Advanced Methods for Collider Physics

TUM-IMPRS Graduate School

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Ex-sheet 03: Colour singlet production at NLO QCD and infrared poles of 1-loop amplitudes

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1 Colour singlet production at NLO QCD

1.1 Real-emission amplitudes for Drell-Yan and Higgs boson production

- Compute the squared matrix element for the scattering processes

$$q(p_a) + \bar{q}(p_b) \to Z(p_Z) + g(k), \qquad q(p_a) + g(p_b) \to Z(p_Z) + q(k) \tag{1}$$

working in conventional dimensional regularisation (CDR), setting $d = 4 - 2\epsilon$. Express the final result in terms of Mandelstam invariants

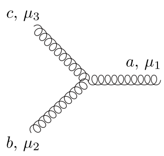
$$\hat{s} = (p_a + p_b)^2, \qquad \hat{t} = (p_a - k)^2, \qquad \hat{u} = (p_b - k)^2\,. \tag{2}$$

The external Z boson is on-shell and has mass m_Z.

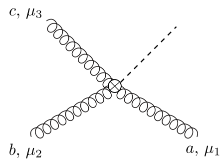
- Working in the heavy-top mass limit, namely adopting the effective Lagrangian

$$\mathcal{L}_{\text{HTL}} = -\frac{\lambda}{4}\, H\, G^{a_1}_{\mu\nu}\, G^{a,\mu\nu}, \qquad \text{with} \qquad \lambda = \frac{\alpha_s}{3\pi v}\left(1 + \left(\frac{5}{2}C_A - \frac{3}{2}C_F\right)\frac{\alpha_s}{2\pi}\right),$$

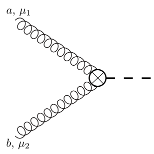
compute the squared matrix elements for all relevant real-emission channels contributing to Higgs boson production at NLO QCD. We recall that the relevant Feynman rules for the interaction vertices, including also those for standard QCD, are



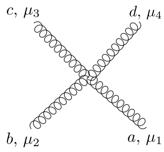
$$= -g_s f^{abc} \mathcal{T}^{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$



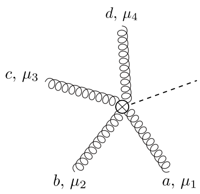
$$= -\lambda f^{abc} \mathcal{T}^{\mu_1\mu_2\mu_3}(p_1, p_2, p_3)$$



$$= -i\lambda\delta^{ab}\,(p_2^{\mu_1} p_1^{\mu_2} - g^{\mu_1\mu_2} p_1 p_2)$$



$$= -i g_s^2 \left[f^{acx} f^{bdx} \mathcal{G}^{\mu_1\mu_2\mu_3\mu_4} + f^{adx} f^{bcx} \mathcal{G}^{\mu_2\mu_1\mu_3\mu_4} + f^{abx} f^{cdx} \mathcal{G}^{\mu_1\mu_3\mu_2\mu_4} \right]$$



$$= -i\lambda \left[f^{acx} f^{bdx} \mathcal{G}^{\mu_1\mu_2\mu_3\mu_4} + f^{adx} f^{bcx} \mathcal{G}^{\mu_2\mu_1\mu_3\mu_4} + f^{abx} f^{cdx} \mathcal{G}^{\mu_1\mu_3\mu_2\mu_4} \right]$$

where we introduced the tensors

$$\mathcal{T}^{\mu_1\mu_2\mu_3}(p_1, p_2, p_3) = g^{\mu_1\mu_2}\,(p_1^{\mu_3} - p_2^{\mu_3}) + g^{\mu_2\mu_3}\,(p_2^{\mu_1} - p_3^{\mu_1}) + g^{\mu_3\mu_1}\,(p_3^{\mu_2} - p_1^{\mu_2})\,,$$
$$\mathcal{G}^{\mu_i\mu_j\mu_k\mu_l} = g^{\mu_i\mu_j} g^{\mu_k\mu_l} - g^{\mu_l\mu_i} g^{\mu_j\mu_k}\,,$$

and all the momenta $p_i^{\mu_i}$ in the vertices are taken as incoming.

1.2 Drell-Yan and Higgs one-loop form factor

Compute the one-loop QCD correction to the processes

$$q(p_a) + q(p_b) \to Z(p_Z), \qquad g(p_a) + g(p_b) \to H(p_H) \tag{3}$$

where the H couples to the gluon through a point-like effective vertex (see above). In deriving these results, extract all the relevant scalar one-loop integrals and compute them explicitly.
The Higgs-boson production amplitude, will contain both UV and IR singularities. You can renormalize the former in the so called $\overline{\text{MS}}$ scheme, which amounts to replacing the bare coupling constant α_s^0 with its renormalised counter part α_s as

$$\alpha_s^0 \mathcal{S}_\epsilon \mu^{2\epsilon} = \alpha_s \mu_R^{2\epsilon} \left(1 - \frac{\alpha_s}{4\pi}\frac{\beta_0}{\epsilon} + \mathcal{O}(\alpha_s^2)\right) \tag{4}$$

where β_0 is the first coefficient of the QCD β-function and it is given by $\beta_0 = (11/3)C_A - 4n_f T_F/3$ and μ_R is the renormalisation scale.

1.3 Contribution from quark-gluon scattering to the Drell-Yan cross section

Extend the discussion seen in class for the NLO QCD corrections to Z boson production to the case of the $q(p_a)g(p_b)$ scattering channel.

- Derive the differential cross-section in the variables z and λ seen in class. What are the differences in the $z \to 1$ and $z \to 0$ limit of the cross-section wrt the $q\bar{q}$ channel?
- Integrate over λ and write down the inclusive cross-section.
- Discuss the small p_T behaviour of the p_T differential cross section and compare it with the $q\bar{q}$ case.

In deriving this result you will need the Altarelli-Parisi splitting kernel

$$P_{qg}(z) = T_F\left[z^2 + (1-z)^2\right] . \tag{5}$$

1.4 NLO QCD corrections to Higgs-boson production

Using the amplitudes computed in exercises 1.1 and 1.2, and using the same phase-space parametrisation adopted in the Drell-Yan case, derive the double differential cross-section for Higgs boson production. In deriving this result you will need the Altarelli-Parisi splitting kernels

$$\begin{aligned} P_{gq}(z) &= C_F\left[\frac{1+(1-z)^2}{z}\right] , \\ P_{gg}(z) &= C_A\left[\frac{2z}{(1-z)_+} + \frac{2(1-x)}{z} + 2z(1-z)\right] + \frac{\beta_0}{2}\delta(1-z) . \end{aligned} \tag{6}$$

As in the previous exercise, discuss the main features of the cross section, in particular the $z \to 0$ and $z \to 1$ limit of the cross-section. What is the main difference arising from having initial-state gluons? How does the cross-section behave in the small p_T limit. What differs wrt the $q\bar{q}$ case in Drell-Yan production?

2 Infared poles of 1-loop QCD amplitudes

2.1 Color bases of low-multiplicity QCD amplitudes

Consider the $2 \to 2$ scattering processes

$$q_{i_1}\, \bar{q}_{i_2} \to Q_{i_3}\, \bar{Q}_{i_4}, \qquad q_{i_1}\, \bar{q}_{i_2} \to g^a\, g^b, \qquad g^a\, g^b \to g^c\, g^d \tag{7}$$

where the subscripts i_n and superscript a, b refer to color indices in the fundamental and adjoint representation.

- How are these amplitudes formally expressed in color space? Can you estimate a priori, i.e. without an explicit calculation what is the dimensionality of the basis?
- Derive the color bases spanning the complete color space for each amplitude. Express your results in terms of the so-called *tracebasis*, namely where all elements in the color space are written as product of $SU(N_c)$ operators T^a_{ij}. You may find the follow identities useful

$$f^{abc} = -2i\mathrm{Tr}\left(T^c\, \left[T^a, T^b\right]\right) \qquad d^{abc} = +2\mathrm{Tr}\left(T^c\, \left\{T^a, T^b\right\}\right), \tag{8}$$

  where d^{abc} is the totally symmetric structure constant
- Which elements in the color bases contribute at tree-level for each amplitude, namely are there any vanishing partial amplitudes at LO?
- How does the picture change if we add an extra gluon to each of the above amplitudes? After discussing the dimensionality of the bases, extract them explicitly.