Advanced Methods for Collider Physics

TUM-IMPRS Graduate School

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Day 5: LHC Phenomenology

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1 Introduction

The following problems include some simple calculations related to the lecture content given in the second week of the block course "Advanced Methods for Collider Physics". They cover a few topics related to the phenomenology of LHC physics, focussing on collinear factorisation and its application to scattering rate predictions. Some of the problems can be solved by hand, and others may be more easily solved using Mathematica or similar. Feel free to try the problems you wish and solve them how you prefer.
If you notice any errors in the problems/errors, please let me know.

2 The strong coupling

The beta function governing the running of the strong coupling constant is defined through

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} \alpha_s^{(n_f)}(\mu) = \beta^{(n_f)}\left(\alpha_s^{(n_f)}\right) = -\beta_0 \left(\alpha_s^{(n_f)}(\mu)\right)^2 + \mathcal{O}(\alpha_s^3)\,. \tag{1}$$

The first term in the perturbative expansion of β is β_0, and it is given by

$$\beta_0 = \frac{11C_A - 2n_f}{2\pi}\,. \tag{2}$$

Find an analytic solution for the running coupling by solving the above differential equation.

Using the analytic solution obtained in 1a, obtain a value for $\alpha_s(m_z = 91\ \text{GeV})$. As a boundary condition use that $\alpha_s(m_c = 1.5\ \text{GeV}) = 0.4269246842638939$, and that $m_b = 5$ GeV. [Hint: adjust the number of active flavours at the b-quark mass threshold]

3 Splitting functions, PDF evolution, etc.

In the lectures we discussed the DGLAP equation related to PDF evolution.

Write down the differential matrix equation in the space of quarks and gluons, and try to obtain the first-order solution to this equation for the evolution of a quark PDF [Hint: ignore the (higher-order) μ dependence of the PDFs and α_s when solving the diff. equation.]

The plus distribution, a generalised function, is defined according to

$$\int_0^1 \mathrm{d}x \frac{f(x)}{[1-x]_+} = \int_0^1 \mathrm{d}x \frac{f(x) - f(1)}{1-x}\,, \tag{3}$$

where $f(x)$ is a smooth function. These distributions are often encountered in perturbative calculations, and in many cases it is convenient/efficient to perform the integral in a restricted range $\int_y^1 \mathrm{d}x$.

Show how the integral of the plus distribution should be modified in this case. [Hint: When the integration of the plus term does not include the end-point, make the replacement $(1-x)_+ \to (1-x)$.]

One often encounters calculations which involve convolutions of splitting functions. For example, such convolutions take the form

$$\left[P^{(0)}_{i\to j}\otimes P^{(0)}_{j\to k}\right](z) = \int_z^1 \frac{dy}{y} P^{(0)}_{i\to j}(y) P^{(0)}_{j\to k}(z/y) \tag{4}$$

Calculate the convolution for the cases $P_{q\to g}\otimes P_{g\to q}$, and $P_{q\to q}\otimes P_{g\to q}$. The results can be compared to those in A.17 of [1].

For reference, the required splitting functions are

$$\begin{aligned} P^{(0)}_{q\to q}(z) =& C_F\left(\frac{1+z^2}{1-z}\right)_+ ,\\ P^{(0)}_{q\to g}(z) =& C_F\frac{1+(1-z)^2}{z} ,\\ P^{(0)}_{g\to q}(z) =& T_R\left(z^2+(1-z)^2\right) . \end{aligned} \tag{5}$$

4 Phase space and decay rates

In the lectures we discussed a recursive approach to implementing an n-body final-state phase space (in four space-time dimensions). As a simple application, we will consider the decay of a top-quark in various approximations.

Analytically calculate the decay rate for the process $t\to W^+b$, then make a numerical estimate of its partial width and lifetime. A calculation of the averaged/summed squared amplitude is given by

$$\overline{\sum}|M|^2 = \frac{2G_F|V_{tb}|^2}{\sqrt{2}} m_t^4\left(x^2\left(1+y^2\right)+\left(1-y^2\right)^2-2x^4\right) , \tag{6}$$

where $x = m_W/m_t$ and $y = m_b/m_t$.

Now consider the off-shell process $t\to(\ell^+\nu)\,b$ which involves three particles in the final state. Instead of attempting the analytic integration, devise a strategy to obtain the decay rate (potentially differentially in some of the outgoing particle kinematics) for this $1\to 3$ process.

References

[1] J. R. Gaunt, M. Stahlhofen, and F. J. Tackmann, *The Quark Beam Function at Two Loops*, *JHEP* **04** (2014) 113, [arXiv:1401.5478].