Scattering Amplitudes in Quantum Field Theory SS 2022

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https://www.ph.nat.tum.de/ttpmath/teaching/ss-2022/

Sheet 04: Higgs production via gluon fusion
Discussion: 08/07/2022

Introduction

In this exercise we will consider the scattering amplitude for the fusion of two gluons into a Higgs via the coupling to a top quark loop, which is the dominant production channel for the Higgs at the LHC. We will use generalised unitarity techniques for the reduction of the amplitude in terms of master integrals, along with the results from the previous exercise sheet for these integrals, to write an explicit formula for the amplitude. A diagrammatic depiction of this scattering process can seen in Figure 1.

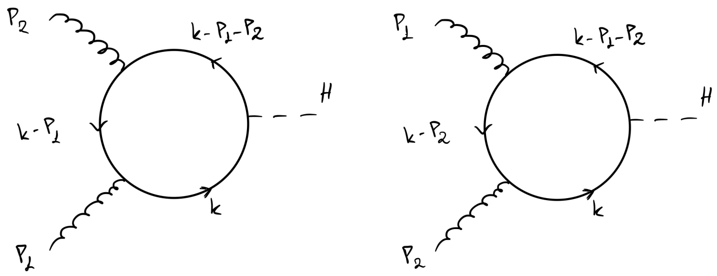

Figure 1: Feynman diagrams for $gg \to H$.

Amplitude for $gg \to H$

We call h_1, h_2 the polarisations of the gluons of momenta p_1, p_2 respectively. The amplitude can be written as follows,

$$\mathcal{M}^{a_1 a_2}_{h_1 h_2} = \delta_{a_1 a_2} g_H \alpha_S \mathcal{A}_{h_1 h_2}, \quad \mathcal{A}_{h_1 h_2} = \int \frac{\mathrm{d}^D k}{(2\pi)^D} A_{h_1 h_2}(p_1, p_2, k) \tag{1}$$

where g_H is the Higgs-Top Yukawa coupling constant and α_S denotes the strong coupling constant. The kinematics of this process is:

$$p_1^2 = p_2^2 = 0, \ (p_1 + p_2)^2 = m_H^2, \tag{2}$$

where m_H is the Higgs mass. In the following we will denote the fermion mass with m. We will also define the following set of propagators:

$$\begin{aligned} &D_0 = k^2 - m^2, \ D_1 = (k - p_1)^2 - m^2, \\ &D_2 = (k - p_2)^2 - m^2, D_3 = (k - p_1 - p_2)^2 - m^2. \end{aligned} \tag{3}$$

Note that, even though generalised unitarity is often used to avoid the use of Feynman diagrams altogether, here we will make use of it in order to reduce the amplitude to master integrals, starting from a representation in terms Feynman diagrams.

1. Using the diagrammatic representation of Figure 1, the massive fermion propagator $\frac{\not{k}+m}{k^2-m^2}$ and the conditions on the external gluons $p_i \cdot \varepsilon_j = 0,\ \forall\, i, j = 1, 2$, write the amplitude at the integrand level as follows

$$A_{h_1h_2} = \frac{\mathrm{Tr}\big[(\not{k}+m)\not{\varepsilon}_1(\not{k}-\not{p}_1+m)\not{\varepsilon}_2(\not{k}-\not{p}_1-\not{p}_2+m)\big]}{D_0D_1D_3} + (1 \leftrightarrow 2), \tag{4}$$

where the contribution of the second diagram is captured by the swapping of labels $(1 \leftrightarrow 2)$. What is the physical meaning of the four conditions $p_i \cdot \varepsilon_j = 0,\ \forall\, i, j = 1, 2$? Could we make a different choice?

2. Perform the trace and express the resulting scalar products between the loop momentum and the external momenta in terms of the propagators in eq. (3) to write $A_{h_1h_2}$ as follows,

$$A_{h_1h_2} = 4m\left[\frac{4(\varepsilon_1 \cdot k)(\varepsilon_2 \cdot k) - \frac{m_H^2}{2}(\varepsilon_1 \cdot \varepsilon_2)}{D_0D_1D_3} - \frac{(\varepsilon_1 \cdot \varepsilon_2)}{D_0D_3} + (1 \leftrightarrow 2)\right]. \tag{5}$$

3. Starting from the considerations made in the lecture, write down the most general decomposition for this three-point amplitude at the *integrand* level,

$$A_{h_1h_2} = \frac{c_1(k)}{D_0D_1D_3} + \frac{c_2(k)}{D_0D_2D_3} + \frac{b_{03}(k)}{D_0D_3} + \ldots. \tag{6}$$

Taking into account that the $gg \to H$ amplitude is ultraviolet-finite, what other terms apart from those explicitly given in (6) are allowed?

4. Compute the triple cut in (5) but *at variance with what was done in class*, keep also ϵ-dimensional terms, and show that the triangle coefficient is

$$c_1(k) = 4m(\varepsilon_1 \cdot \varepsilon_2)\left[2m^2 - \frac{m_H^2}{2} - 2\mu^2\right], \tag{7}$$

where $\mu = k \cdot n_\epsilon$.

5. Now focus on the first bubble coefficient $b_{03}(k)$. Show that it vanishes by computing the respective double cut.

6. Using the results in the previous two points, argue that the final form of the amplitude is

$$\mathcal{A}_{h_1h_2} = 8m(\varepsilon_1 \cdot \varepsilon_2)\left[\left(2m^2 - \frac{1}{2}m_H^2\right)I_3 - 2I_3(\mu^2)\right] \tag{8}$$

where

$$\frac{-i(4\pi)^{D/2}}{\mathrm{e}^{\epsilon\gamma_E}} I_3 = I(1, 1, 1). \tag{9}$$

For the definition of $I(1, 1, 1)$ see eq.(1) of **exercise sheet 03**. Notice that the funny prefactor in eq. (9) has the role of adjusting the integration measure from the unphysical one used in the previous exercise sheet, to the physical one.

7. Starting from your final result in eq. (8), discuss what happens for different choices of the helicities of the external gluons. You should find that some of the helicity amplitudes give an identically zero result. Can you justify physically why this should be the case?