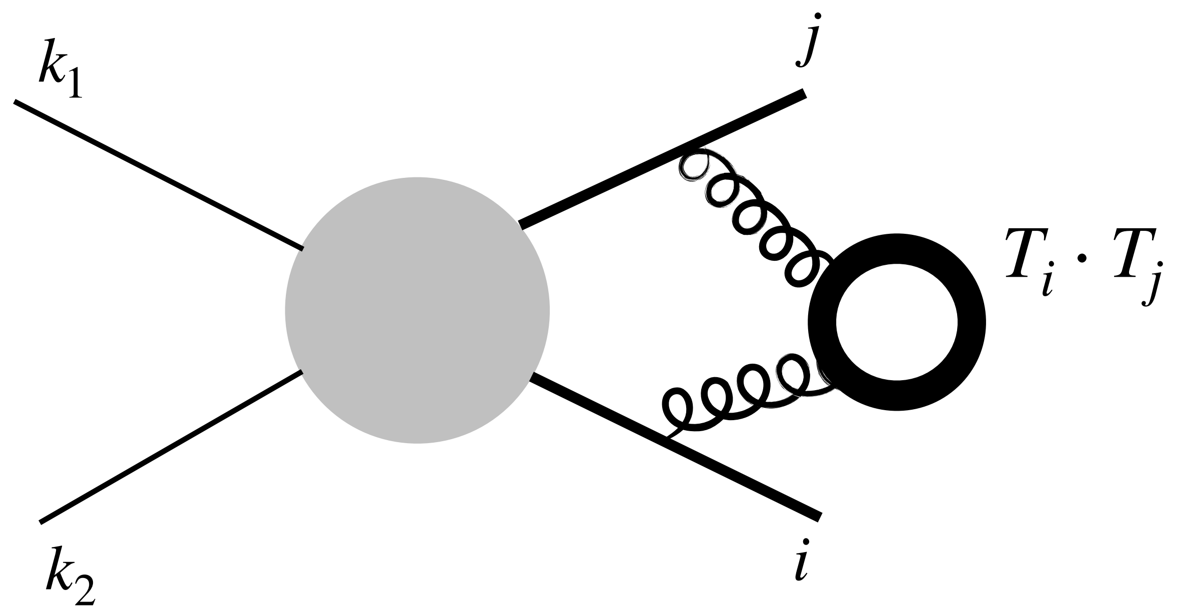

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$$\text{with} \quad S^{(2)}\left(\frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon\right) = T_R \left(\frac{\mu^2}{m^2}\right)^{2\epsilon}\left(-\frac{4}{3\epsilon^2} + \frac{20}{9\epsilon} - \frac{112}{27} - \frac{4\zeta_2}{3}\right) \ln\left(\frac{-s_{ij}}{m^2}\right)$$

for the specific cases of $\mathbf{Q\bar{Q}W}$ and $\mathbf{Q\bar{Q}H}$ production we can reconstruct the massive amplitudes, up to power corrections in the heavy-quark mass, by exploiting the corresponding (known) massless amplitudes

[Abreu et al. (2021)] [Badger et al. (2021)]

WQQAmp: a massive C++ implementation

- **idea:** exploit the recently computed leading-colour **massless two-loop 5-point amplitudes** for W+4 partons [Abreu at al. (2022)]

$$u/d(p_1) + \bar{d}/\bar{u}(p_2) \rightarrow W^{\pm}(p_3) + Q(p_4) + \overline{Q}(p_5)$$

evaluation of the LC massless one-loop bare amplitudes and two-loop finite remainders at $\mu = 1$

- mapping of the massive kinematics
- crossing of the partonic channel
- evaluation of the MIs [Chicherin, Sotnikov, Zoia (2021)]

→ restore all UV and IR poles

→ add dependence on the dimensional scale μ

→ perform one-loop and two-loop UV renormalisation

→ MASSIFICATION [Moch, Mitov (2007)]

→ remove IR poles of the massified amplitudes [Ferroglia et al. (2009)]

→ output: one-loop and two-loop massive finite remainders in minimal subtraction scheme

evaluation time of $\mathcal{O}(4s)$ per phase space point
[double precision for rational coefficients and MIs]

VALIDATION and CHECKS:

- stability of the two-loop massless amplitudes
- one-loop amplitudes in LCA tested against MCFM
- cancellation of the massified poles in LCA

WQQAmp: a massive C++ implementation

- idea: exploit the recently computed leading-colour **massless two-loop 5-point amplitudes** for W+4 partons [Abreu at al. (2022)]

$$
\begin{aligned}
\mathcal{M}_m^{(2),\mathrm{fin}} = {} & \mathcal{M}^{(2),\epsilon^0}_{(m=0)} + \left[Z^{(1),1/\epsilon^2}_{[Q]} - \mathbf{Z}^{(1),1/\epsilon^2}_{m\ll\mu_h} \right]_{\mathrm{LC}} \mathcal{M}^{(1),\epsilon^2}_{(m=0)} + \left[Z^{(1),1/\epsilon}_{[Q]} - \mathbf{Z}^{(1),1/\epsilon}_{m\ll\mu_h} \right]_{\mathrm{LC}} \mathcal{M}^{(1),\epsilon}_{(m=0)} \\
& + Z^{(1),\epsilon^0}_{[Q]} \mathcal{M}^{(1),\epsilon^0}_{(m=0)} + Z^{(1),\epsilon}_{[Q]} \mathcal{M}^{(1),1/\epsilon}_{(m=0)} + Z^{(1),\epsilon^2}_{[Q]} \mathcal{M}^{(1),1/\epsilon^2}_{(m=0)} \\
& + \left(Z^{(2),\epsilon^0}_{[Q]} - Z^{(1),\epsilon}_{[Q]} \mathbf{Z}^{(1),1/\epsilon}_{m\ll\mu_h} - Z^{(1),\epsilon^2}_{[Q]} \mathbf{Z}^{(1),1/\epsilon^2}_{m\ll\mu_h} \right) \mathcal{M}^{(0)}_{(m=0)}
\end{aligned}
$$

free from $\log(m^2)$ terms, higher ϵ orders of the one-loop massless amplitude required

MASSIFICATION [Moch, Mitov (2007)]

remove IR poles of the massified amplitudes [Ferroglia et al. (2009)]

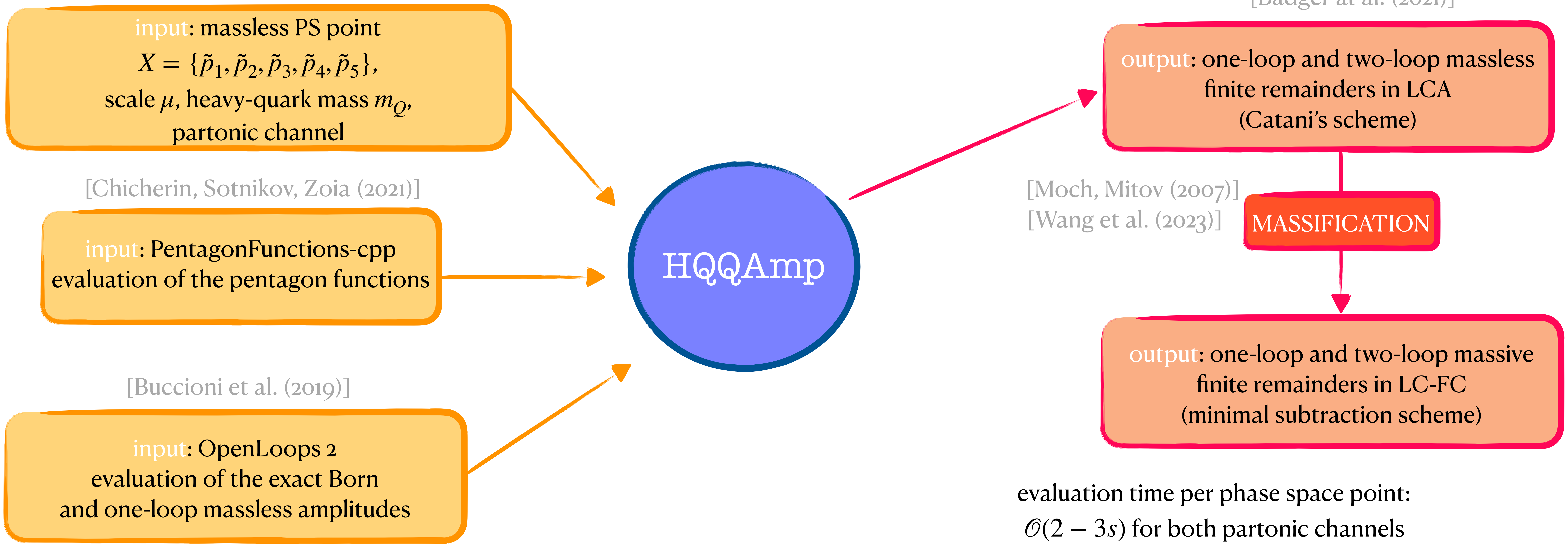
output: one-loop and two-loop massive finite remainders in minimal subtraction scheme

N.B. application of the massification at the level of UV renormalised amplitudes

HQQAmp: a massive C++ implementation

- idea: similarly to WQQAmp, implement the one-loop and two-loop massless amplitudes of [Badger at al. (2021)] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**
- different workflow and possibility of choosing the **precision** for the MIs and relative coefficients

$$q/g(p_1) + \bar{q}/g(p_2) \to H(p_3) + Q(p_4) + \overline{Q}(p_5)$$



evaluation time per phase space point:
$\mathcal{O}(2-3s)$ for both partonic channels
[quadruple (double) precision for the coefficients (MIs)]

cross-checked against an independent implementation by C.Biello

HQQAmp: a massive C++ implementation

- idea: similarly to WQQAmp, implement the one-loop and two-loop massless amplitudes of [Badger at al. (2021)] in a **C++ library** for the efficient numerical evaluation of the **massive amplitudes**
- different workflow and possibility of choosing the **precision** for the MIs and relative coefficients

$$|\mathcal{M}_m^{\text{fin}}\rangle = \mathbf{Z}^{-1}_{m\ll\mu_h}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon\right) Z^{(m|0)}_{[Q]}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon\right) Z^{(m|0)}_{[c]}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \epsilon\right)$$
$$\times\, \mathbf{S}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \frac{\mu^2}{m^2}, \epsilon\right) \mathbf{Z}_{(m=0)}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{s_{ij}}, \epsilon\right) |\mathcal{M}^{\text{fin}}_{(m=0)}\rangle + \mathcal{O}\left(\frac{m}{\mu_h}\right)$$
$$= \mathcal{F}_{[c]}\left(\alpha_s^{(n_f)}, \frac{\mu^2}{m^2}, \frac{\mu^2}{s_{ij}}\right) |\mathcal{M}^{\text{fin}}_{(m=0)}\rangle + \mathcal{O}\left(\frac{m}{\mu_h}\right)$$

it is an operator in colour space and it encodes all mass logarithms!

$$|\mathcal{M}_m^{(1),\text{fin}}\rangle = |\mathcal{M}^{(1),\text{fin}}_{(m=0)}\rangle + \mathcal{F}^{(1)}_{[c]}\,|\mathcal{M}^{(0)}_{(m=0)}\rangle$$
$$|\mathcal{M}_m^{(2),\text{fin}}\rangle = |\mathcal{M}^{(2),\text{fin}}_{(m=0)}\rangle + \mathcal{F}^{(1)}_{[c]}\,|\mathcal{M}^{(1),\text{fin}}_{(m=0)}\rangle + \mathcal{F}^{(2)}_{[c]}\,|\mathcal{M}^{(0)}_{(m=0)}\rangle$$

massless two-loop contribution in LCA. All remaining terms are "promoted" to FC

no need to implement the higher ϵ orders of the massless one-loop amplitude

[Badger at al. (2021)]

output: one-loop and two-loop massless finite remainders in LCA (Catani's scheme)

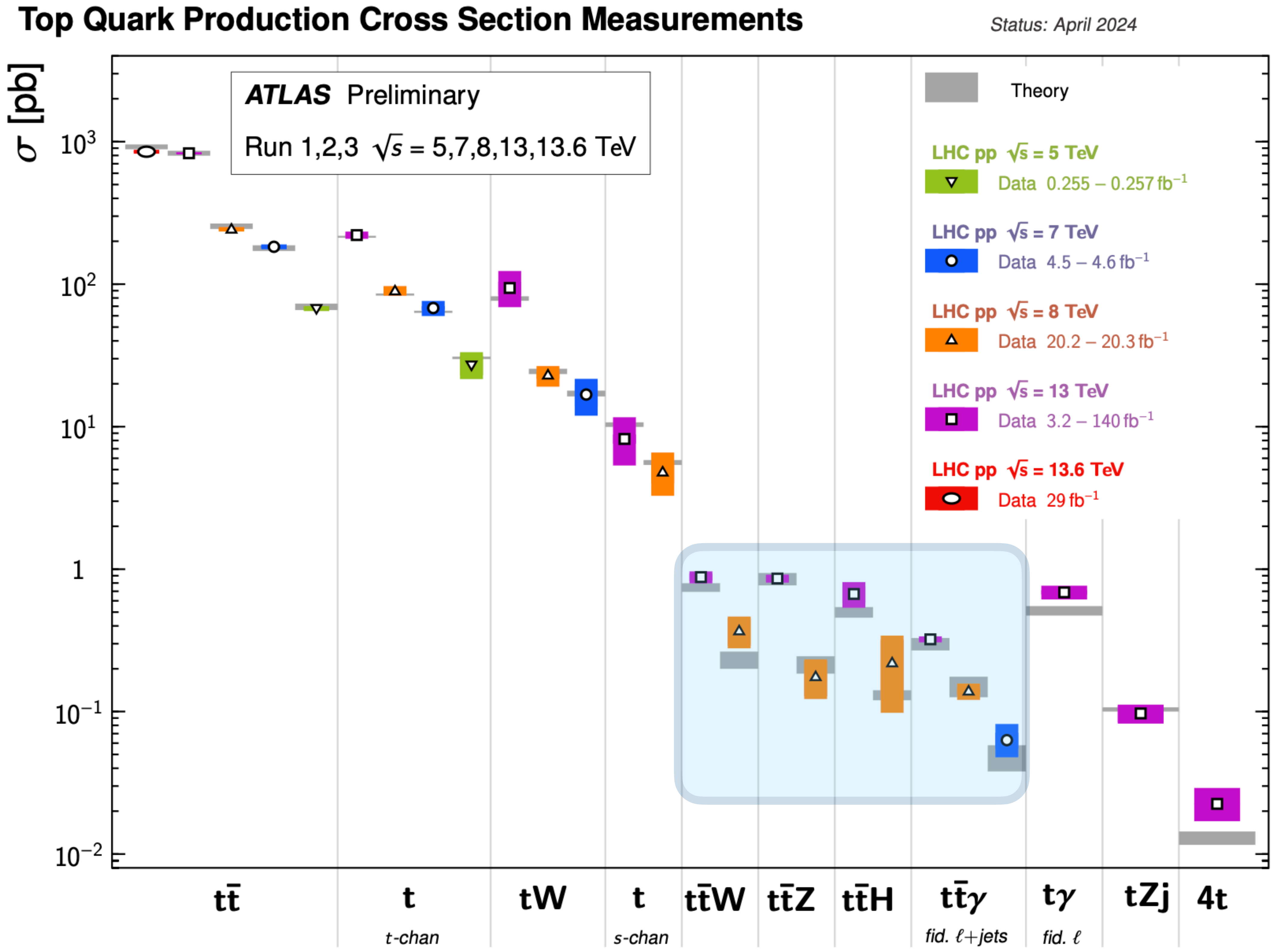
[Wang et al. (2023)] MASSIFICATION

output: one-loop and two-loop massive finite remainders in LC-FC (minimal subtraction scheme)

Yukawa renormalised ON-SHELL

N.B. application of the massification directly on the finite remainders

$t\bar{t}$ production with an EW or Higgs boson



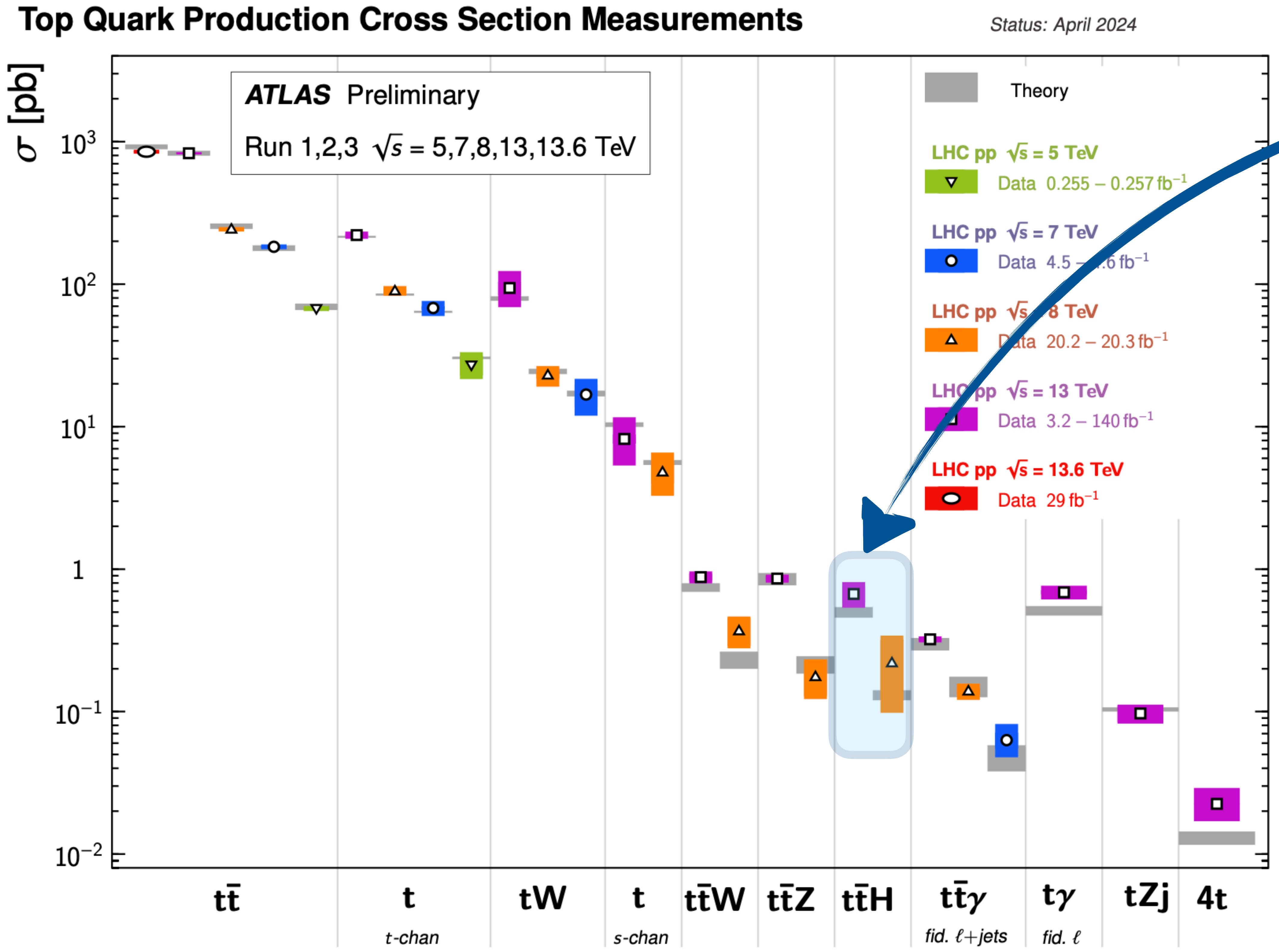
[ATL-PHYS-PUB-2024-006 (2024)]

- the cross section is at least **two orders** of magnitude **smaller** than in the case of $t\bar{t}$ production but …
- they have been measured experimentally with 10-20% uncertainties
- these processes are **crucial** for characterising the interactions of top quarks with gauge and Higgs bosons

Why is $t\bar{t}H$ production interesting ?

motivations:

- the study of the Higgs boson is **one of the priorities** in the LHC experimental program, after its discovery in 2012
- the Higgs boson couplings to SM particles are proportional to their masses: **special role played by the top quark!**



[ATL-PHYS-PUB-2024-006 (2024)]

only about 1% of the Higgs bosons are produced in association with a top-quark pair (first observation in 2018) but... the production mode $pp \to t\bar{t}H$ allows for a direct measurement of the top-quark Yukawa coupling

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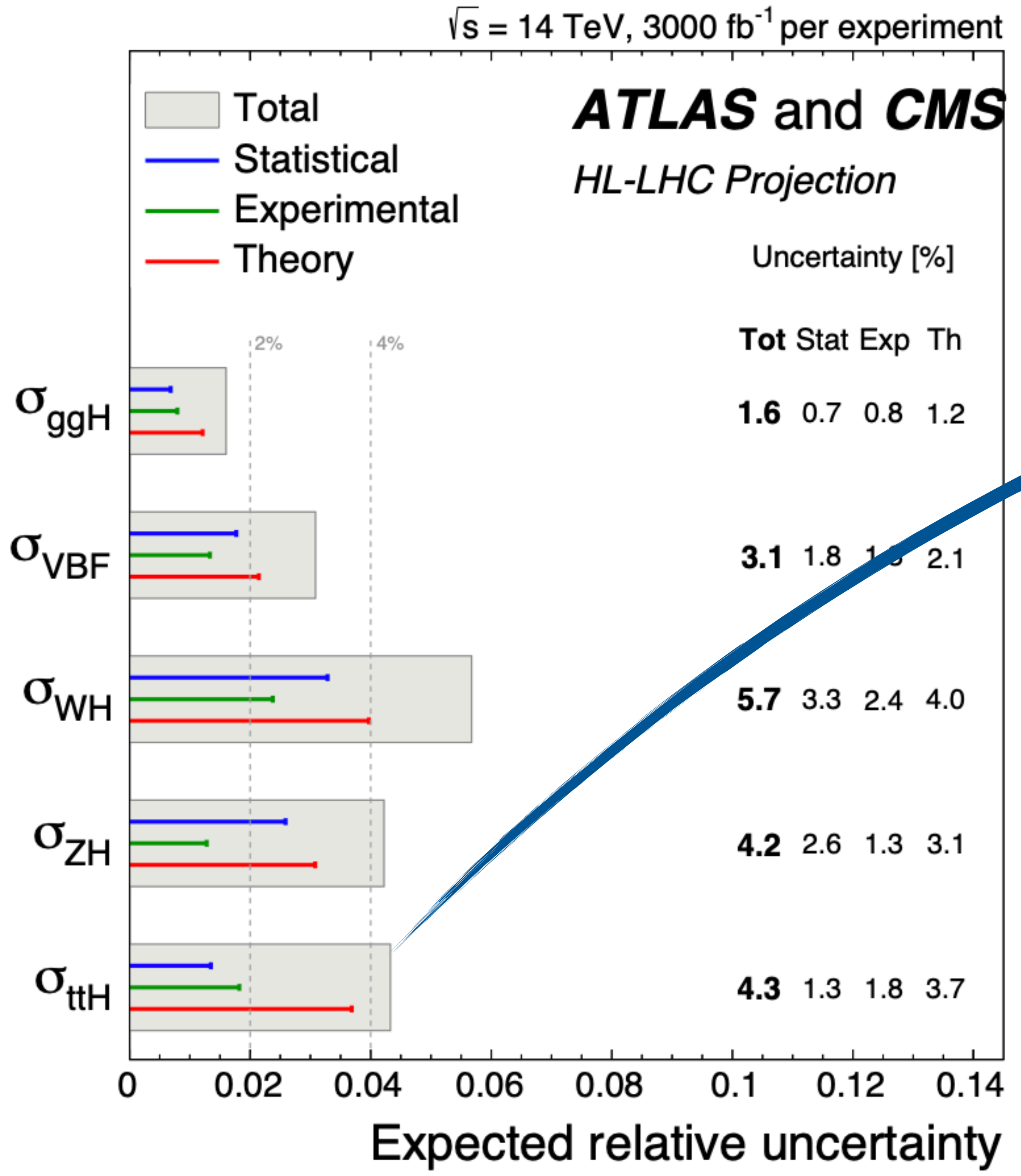

[Report from WG2 (2019)]

the current experimental accuracy is $\mathcal{O}(20\%)$ but, according to the HL-LHC projections, it is expected to go down to $\mathcal{O}(2\%)$

the extraction of the $t\bar{t}H(H \to b\bar{b})$ signal is limited by the theoretical uncertainties in the modelling of the backgrounds, mainly $t\bar{t}b\bar{b}$ and $t\bar{t}$ + light-flavour jets

moreover, NLO QCD + EW theory predictions equipped with NNLL soft-gluon resummation are affected by $\mathcal{O}(10\%)$ uncertainty

Theoretical predictions for $t\bar{t}H$

state of the art:

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas (2001,2003) [Reina, Dawson, Wackeroth, Jackson, Orr (2001,2003)]
- ☑ **NLO EW** corrections (*on-shell top quarks*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ **NLO QCD** corrections (*leptonically decaying top quarks*) [Denner, Feger (2015)] [Stremmer, Malgorzata (2022)]
- ☑ **NLO QCD + EW** corrections (*off-shell top quarks*) [Denner, Lang, Pellen, Uccirati (2017)]
- ☑ current predictions based on **NLO QCD + EW** corrections (*on-shell top quarks*), including **NNLL** soft-gluon resummation [Broggio et al.] [Kulesza et al.]
- ☑ **NNLO QCD** contributions for the **off-diagonal** partonic channels [Catani, Fabre, Grazzini, Kallweit (2021)]
- ☑ **complete NNLO QCD** predictions with approximated two-loop amplitudes [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]
- ☑ + full tower of **EW** corrections [Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2024)]

first NNLO calculation!

Theoretical predictions for $t\bar{t}H$

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- ☑ **NLO QCD** corrections (*on-shell top quarks*)
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main bottleneck

Two-loop amplitudes for ttH production: the quark-initiated Nf-part
Bakul Agarwal, Gudrun Heinrich, Stephen P. Jones, Matthias Kerner, Sven Yannick Klein, Jannis Lang, Vitaly Magerya, Anton Olsson

One loop QCD corrections to $gg \to t\bar{t}H$ at $\mathcal{O}(\epsilon^2)$
Federico Buccioni, Philipp Alexander Kreer, Xiao Liu, Lorenzo Tancredi

Two-loop QCD amplitudes for $t\bar{t}H$ production from boosted limit
Guoxing Wang, Tianya Xia, Li Lin Yang, Xiaoping Ye

HOT TOPIC !!

Two-Loop Master Integrals for Leading-Color $pp \to t\bar{t}H$ Amplitudes with a Light-Quark Loop
F. Febres Cordero, G. Figueiredo, M. Kraus, B. Page, L. Reina

Results: systematic uncertainties

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\|_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\|_{\mathrm{soft}}$	$-2.980(3)$	$2.622(0)$	$-239.4(4)$	$65.45(1)$

- at NLO, difference of **5%** (**30%**) in $q\bar{q}$ (gg) channel

- at NNLO, the hard-virtual contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$ small!

- our prescription to provide a conservative uncertainty is:
  - ☑ apply the approximation at a **different subtraction scale** (vary μ_{IR} by a factor 2 around Q); add the two-loop shift based on the exact tree-level and one-loop $t\bar{t}H$ amplitudes
  - ☑ take into account the NLO discrepancy and multiply it by a **tolerance factor 3**
  - ☑ combine **linearly** the gg and $q\bar{q}$ channels

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FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO} , $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

it is clear that the quality of the final result depends on the size of the contribution we are approximating

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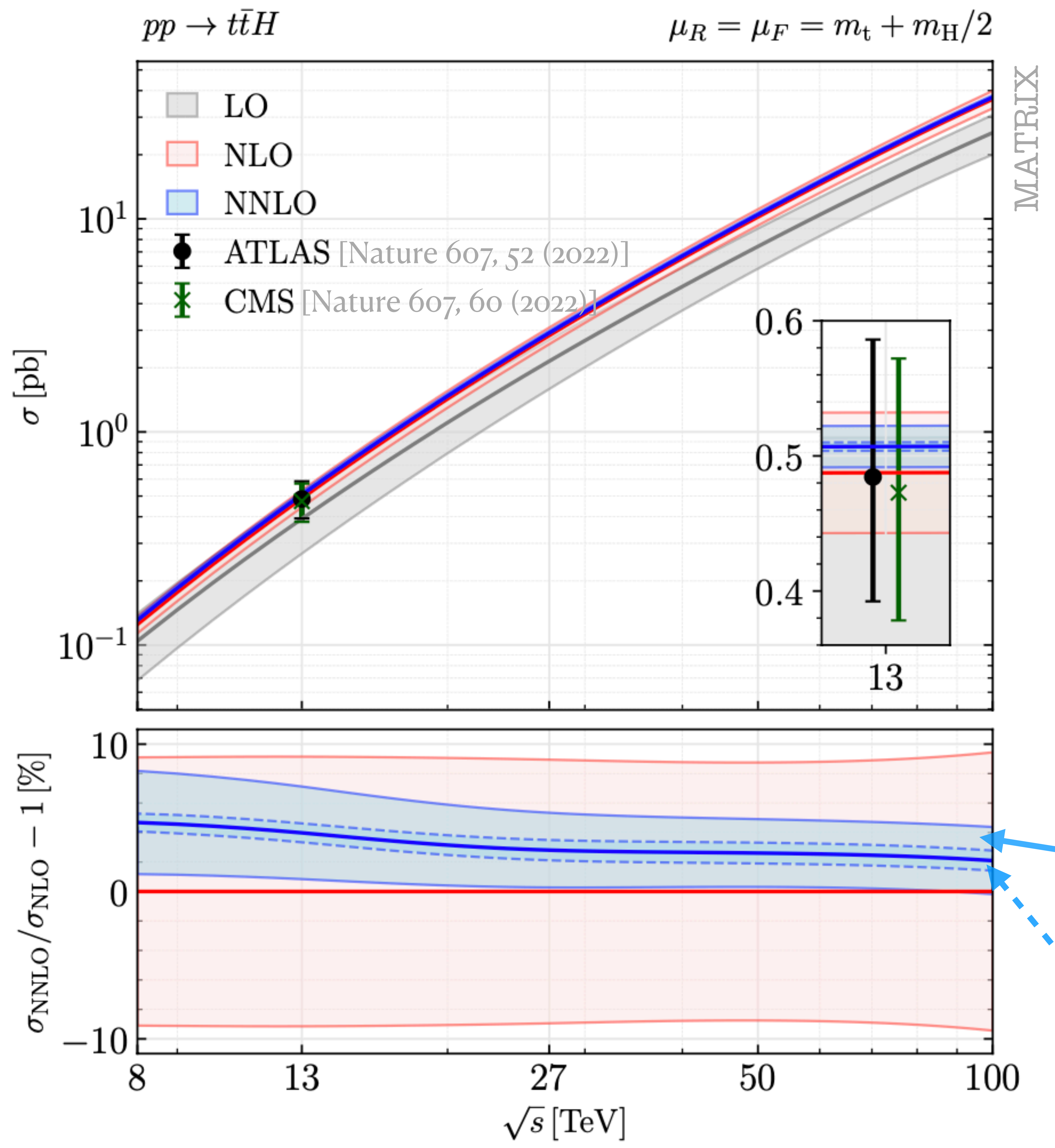
Results: total cross section

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$



σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910\,^{+31.3\%}_{-22.2\%}$	$25.38\,^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875\,^{+5.6\%}_{-9.1\%}$	$36.43\,^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)\,^{+0.9\%}_{-3.0\%}$	$37.20(25)\,^{+0.1\%}_{-2.2\%}$

- at NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- at NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$

nice perturbative convergence with theory uncertainties at $\mathcal{O}(3\%)$

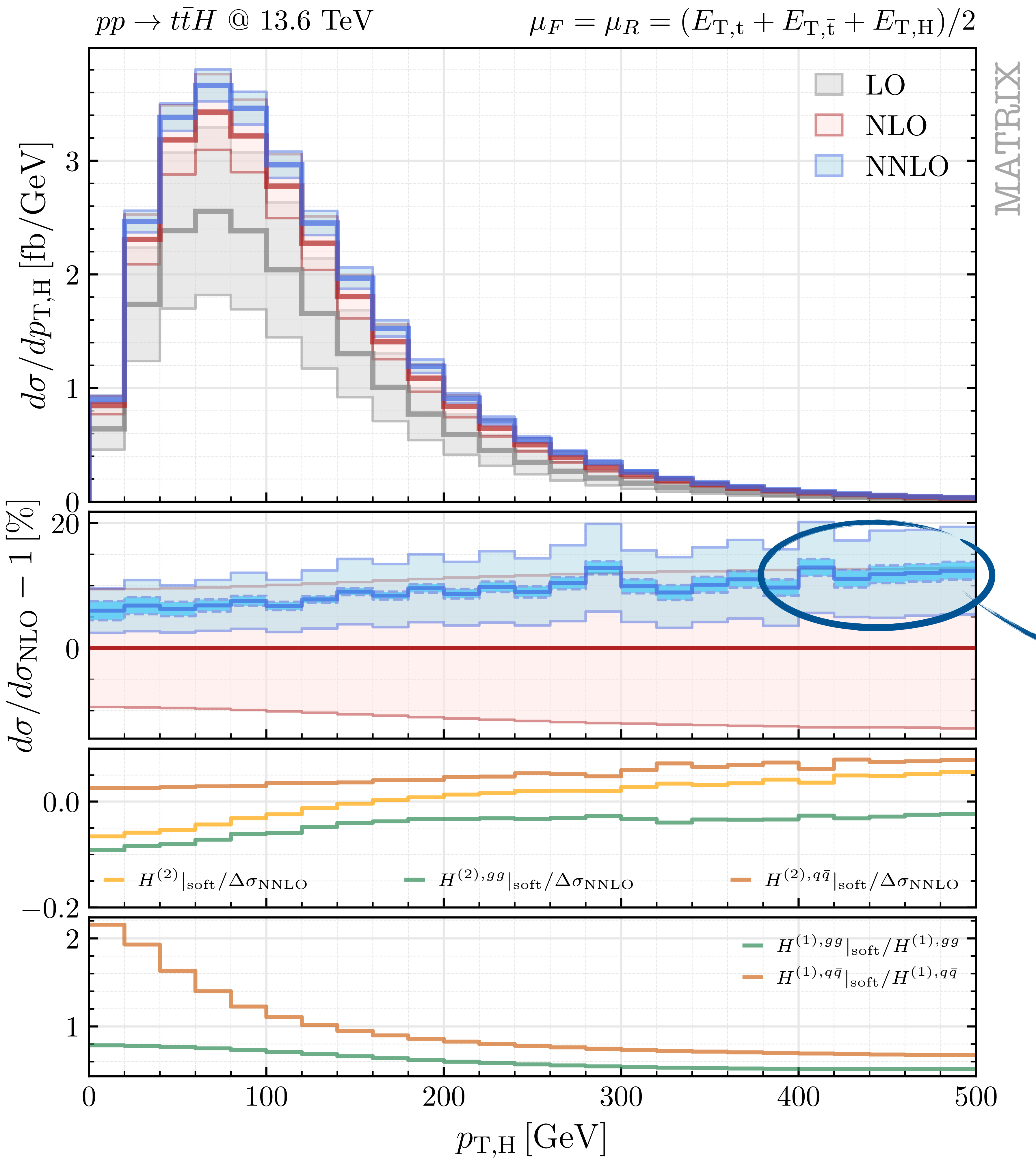
symmetrised 7-point scale variation

systematic + soft-approximation

First differential results: "soft-based"

Higgs transverse momentum

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$



- significant reduction of the perturbative uncertainties
- soft-approximation uncertainty computed on a **bin-by-bin basis** (NLO discrepancy multiplied by a constant tolerance factor 3)

oversimplified procedure ...

- the systematic uncertainties seem to be under control, but are they trustable?

in the tail of the $p_{T,H}$ distribution, far from the region of validity of the soft-approximation, the systematic errors are "artificially" too small

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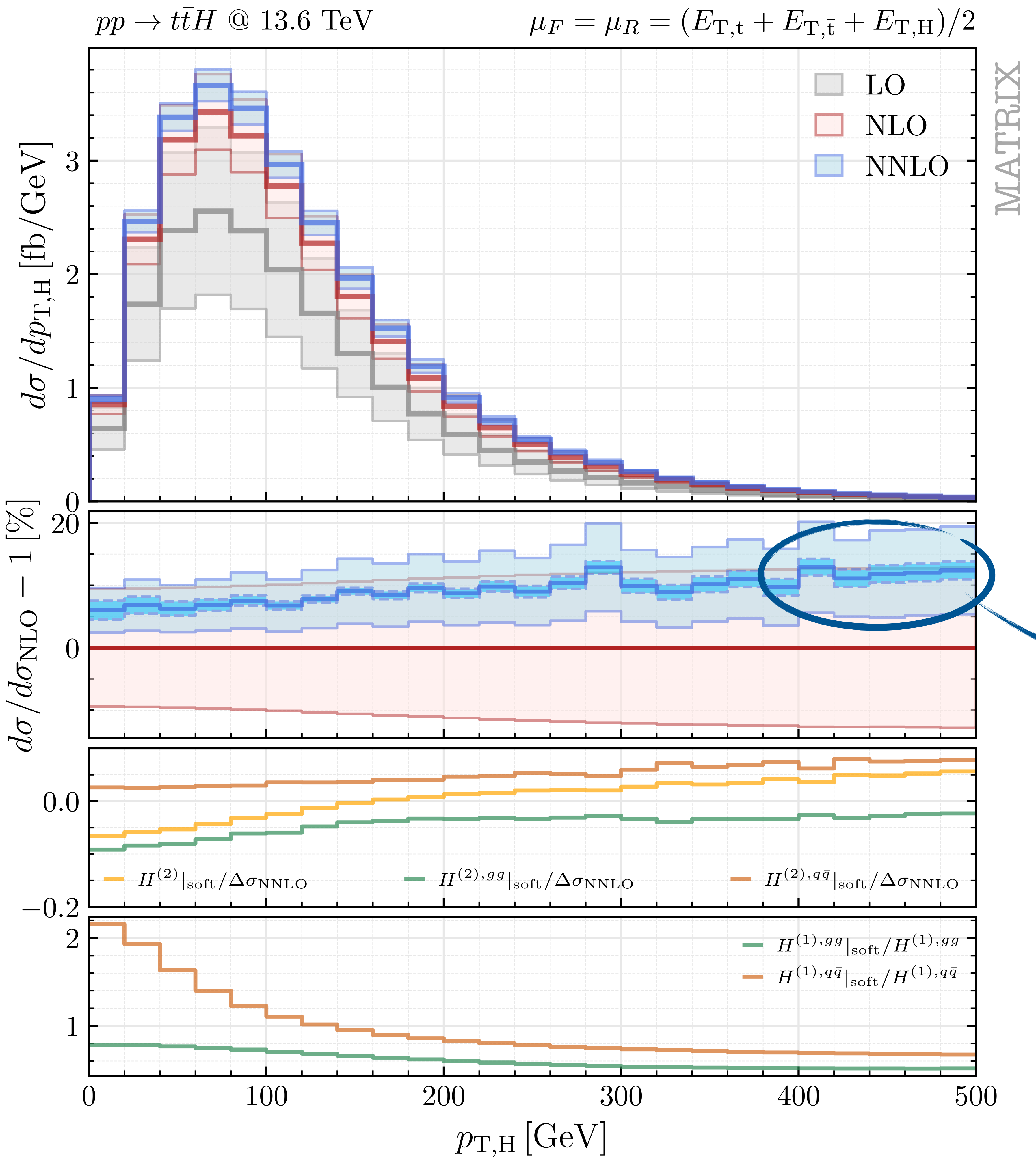

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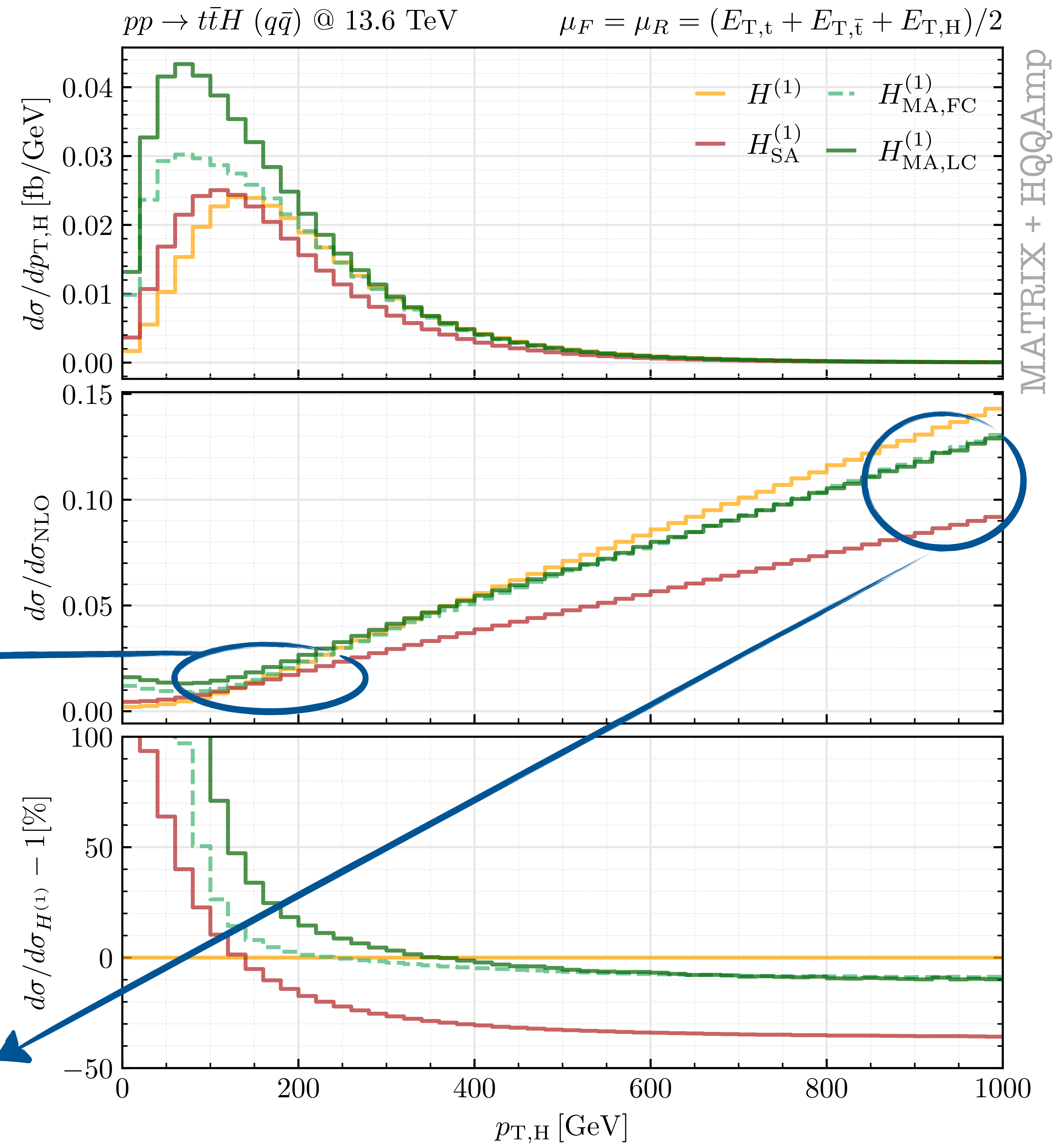
to make our predictions more robust at the differential level we "combine" the SOFT-HIGGS APPROXIMATION with a HIGH-ENERGY expansion

Quality of both approximations at NLO

setup: NNLO NNPDF40, $m_H = 125.09GeV$, $m_t = 172.5GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

different setup!



around the peak:

1. FC-FC massification and soft approximation are nearly equivalent
2. LC-FC massification overestimates the exact result by almost a factor of 2

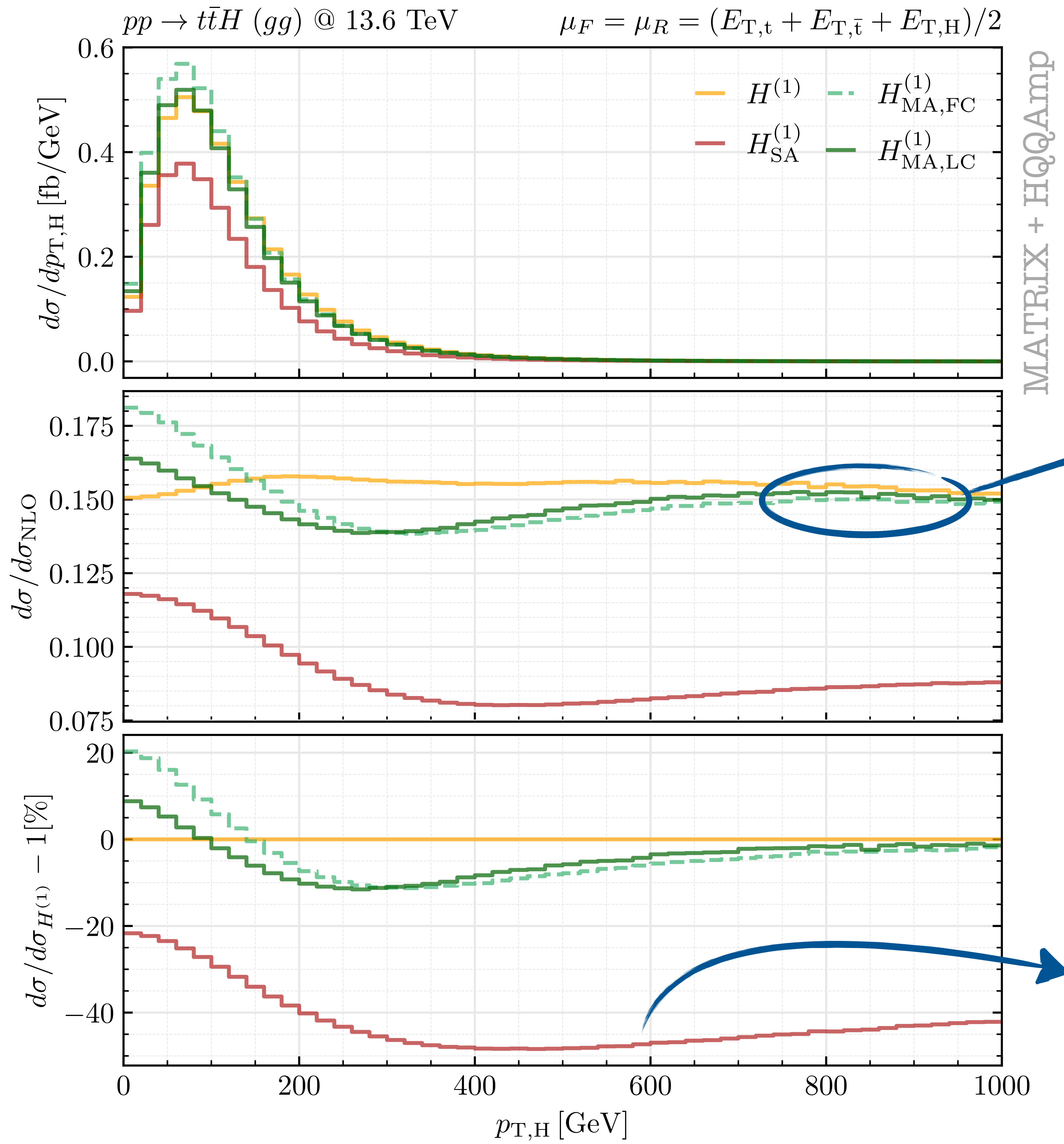
in the high-p_T tail:

1. missing subleading colour contributions are less relevant
2. soft approximation underestimates the exact result: $\mathcal{O}(2\%)$ difference of the NLO cross section

Quality of both approximations at NLO

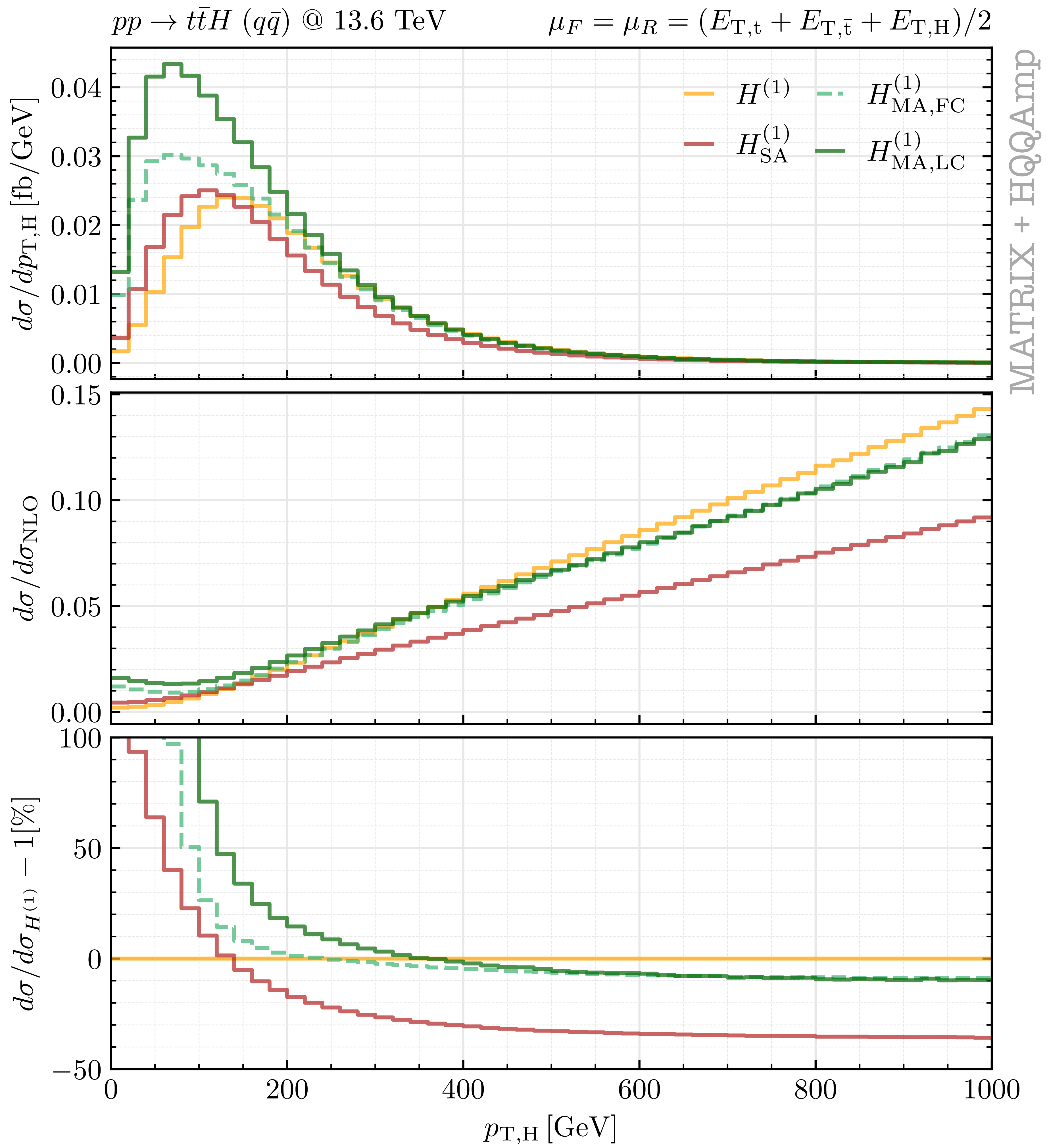
setup: NNLO NNPDF40, $m_H = 125.09GeV$, $m_t = 172.5GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

different setup!



massified results are in good agreement with the exact one-loop, with negligible effects on the NLO cross section in the tail

soft-approximated result is systematically below the exact one-loop, with effects of $\mathcal{O}(8\%)$ of the NLO cross section in the tail



First differential results: “best” $H^{(2)}$ prediction

setup: NNLO NNPDF40, $m_H = 125.09GeV$, $m_t = 172.5GeV$, $\mu_R = \mu_F = (E_{T,t} + E_{T,\bar{t}} + E_{T,H})/2$

H1-based error

$$\delta_{\rm SA}^{H^{(1)}} = 2 \times \left| \frac{\sigma_{H^{(1)}_{\rm SA}}}{\sigma_{H^{(1)}}} - 1 \right| \times \max\left(|\sigma_{H^{(2)}_{\rm SA}}|, |\sigma_{H^{(2)}_{\rm MA}}| \right)$$

$$\delta_{\rm MA}^{H^{(1)}} = 2 \times \max\left(\left| \frac{\sigma_{H^{(1)}_{\rm MA,FC}}}{\sigma_{H^{(1)}}} - 1 \right|, \left| \frac{\sigma_{H^{(1)}_{\rm MA,LC}}}{\sigma_{H^{(1)}}} - 1 \right| \right) \times \max\left(|\sigma_{H^{(2)}_{\rm SA}}|, |\sigma_{H^{(2)}_{\rm MA}}| \right)$$

μ_{IR}-variation error

$$\delta_{\rm SA}^{\mu_{\rm IR}} = \max\left(\left| \sigma_{H^{(2)}_{\rm SA}(\tilde{Q}/2)} + (Q/2 \to Q) - \sigma_{H^{(2)}_{\rm SA}} \right|, \left| \sigma_{H^{(2)}_{\rm SA}(2\tilde{Q})} + (2Q \to Q) - \sigma_{H^{(2)}_{\rm SA}} \right| \right)$$

$$\delta_{\rm MA}^{\mu_{\rm IR}} = \max\left(\left| \sigma_{H^{(2)}_{\rm MA}(\tilde{Q}/2)} + (Q/2 \to Q) - \sigma_{H^{(2)}_{\rm MA}} \right|, \left| \sigma_{H^{(2)}_{\rm MA}(2\tilde{Q})} + (2Q \to Q) - \sigma_{H^{(2)}_{\rm MA}} \right| \right)$$

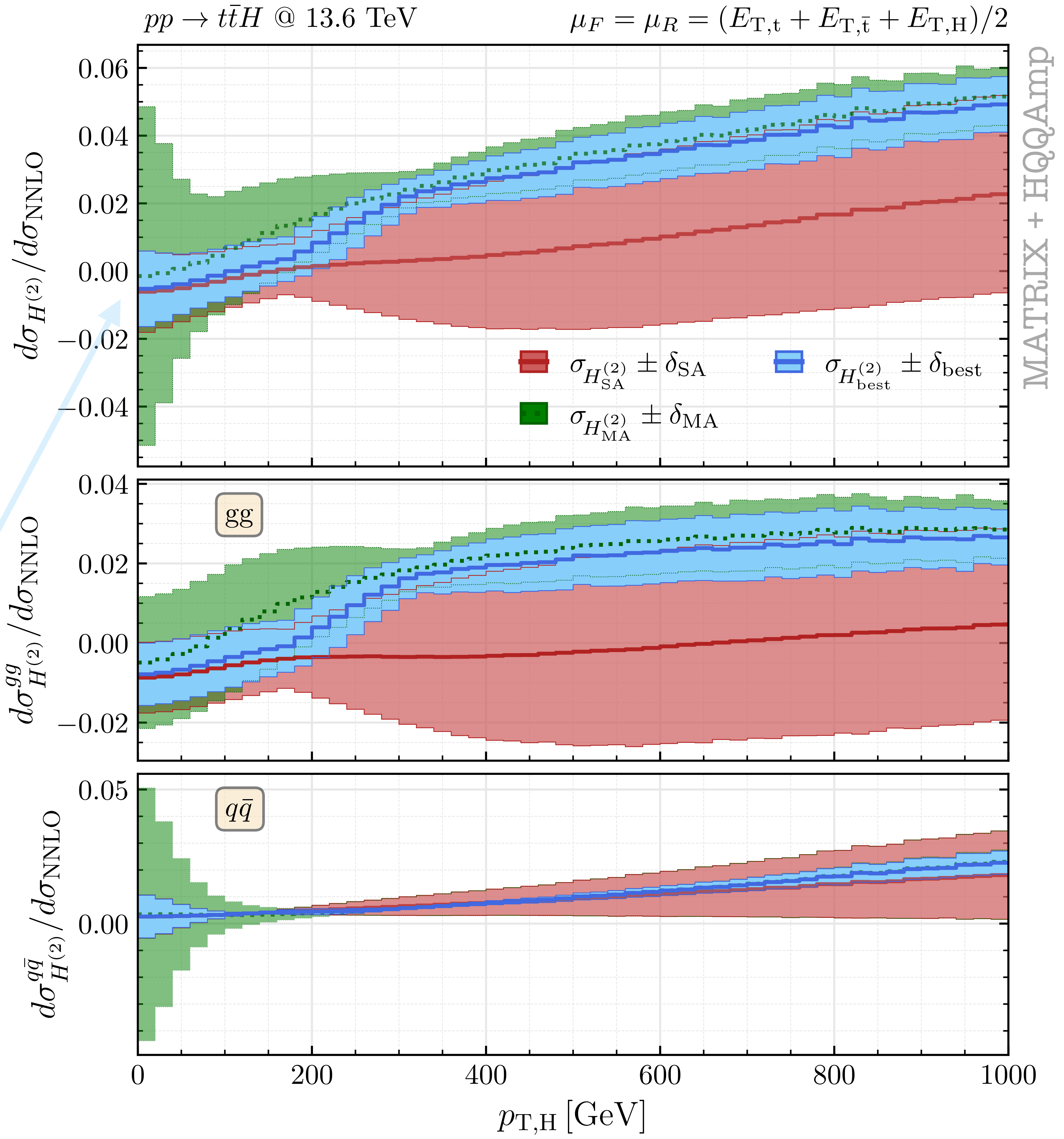
the final systematic error on each approximation and for each partonic channel is obtained by taking the maximum between $\delta^{\mu_{\rm IR}}$ and $\delta^{H^{(1)}}$

“best” for each partonic channel:

$$\sigma_{H^{(2)}_{\rm best}} = \frac{1}{\omega_{\rm SA} + \omega_{\rm MA}} \left(\omega_{\rm SA} \sigma_{H^{(2)}_{\rm SA}} + \omega_{\rm MA} \sigma_{H^{(2)}_{\rm MA}} \right)$$

$$\delta_{\rm best} = \left(\frac{1}{\omega_{\rm SA} + \omega_{\rm MA}} \right)^{1/2}$$

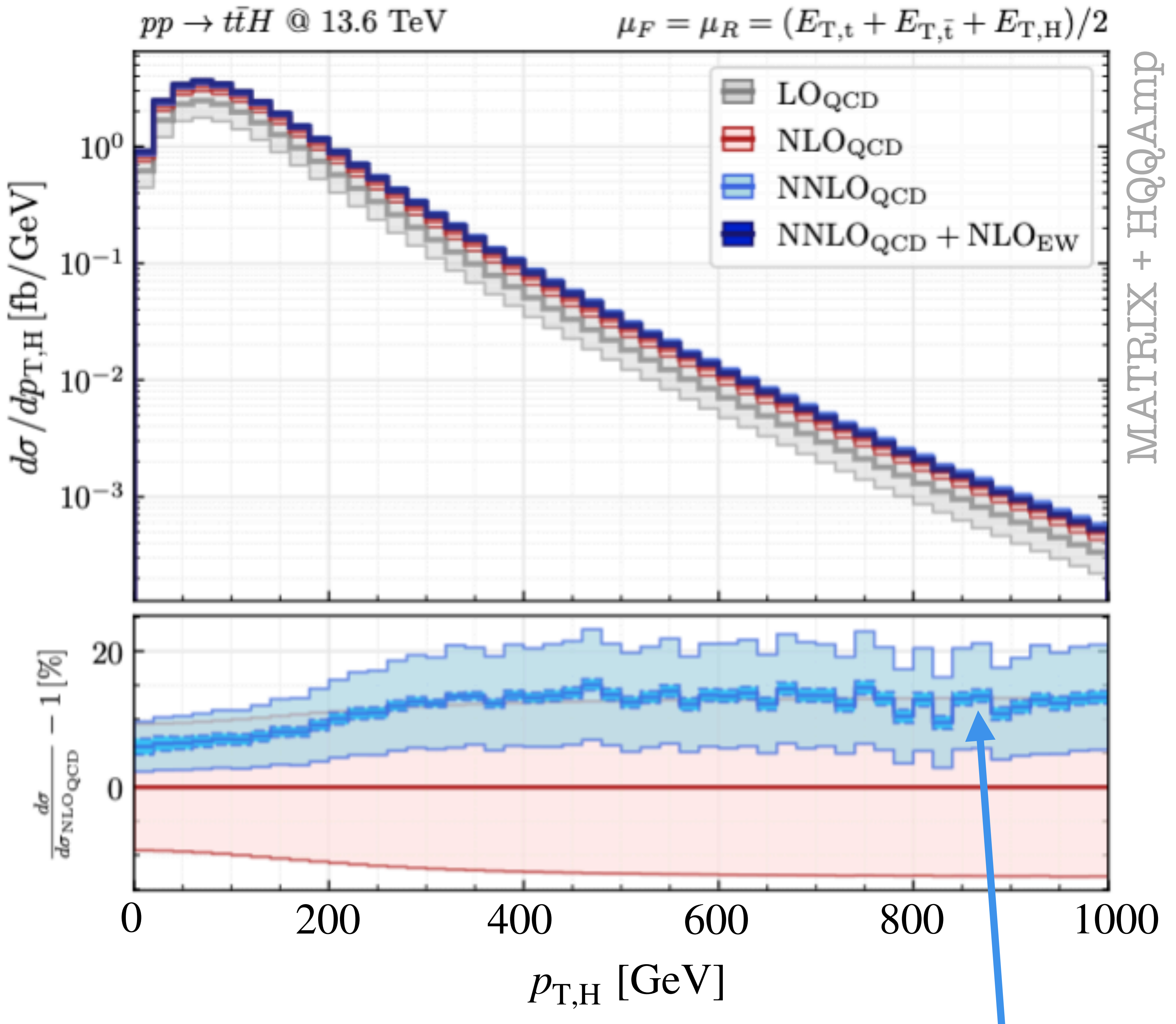
the errors on each channel are finally combined quadratically



1. the “best” prediction nicely interpolates between the two limits
2. the associated error does not vary strongly over the $p_{T,H}$ range
3. the individual soft and massified predictions have overlapping error bands

NNLO QCD + EW corrections

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$



systematic error associated with the "best" prediction for the double-virtual contribution

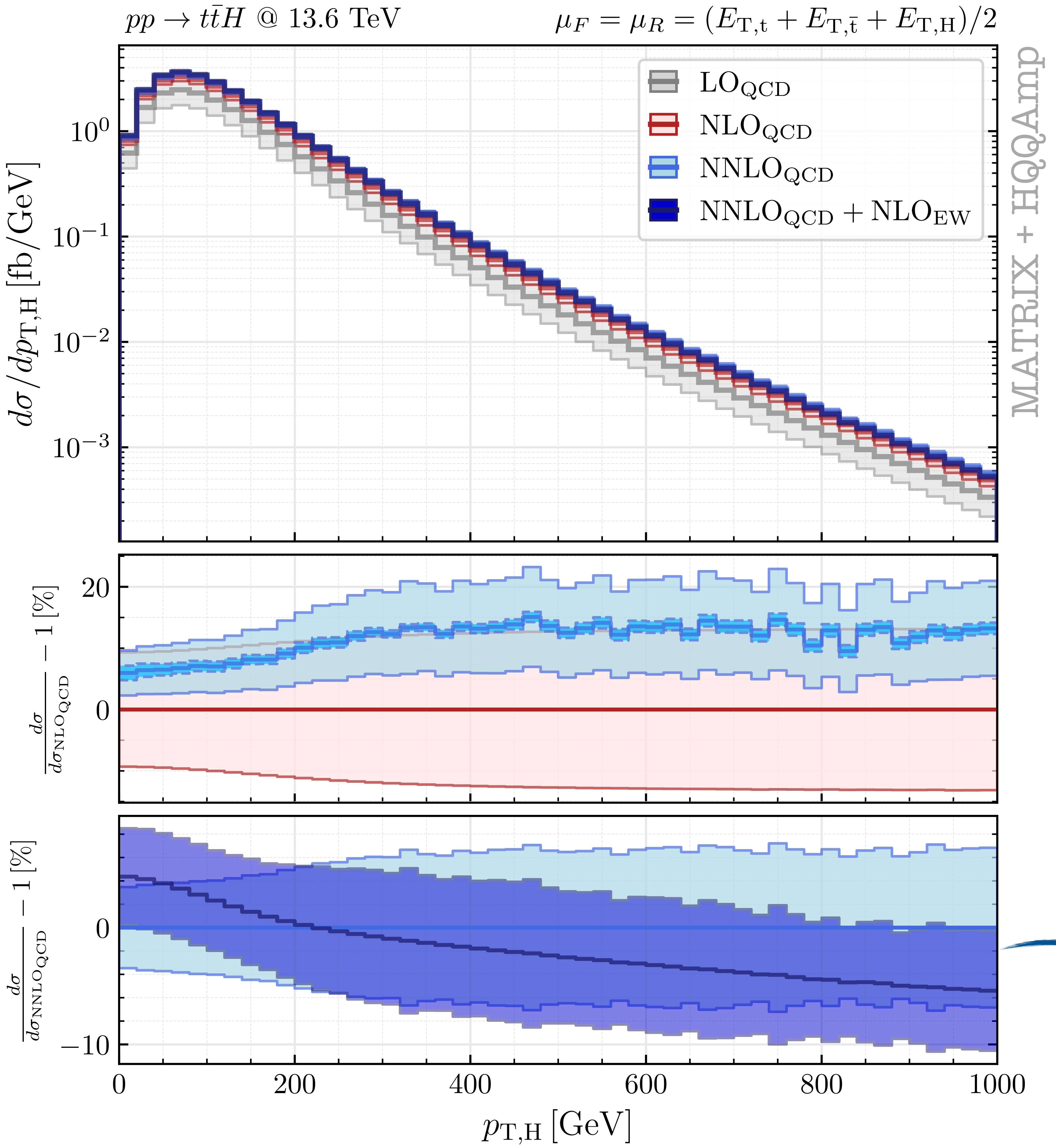
total XS at fixed scale $\mu_R = \mu_F = m_t + m_H/2$

$\sqrt{s} = 13.6$ TeV	σ [fb]
$\mathrm{LO_{QCD}}$	$423.9 \,^{+30.7\%}_{-21.9\%}$(scale)
$\mathrm{NLO_{QCD}}$	$528.9 \,^{+5.7\%}_{-9.0\%}$(scale)
$\mathrm{NNLO_{QCD}}$	$550.3(5) \,^{+0.9\%}_{-3.1\%}$(scale) $\pm 0.8\%$(approx)
$\mathrm{NNLO^{soft}_{QCD}}$	$548.7(5) \,^{+0.8\%}_{-3.0\%}$(scale) $\pm 0.6\%$(approx)

- NNLO QCD predictions based on the soft-approximated and "best" double virtual are **fully compatible**: difference of 0.3 %
- the **systematic uncertainty** based on the refined prescription is **slightly larger**: $\mathcal{O}(0.8\%)$ instead of $\mathcal{O}(0.6\%)$ of the NNLO cross section

NNLO QCD + EW corrections

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$



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$\mathrm{NNLO_{QCD}}$	$550.3(5)\ ^{+0.9\%}_{-3.1\%}$(scale) $\pm 0.8\%$(approx)
$\mathrm{NNLO_{QCD} + NLO_{EW}}$	$561.9(5)\ ^{+1.1\%}_{-3.2\%}$(scale) $\pm 0.8\%$(approx)

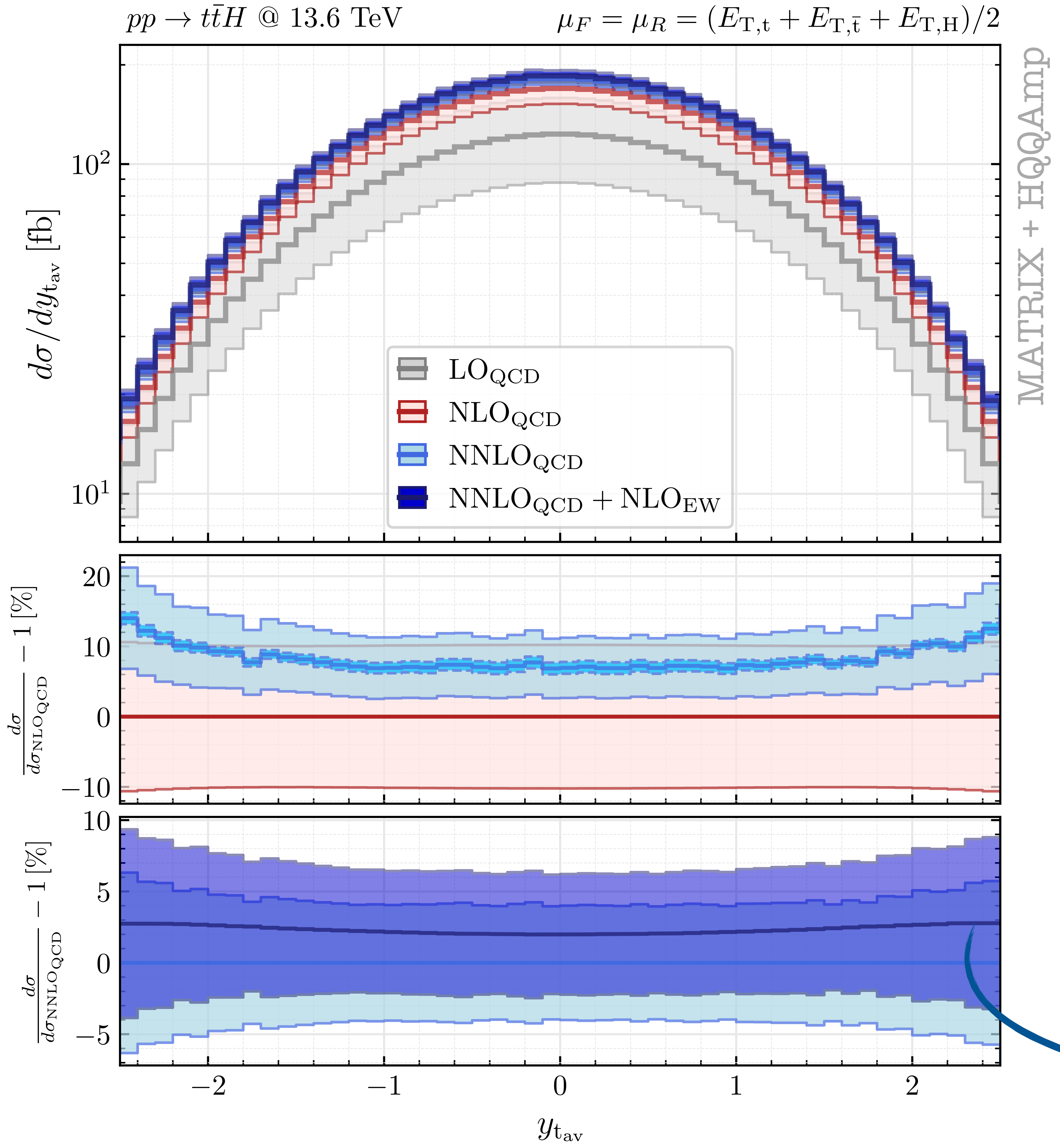
- inclusion of **all subdominant LO** ($\mathcal{O}(\alpha_s \alpha^2)$, $\mathcal{O}(\alpha^3)$) **and NLO** ($\mathcal{O}(\alpha_s^2 \alpha^2)$, $\mathcal{O}(\alpha_s \alpha^3)$, $\mathcal{O}(\alpha^4)$) contributions: $+2\,\%$ at the cross section level

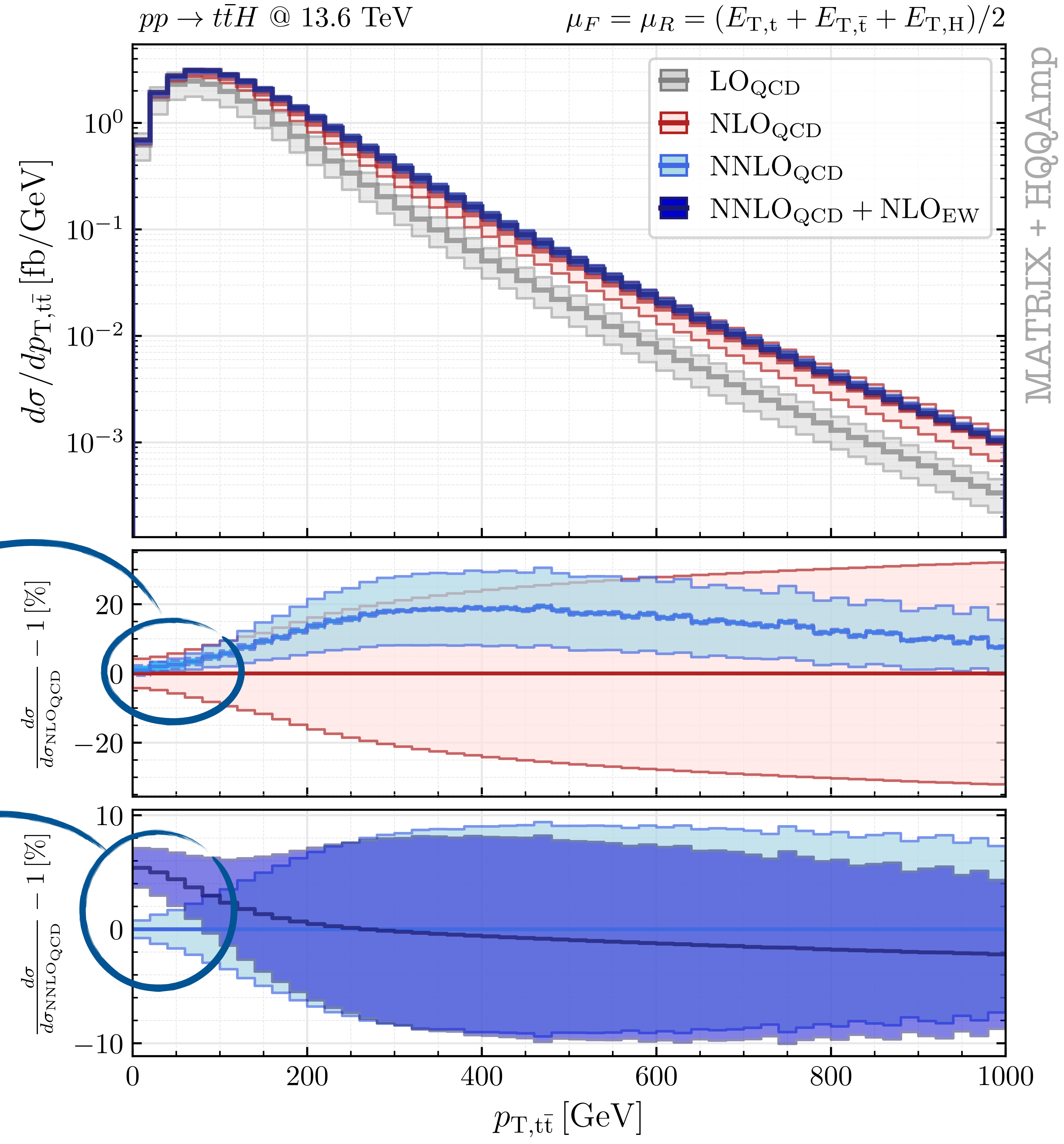
non-negligible impact compared to NNLO scale-variation bands

positive (negative) subdominant LO and NLO corrections in the small (large) $p_{T,H}$ region more distributions ...

NNLO QCD + EW corrections

setup: NNLO NNPDF40_nnlo_as_0118_qed, $m_H = 125.09 GeV$, $m_t = 172.5 GeV$





extreme reduction of the scale uncertainties

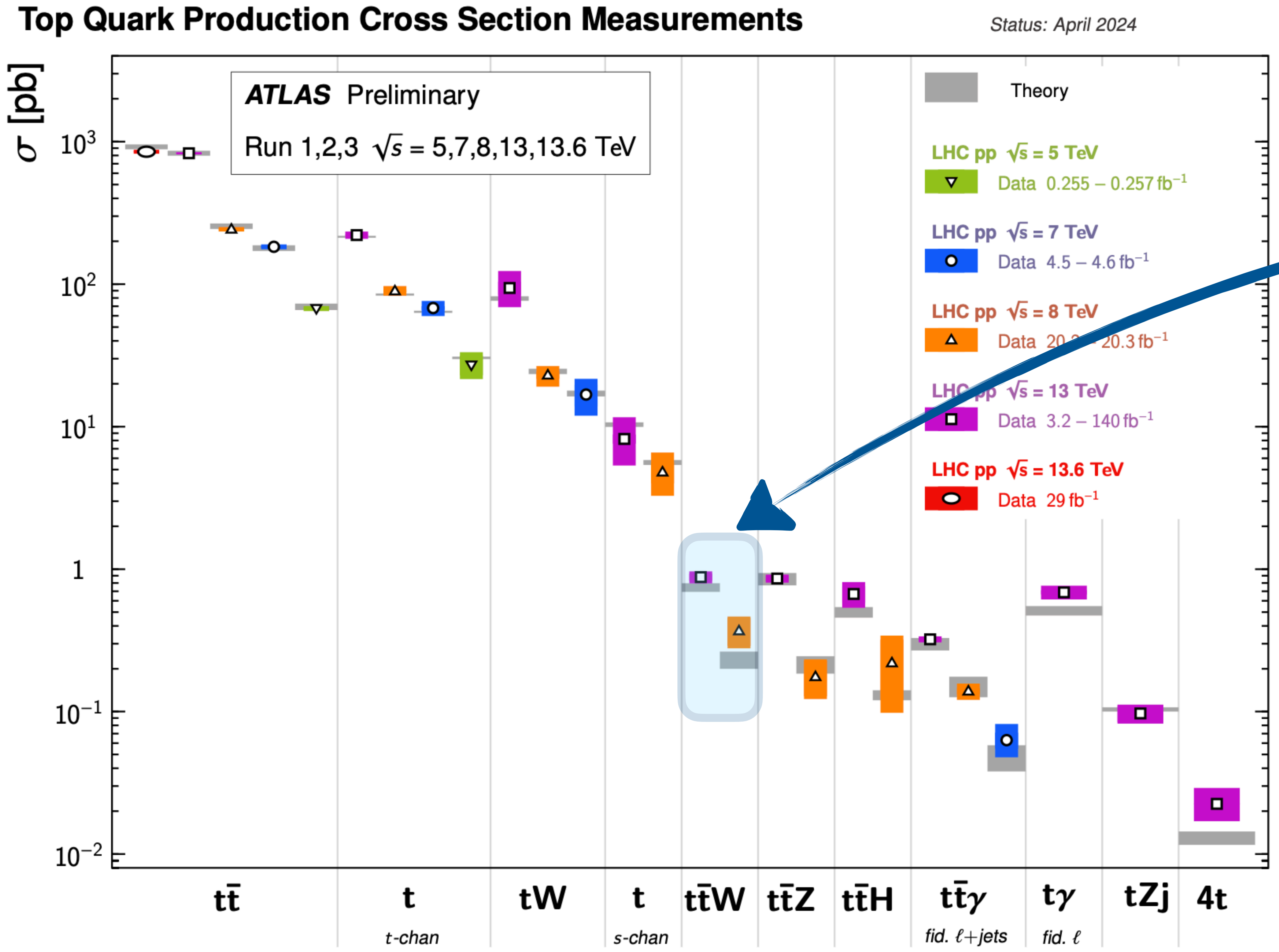
no overlapping bands

constant shift

Why is $t\bar{t}W$ production interesting ?

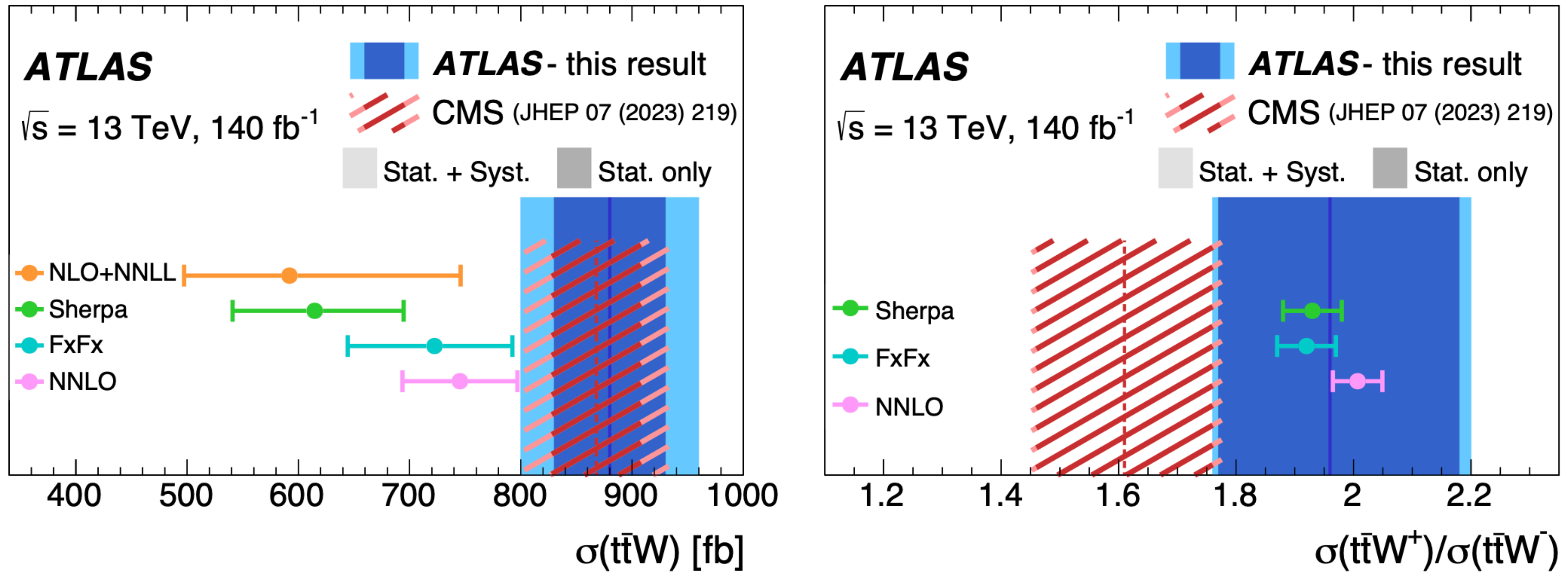
motivations:

- it is among the **most massive SM signatures** at hadron colliders
- **relevant background** for SM processes ($t\bar{t}H$, $t\bar{t}t\bar{t}$) and for BSM searches (in the multi-lepton signature)



[ATL-PHYS-PUB-2024-006 (2024)]

well-known tension between theory and experiments: slight excess at 1-2σ level (confirmed also by indirect measurements)

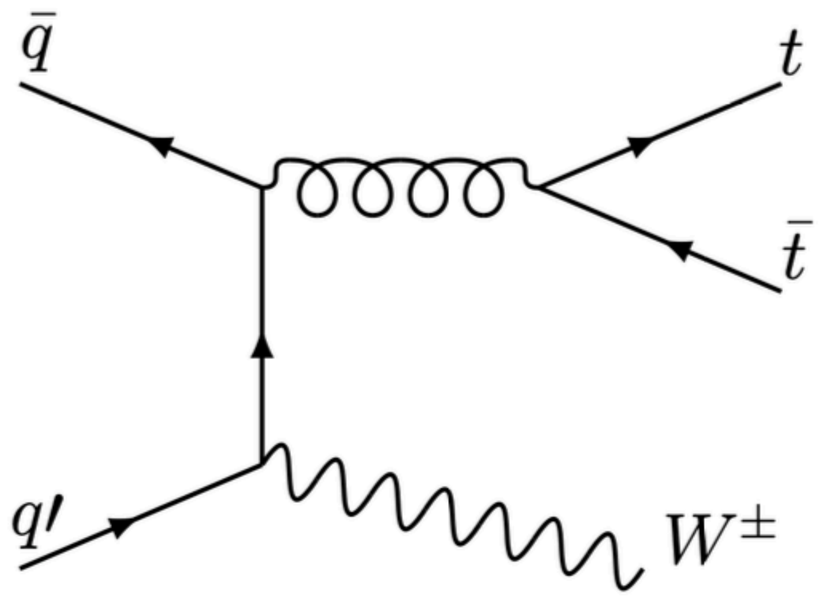


[ATLAS: arXiv 2401.05299]

Why is $t\bar{t}W$ production interesting ?

motivations:

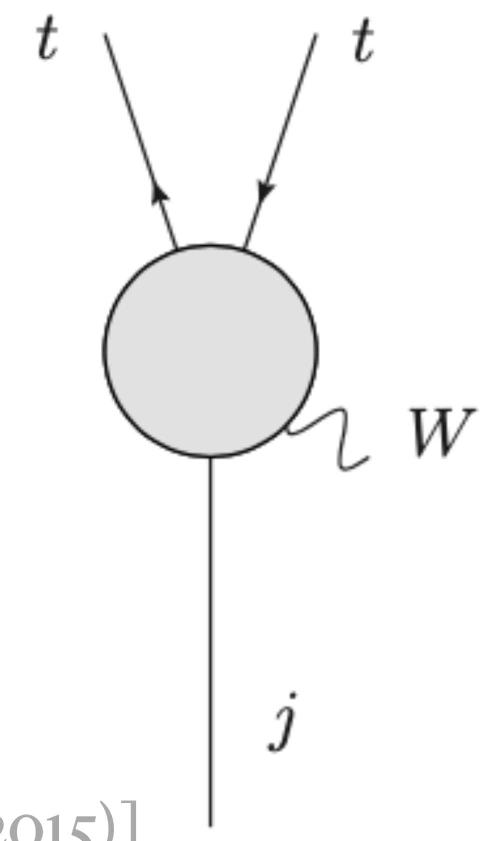
- among the other $t\bar{t}V$ ($V = \{H, Z, \gamma\}$) processes, $t\bar{t}W$ is rather peculiar since the W boson can only be emitted off an **initial-state light quark** (i.e. no gluon fusion at LO)
- different pattern of radiative corrections: both **QCD and EW corrections are relevant**



$\alpha_s^2\alpha$ LO QCD	$\cancel{\alpha_s\alpha^2}$	α^3 LO EW
$\alpha_s^3\alpha$ NLO QCD	$\alpha_s^2\alpha^2$ NLO EW	$\alpha_s\alpha^3$ α^4 subleading NLO EW

opening of the quark-gluon channel

large NLO QCD corrections dominated by configurations where the $t\bar{t}$ pair recoils against a hard jet, accompanied by a relatively soft W boson



[Maltoni, Pagani, Tsinikos (2015)]

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- different pattern of radiative corrections: both **QCD and EW corrections are relevant**

$\alpha_s^2\alpha$ LO QCD

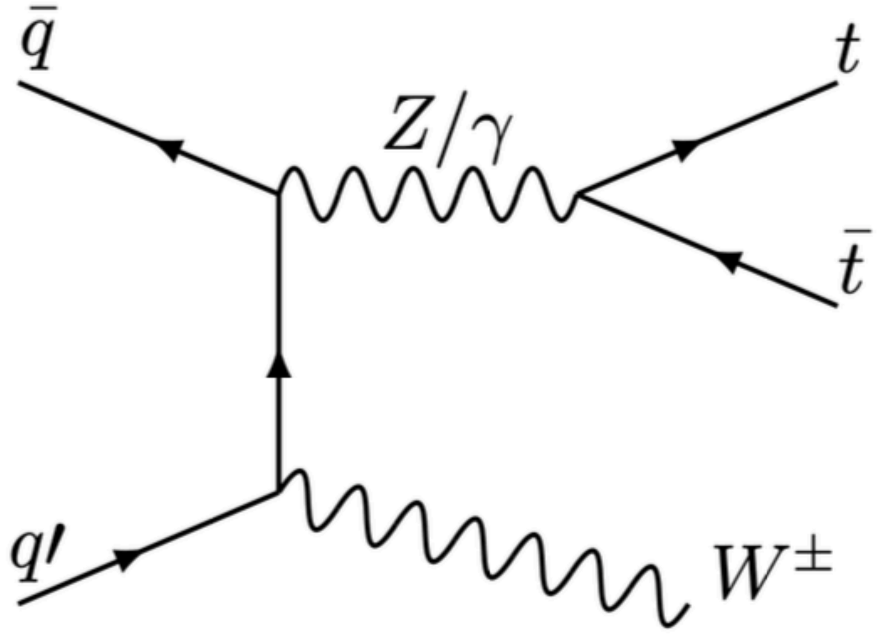
$\cancel{\alpha_s\alpha^2}$

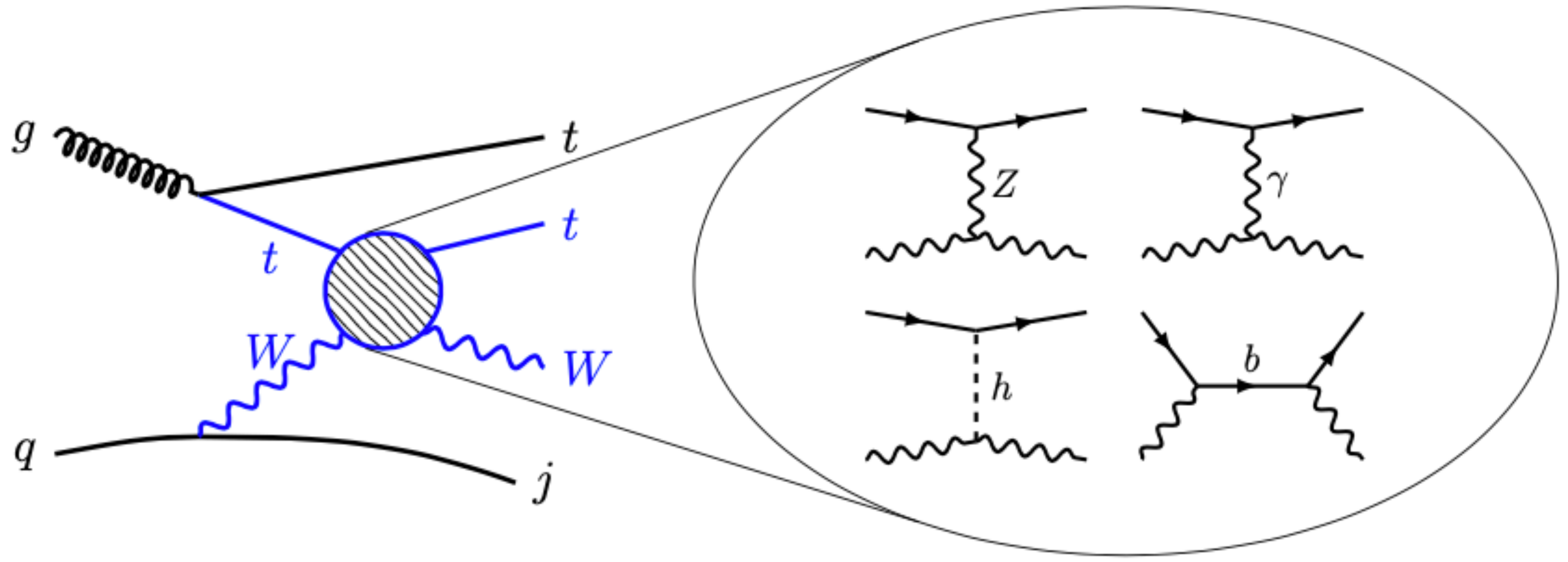
α^3 LO EW

$\alpha_s^3\alpha$ NLO QCD

$\alpha_s^2\alpha^2$ NLO EW

$\alpha_s\alpha^3$ α^4 subleading NLO EW





large positive subleading EW corrections $\mathcal{O}(10\%)$ at the LHC, which partially cancel against negative NLO EW $\mathcal{O}(-5\%)$.
Dominated by the opening of $tW \to tW$ scattering diagrams

[Frederix, Pagani, Zaro (2017)]

Theoretical predictions for $t\bar{t}W$

state of the art:

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Badger, Campbell, Ellis (2010-2012)]
- ☑ **NLO QCD + EW** corrections (*on-shell top quarks and W*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ inclusion of **soft gluon resummation at NNLL** [Broggio et al. (2016)] [Kulesza et al. (2019)]
- ☑ **NLO QCD** corrections (*full off-shell process, three charged lepton signature*) [Bevilacqua et al. (2020)] [Denner, Pellicciolі (2020)]
- ☑ combined **NLO QCD + EW** corrections (*full off-shell process, three charged lepton signature*) [Denner, Pellicciolі (2021)]
- ☑ experimental measurements are usually compared with **NLO QCD + EW** (*on-shell*) predictions supplemented with **multi-jet merging** [Frederix, Tsinikos (2021)]

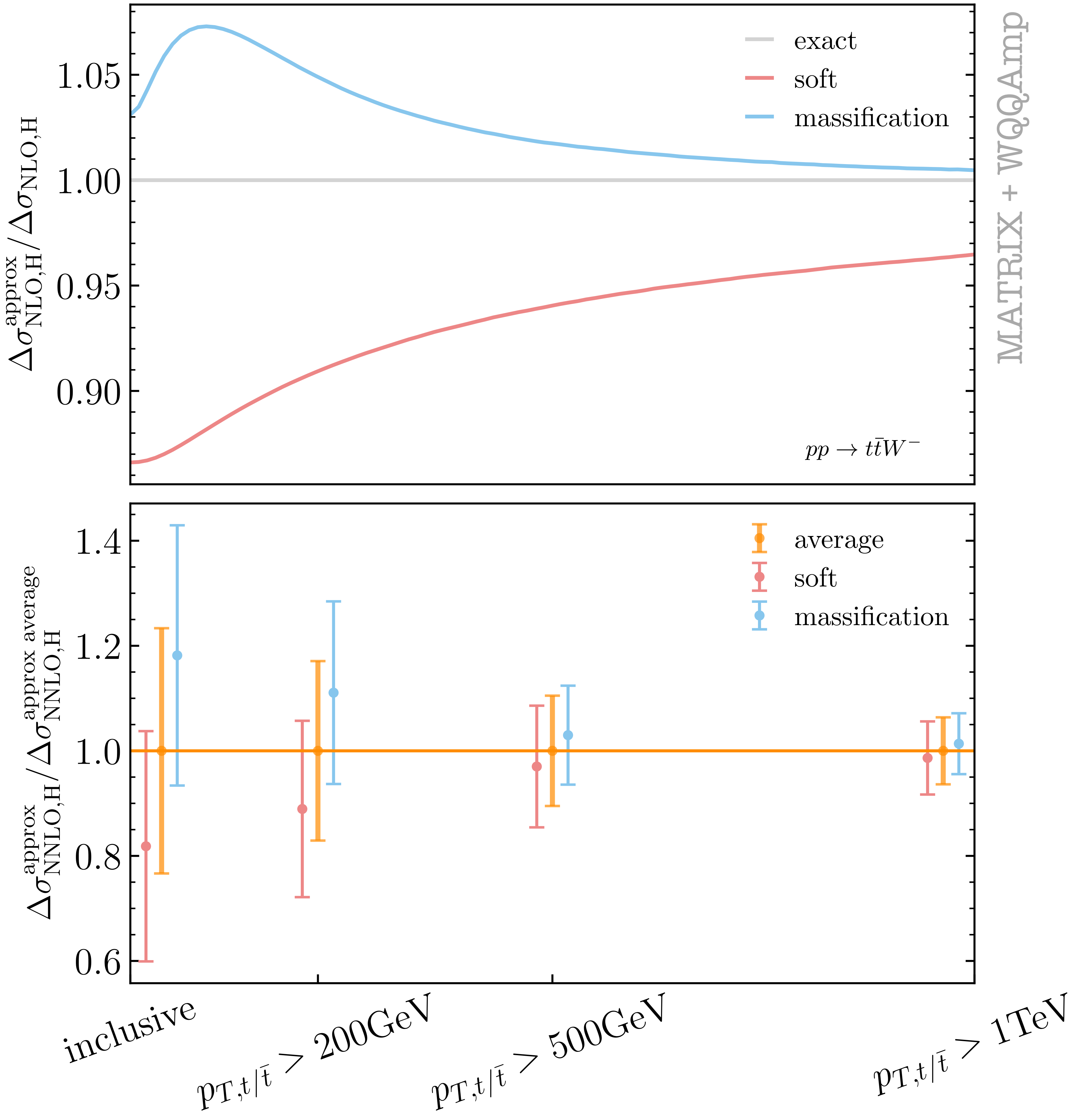
- ☑ **complete NNLO QCD + NLO EW** (*on-shell*) with approximated two-loop amplitudes

[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (2023)]

first NNLO calculation!

Results: estimate for $H^{(2)}$

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- at NLO both approaches show a **remarkable good agreement** with the exact virtual coefficient (discrepancy within 15%)

  agreement improved by the LO reweighting!

- at NNLO we define our best prediction as the arithmetic average of the two approximated results

- the **conservative systematic uncertainty** on the approximated two-loop contribution is defined by linearly combining the uncertainties on the two approximations

  the uncertainty on each approximation is computed as the maximum between the NLO discrepancy and effects due to μ_{IR} scale variation

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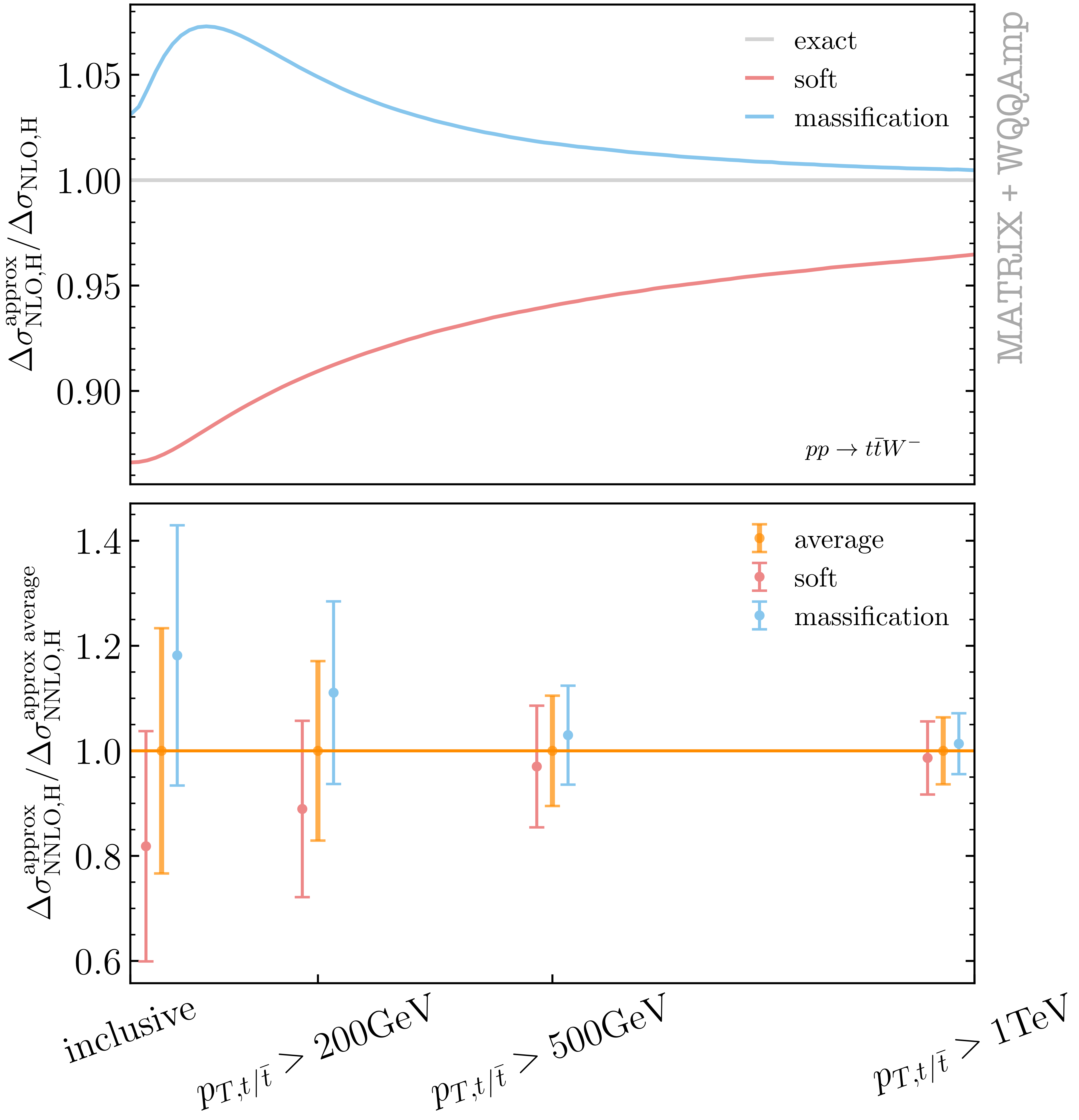

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  the uncertainty on each approximation is computed as the maximum between the NLO discrepancy and effects due to μ_{IR} scale variation

- the two-loop contribution turns out to be **6-7%** of the NNLO cross section

  relatively sizeable!

FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO} , $\pm 25\,\%$ on $\Delta\sigma_{NNLO,H}$

intermezzo: other ways of combining SA and MA

"best": refined procedure adopted also for $t\bar{t}H$

$$H^{(2)} \sim \frac{1}{\omega_{\rm SA}+\omega_{\rm MA}}\left(\omega_{\rm SA}H^{(2)}_{\rm SA}+\omega_{\rm MA}H^{(2)}_{\rm MA}\right)$$

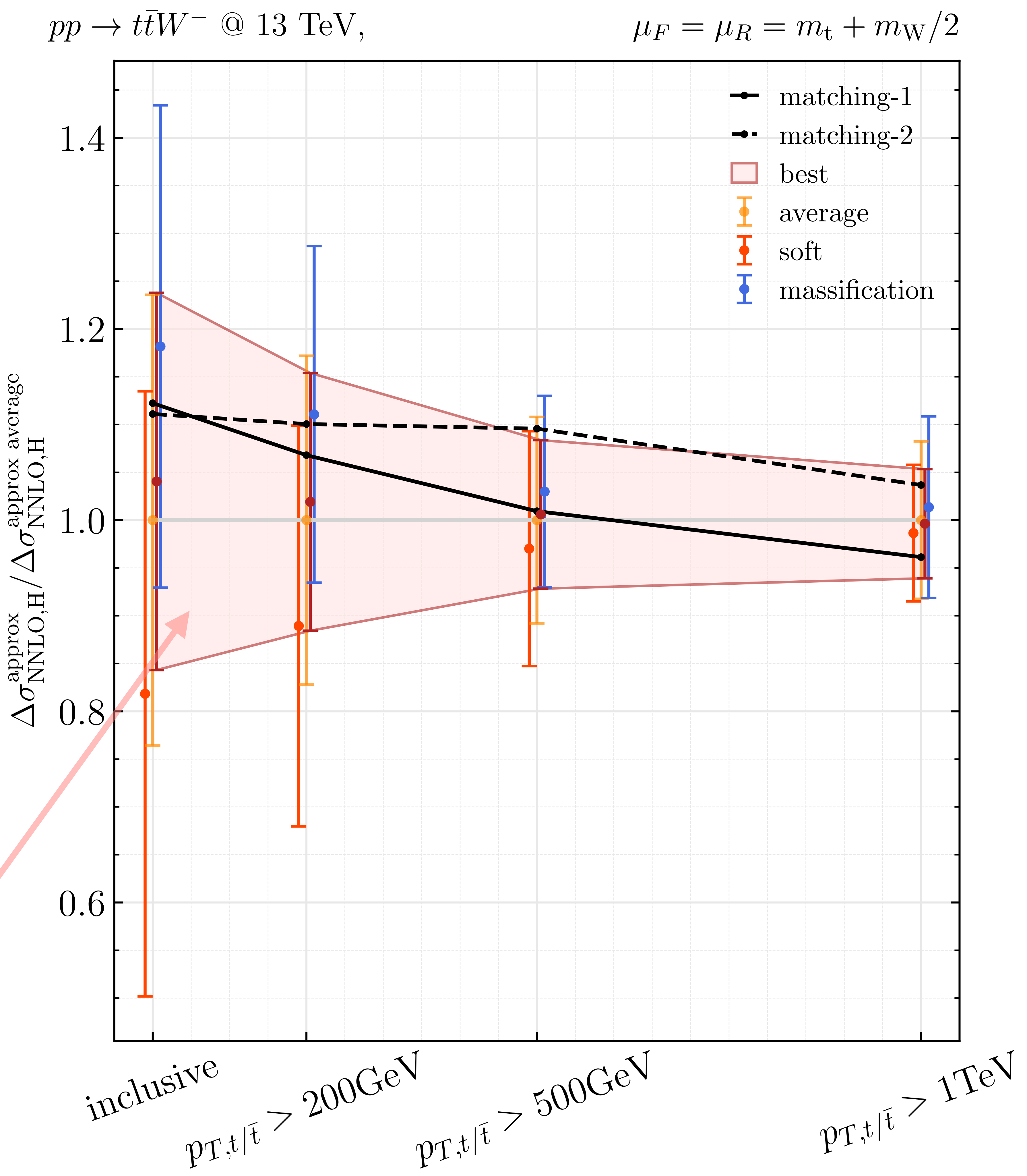
"matching-1"

$$H^{(2)} \sim H^{(2)}_{\rm MA} + H^{(2)}_{\rm SA} - H^{(2)}_{\rm SA\to MA}$$

different treatment of top-quark loops

"matching-2"

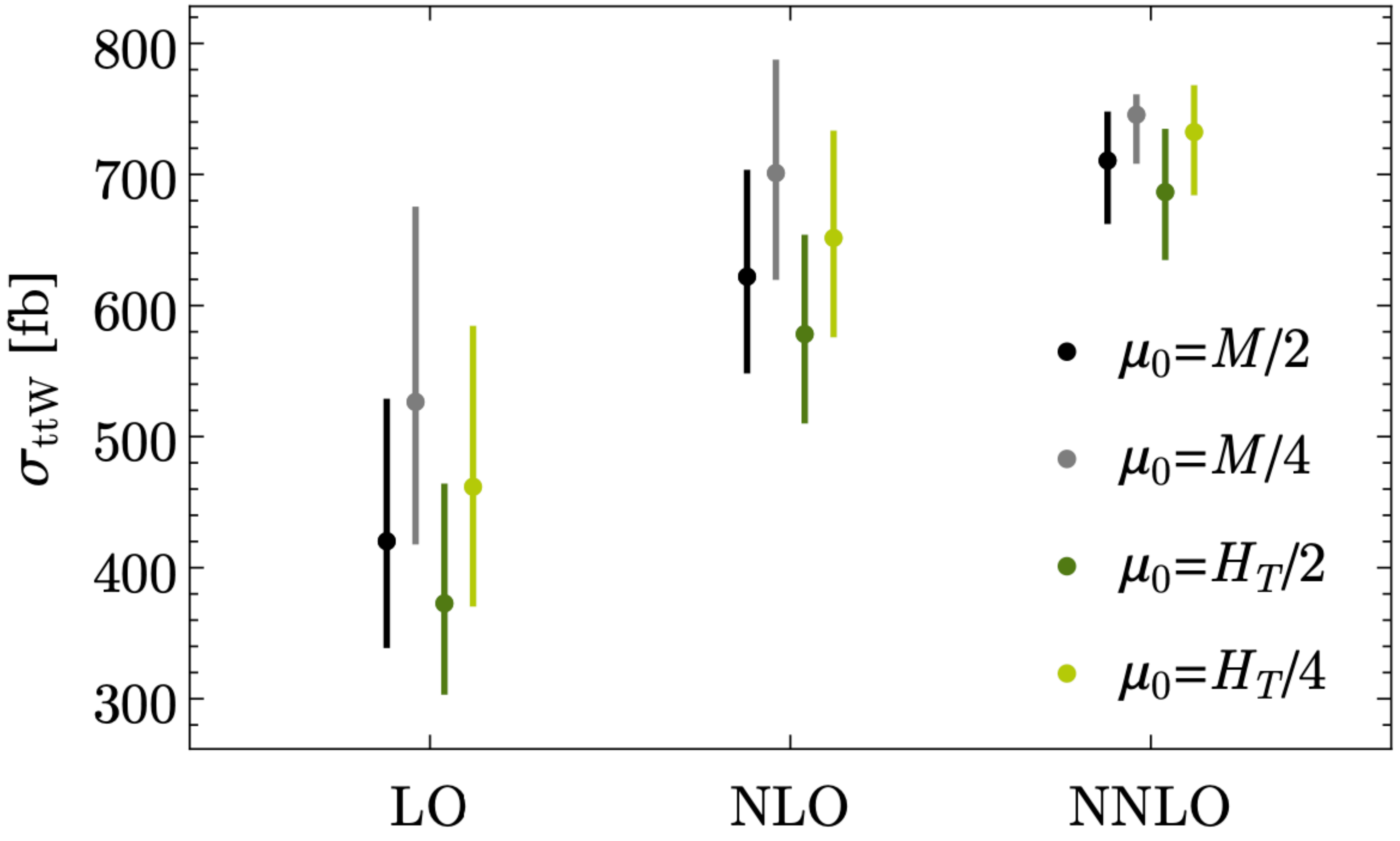
$$H^{(2)} \sim H^{(2),\rm ntl}_{\rm MA} + (H^{(2)}_{\rm SA} - H^{(2)}_{\rm SA\to MA}) + (H^{(2)}_{\rm SA} - H^{(2),\rm ntl}_{\rm SA\to MA})$$



1. the "best" and "average" predictions are almost identical (few % effects in the inclusive case)
2. larger effects from the "matching"
3. the various ways of combining SA and MA are however compatible within the quoted systematic uncertainties

Results: other sources of uncertainties

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$



$M \equiv 2m_t + m_W$

$H_T \equiv m_T(W) + m_T(t) + m_T(\bar{t})$

- **perturbative scale uncertainties**:
  - 7-point scale variation around the central scale $\mu_0 = M/2$
  - choice of other possible central scales
  - better convergence for smaller scales (exclude $\mu_0 = H_T/2$)
  - **symmetrisation** of the $M/2$ scale uncertainty

we rely on our perturbative scale uncertainties also because NNLO corrections are not dominated by new opening channels

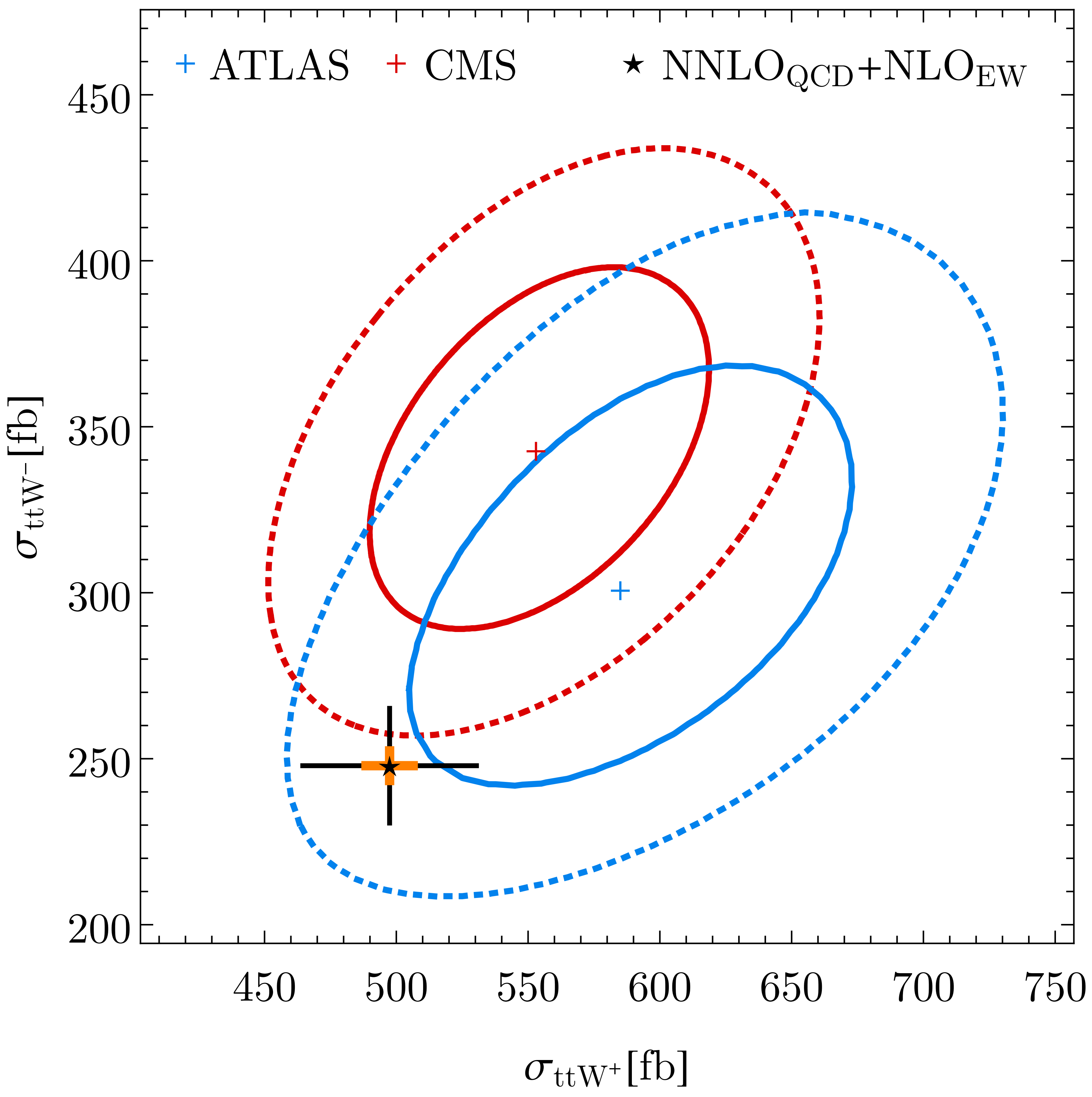
- **PDF** and α_s **uncertainties**: $\sim 2\,\%$

  (computed with the new MATRIX+PineAPPL implementation) [Devoto, Jezo, Kallweit, Schwan (in preparation)]

- statistical uncertainties: negligible

Results: comparison with data

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385GeV$, $m_t = 173.2GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
LO$_{\mathrm{QCD}}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
NLO$_{\mathrm{QCD}}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
NNLO$_{\mathrm{QCD}}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
NNLO$_{\mathrm{QCD}}$+NLO$_{\mathrm{EW}}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

- NNLO QCD corrections lead to
  - moderately higher rates (**+15%**)
  - reduction of the perturbative scale uncertainties
- inclusion of **all subdominant LO and NLO** contributions ($\mathcal{O}(\alpha^3)$, $\mathcal{O}(\alpha_s^2\alpha^2)$, $\mathcal{O}(\alpha_s\alpha^3)$, $\mathcal{O}(\alpha^4)$) labelled as NLO$_{\mathrm{EW}}$ (**+5%**)
- the **tension stays** at the 1σ (ATLAS) and 2σ (CMS) level respectively
- our result is **compatible** with FxFx: $\sigma_{t\bar{t}W}^{\mathrm{FxFx}} = 722.4^{+9.7\%}_{-10.8\%}$ fb

[ATLAS-CONF-2023-019]

[CMS: arXiv 2208.06485]

Results: comparison with data

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$

updated ATLAS measurement

$$\sigma_{\text{ATLAS}} = 880 \pm 50\,(\text{stat.}) \pm 70\,(\text{syst.}) = 880 \pm 80 \text{ fb}$$

	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
LO_{QCD}	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
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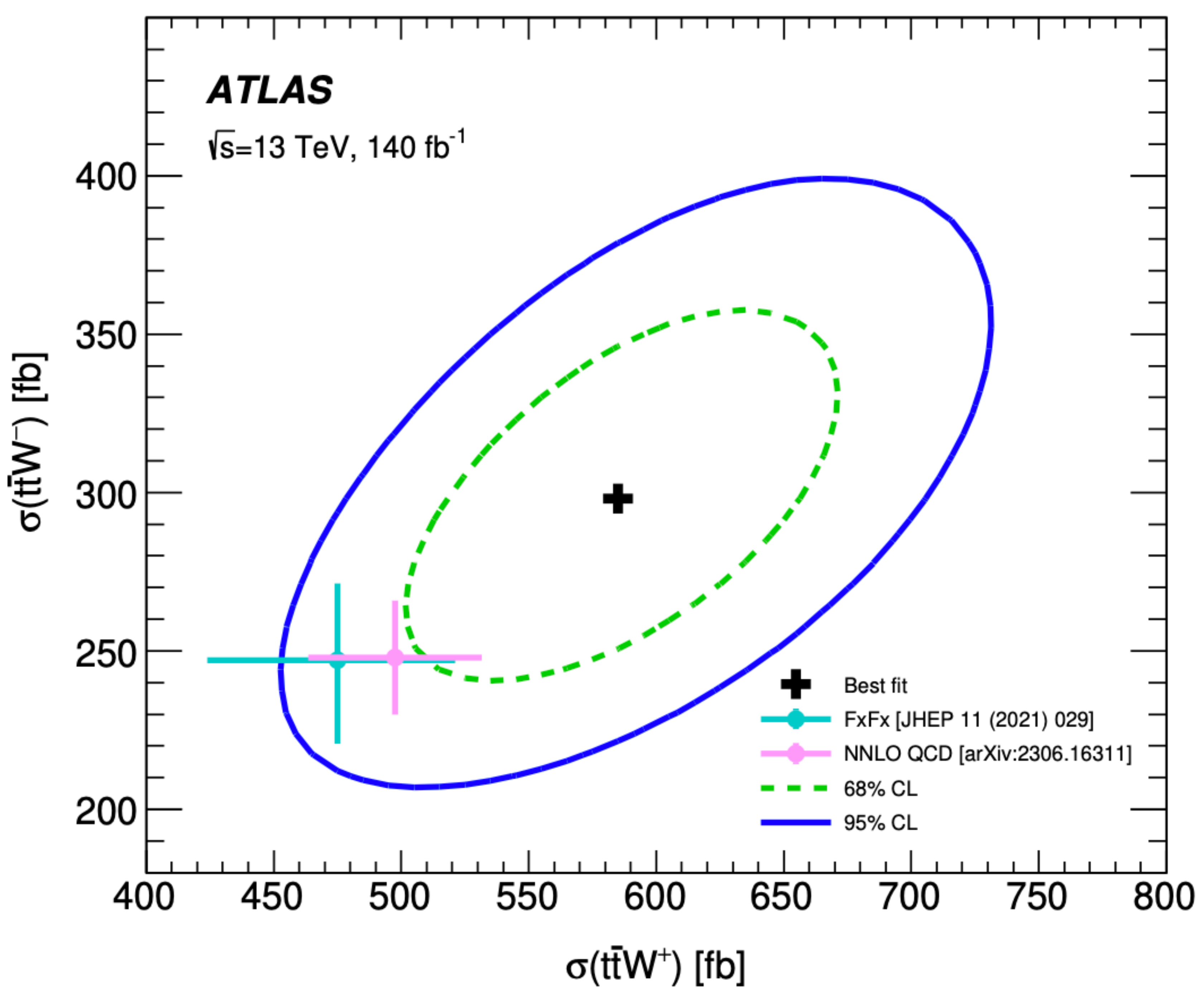
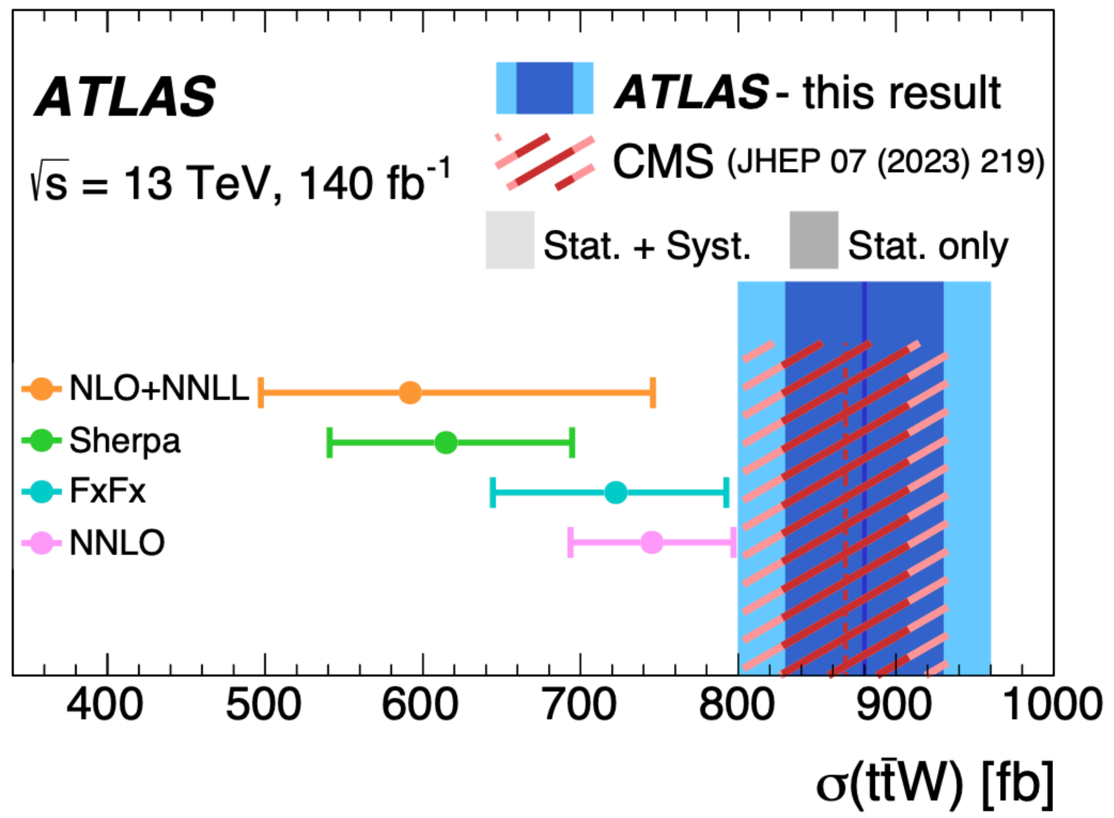
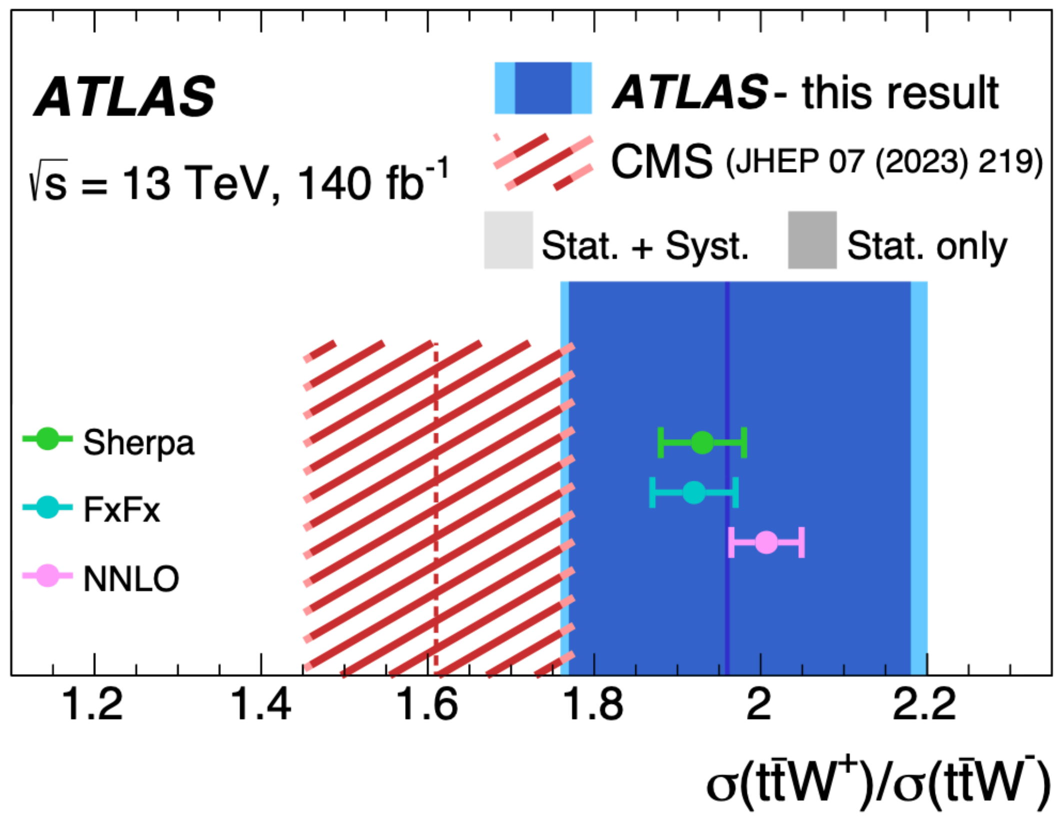
[ATLAS: arXiv 2401.05299]



- the updated measurement is **compatible** with our prediction at the level of 1.4σ
- good agreement also for the ratio

$$\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-) = 1.96 \pm 0.21\,(\text{stat.}) \pm 0.09\,(\text{syst.}) = 1.96 \pm 0.22$$





Summary

- As the LHC has entered its "precision" phase, more accurate theoretical predictions are of paramount importance
- the current frontier is represented by NNLO corrections for $2 \to 3$ processes with **several massive external legs**

main bottleneck: two-loop amplitudes

- the associated production of a Higgs or W boson with a top-quark pair ($t\bar{t}H$, $t\bar{t}W$) belongs to this category

- strategy: develop physically motivated, reasonable and reliable **approximations** for the double-virtual contribution

SOFT-BOSON APPROXIMATION MASSIFICATION

possible thanks to remarkable progress in two-loop 5-point scattering amplitudes!

- the quantitative impact of the genuine two-loop contribution, in our computation, is relatively **small** (~1% on σ_{NNLO}) for $t\bar{t}H$ and **moderate** (~6-7% on σ_{NNLO}) for $t\bar{t}W$
- however, we have achieved **good control** of the systematic uncertainties and a **reduction** of the perturbative uncertainties
- we produced results for the **total cross section**:
  - $t\bar{t}H$: moderate NNLO ($+4\,\%$) and EW ($+2\,\%$) corrections
  - $t\bar{t}W$: the inclusion of NNLO QCD + NLO EW corrections cannot "solve" the tension with the data ($\sim 1\sigma$ - 2σ)
- we have shown **first differential results** for $t\bar{t}H$

Summary

the journey towards a complete NNLO prediction (based on exact two-loop amplitudes) is still long ...but interesting times are ahead!

THANK YOU!