SCATTERING AMPLITUDES

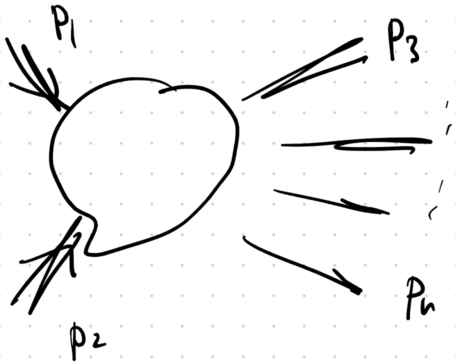
1- Intro and Poincaré

WS 2021

Technical University

Munich

SCATTERING AMPLITUDES



scattering of particles

with momenta p_i, σ_i

spins

etc

A amplitude $\Rightarrow$ probability amplitude

$$\sigma = \int |A|^2 \, dPs$$ cross-section

these objects are very beautiful math. obj

they have many nice properties.

the way we compute them with Feyn. Diag.

OBSCURES many of these properties. First

one is transf under Poincaré Transf.

SPACE-TIME SYMMETRIES, PARTICLES AND S-AMPL.

Amplitude computed from S-matrix

$$S = \langle \Psi^{out}_{p_1'\sigma_1' \dots p_n'\sigma_n'} | \Psi^{in}_{p_1\sigma_1 \dots p_n\sigma_m} \rangle$$

multi particle states

- One Particle states are defined as irreducible representations of POINCARE'

Lorentz + Translations

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + b^\mu$$

an element of P is $g(\Lambda, b)$

$$g(b,\Lambda) = \begin{bmatrix} & \Lambda^\mu{}_\nu & & & \begin{matrix} b^0 \\ b^1 \\ b^2 \\ b^3 \end{matrix} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Poincaré Group : Group of symmetries of NATURE, embedded in QUANTUM FIELD THEORY QFT

$\mathcal{P}$

- Physical State $\boxed{|\psi\rangle}$ which represents some kind of particle

they can be defined as set of states that only mix among themselves under $\mathcal{P}$.

$$|\psi_i\rangle \xrightarrow{\mathcal{P}} P_{ij} \, |\psi_j\rangle$$

* no subset must transform only among themselves

$\Longrightarrow$ IRREDUCIBLE REPR.

* matrix elements must be Poincaré Invariant

$$\mathcal{M} = \langle\psi_1| P^\dagger P |\psi_2\rangle = \langle\psi_1|\psi_2\rangle$$

$P^\dagger = P^{-1}$ UNITARY REPR.

IRREDUCIBLE REPRESENTATIONS are classified through the irreps of LITTLE GROUP of the particle's momentum p_μ

$$\boxed{W^{\mu\nu} p_\nu = p^\mu}$$

[WIGNER]

ONE PARTICLE STATES

$$U(\Lambda, a)\Psi_{p,\sigma} = e^{-ia\cdot p} \sum_{\sigma'} D^{(j)}_{\sigma\sigma'}(W(\Lambda, p))\, \Psi_{\Lambda p,\sigma'}$$

translations

Little Group of Poincaré for momentum p^μ

4

three different cases are physically relevant

- $p^\mu = 0$ $\Rightarrow$ Vacuum, nothing happens

- $p^2 \neq 0$; $p^2 > 0$ $\Rightarrow$ massive particles

Little group $SO(3)$ $\left(p^\mu = (m, 0, 0, 0) \right)$

representations spin j ; $(2j+1)$ states

$j =$ integer
half-integer

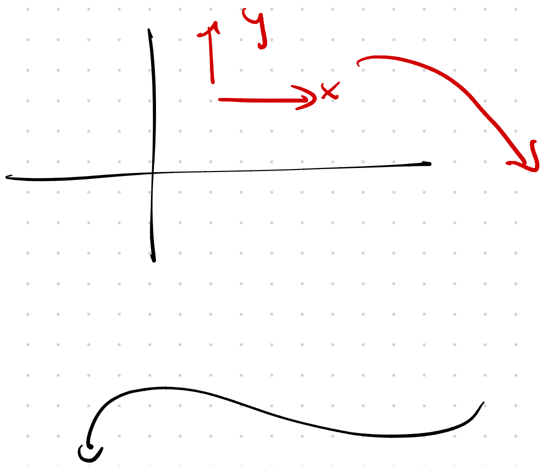
usual particles we know $\Rightarrow$ Little group tells us particles simply transform under $SO(3)$, like in Non Relativistic QM !

$$D^j_{\sigma\sigma'}(W(\Lambda, p)) = (2j+1)\text{-dim unitary matrices that represent } \underline{\underline{SO(3)}}$$

• $p^2 = 0$ light-like, massless particles

little group $p^\mu = (E, 0, 0, E)$

is $ISO(2)$ = Euclidean group of rot + transl in plane!

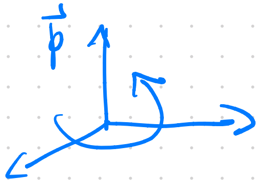


translations are represented with continuous eigenvalues!

We do not see continuous eigenvalues associated to particles!

Degenerate $ISO(2) \simeq SO(2)$ representations?

$\vec{p}$



this $SO(2)$, so rot around $\vec{p}$

$\Downarrow$

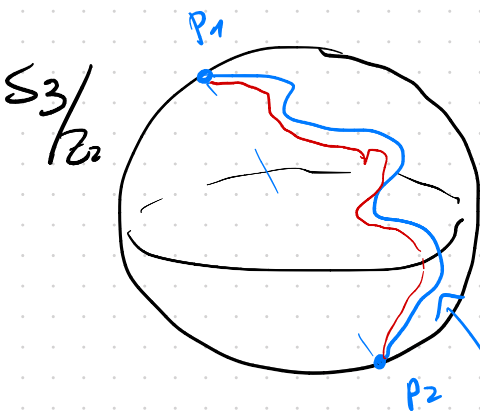
$$D^{(j)}_{\sigma\sigma'}(W(\Lambda, p)) = e^{ih\theta(\Lambda, p)} \Rightarrow$$

h = helicity is in principle also continuous

$\Rightarrow$ Since Lorentz Group (and so Poincaré)

is NOT simply connected, Topologically,

one can show that h must be n or $\frac{n}{2}$!



$p_1 \sim p_2$ topology Poincaré group

$$\mathbb{R}_4 \times \mathbb{R}_3 \times S_3/\mathbb{Z}_2$$

this path cannot be reduced to a point !

but if I go back and forth twice, it is

$$\Rightarrow e^{4\pi i h} = \text{must be trivial} = 1$$

$$\Longrightarrow h = \left\{ n \text{ or } \frac{n}{2} \right\} \; !$$

$n \in \mathbb{N}$

IMPORTANT : why photon $h = \pm 1$

and not just + or − ?

$\Rightarrow$ PARITY ! , invariance under discrete symmetries

swaps $\vec{p} \to -\vec{p}$, flips helicity

$h = \pm 1$ doublet under P in $O(1,3)$

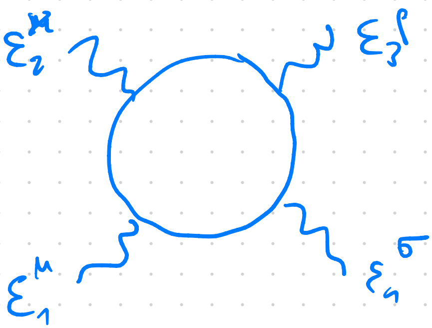
in a theory that is NOT parity invariant, we don't have doublets ! For example, if neutrinos had been massless $\nu = \pm \frac{1}{2}$ separate STATES

L , R neutrinos different PARTICLES

back to $S = \langle \Psi^{out}_{p'_1\sigma'_1 \dots p'_n\sigma'_n} | \Psi^{in}_{p_1\sigma_1 \dots p_m\sigma_m} \rangle$

this should transform as dictated by multiparticle states! $\Longrightarrow$ LITTLE GROUP

the way we usually compute S is through exp in Feynman Diagrams



$$= \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_3^\rho \varepsilon_4^\sigma \, F_{\mu\nu\rho\sigma}$$

Amputated Green Function

transforms as Lorentz TENSOR!

Pol vectors

Fix it by contracting Lorentz indices and bringing back Little Group!

VERY IMPORTANT : in QFT we usually work with non-observable FIELDS.

Fields do not need to transform as UNITARY IRREPS OF POINCARÉ GROUP $\Rightarrow$ THEY DON'T!

$$\psi^{a}(x) = \sum_{\lambda} \int d\tilde{p} \left[b(p,\lambda) \underbrace{u^{a}(p,\lambda) e^{ipx}} + h.c \right]$$

Field transforms as FINITE IRREP of LORENTZ (not unitary)

PARTICLE STATES transform as INFINITE UNITARY IRREPS of POINCARÉ

Connection!
$u^{a}(p,\lambda)$ wave F. a transforms as Lorentz
and as little group
$\Downarrow$
POLARISATIONS

in SCATT AMPL

So now we are left with this goal:

1- we need to understand Lorentz, In order to represent ε^{μ}, u^{a} etc

$\Rightarrow$ represent properly S-amplitude

irreps of LORENTZ GROUP

2- We want to use a "notation" that makes the "right" transformation under LITTLE GROUP straight forward

Spinor - Helicity Notation

two (many?) birds with one stone, this notation will be good also for other properties!