# Scattering Amplitudes in Quantum Field Theory WS 2021/2

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https://www.ph.tum.de/academics/org/cc/course/950594976/

**Sheet 04: Spinor-helicity formalism and color ordering** (12/11/2021)

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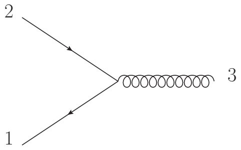
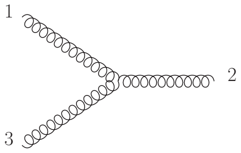
In this exercise sheet, we use various techniques introduced in the lecture to efficiently compute the tree-level amplitude for the scattering of two quarks and two gluons, $q\bar{q}gg \to 0$.

## QCD Feynman rules

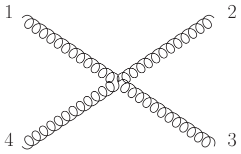
We use the following set of Feynman rules




$$= \mathrm{i} g_s t^{a_3}_{i_1 i_2} \gamma^{\mu_3} \,, \tag{1}$$




$$= g_s f^{a_1 a_2 a_3} \left[ g^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + \mathrm{cycl}(123) \right] \,, \tag{2}$$




$$= -\mathrm{i} g_s^2 \left\{ f^{a_1 a_2 b} f^{a_3 a_4 b} (g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} - g^{\mu_1 \mu_4} g^{\mu_2 \mu_3}) + \mathrm{cycl}(234) \right\} , \tag{3}$$

where all momenta are incoming.

## Amplitude $q\bar{q}gg$

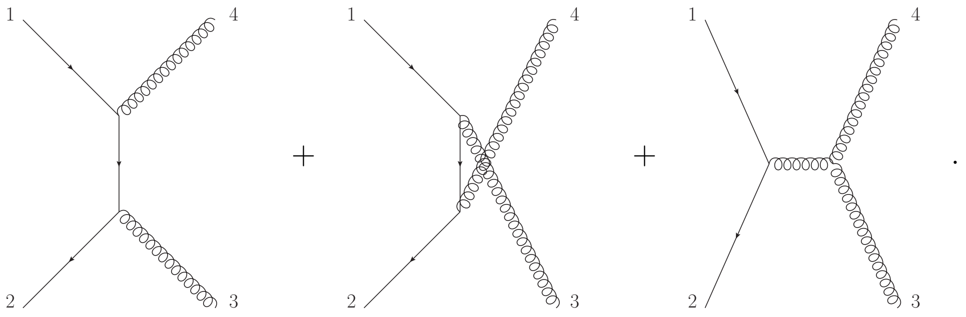
We choose all momenta to be incoming, i.e. we consider

$$q_{\lambda_1, i_1}(p_1) \; + \; q_{\lambda_2, i_2}(p_2) \; + \; g_{\lambda_3, a_3}(p_3) \; + \; g_{\lambda_4, a_4}(p_4) \to 0 \,, \tag{4}$$

where $\lambda_j$, $i_j$, and $a_j$, denote helicity indices, color indices in the fundamental, and color indices in the adjoint representation for the particle of momentum $p_j$, respectively. Momentum conservation implies

$p_{1234} = 0$, where $p_{i\ldots j} \equiv \sum_{n=1}^{j} p_n$. We find

$$i\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \quad (5)$$




### Part a

Use the fact that $[t^a, t^b]_{ij} = i f^{abc} t^c_{ij}$ to obtain the decomposition in color-ordered amplitudes

$$\mathcal{M}_{\lambda_1\lambda_2\lambda_3\lambda_4} = \mathcal{M}^{(1)}_{\lambda_1\lambda_2\lambda_3\lambda_4} t^{a_3}_{i_1 k} t^{a_4}_{k i_2} + \mathcal{M}^{(2)}_{\lambda_1\lambda_2\lambda_3\lambda_4} t^{a_4}_{i_1 k} t^{a_3}_{k i_2} \, . \quad (6)$$

Find a relation between $\mathcal{M}^{(1)}$ and $\mathcal{M}^{(2)}$.

### Part b

1. Enumerate all helicity configurations allowed for the external particles. How many independent helicity amplitudes are there?
2. Prove that all helicity amplitudes where the external gluons have the same helicity, are zero at tree-level.
3. Compute the independent color-stripped amplitudes $\mathcal{M}^{(j)}$ that contribute.

### Part c

Using the result of part b, compute the amplitude squared and summed over helicities. You should find the following result

$$\sum_{\lambda_{1,2}=L,R} \sum_{\lambda_{3,4}=\pm} |\mathcal{M}|^2 = g_s^4 \left[ 2\frac{(N^2-1)^2}{N} \frac{s_{13}^2 + s_{14}^2}{s_{13}s_{14}} - 4N(N^2-1)\frac{s_{13}^2 + s_{14}^2}{s_{12}^2} \right] , \quad (7)$$

where $s_{ij} = (p_i + p_j)^2$.