Advanced Quantum Mechanics SS 2024

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Sheet 03: Time dependent perturbation theory (to be handed in by 05.06.2024)

1 Density of states

Consider a single free particle in a box of length L with periodic boundary condition. As you should know from Quantum mechanics 1, momentum components will be quantised according to

$$p_i = \frac{2\pi\hbar}{L} n_i, \qquad n_i \text{ an integer} \tag{1}$$

We define the density of states as

$$\frac{d^3n}{dE} = \rho(E)d\Omega. \tag{2}$$

1. How is this relevant for the application of Fermi Golden Rule?

Compute $\rho(E)$ for

- 1. A non relativistic particle;
- 2. A relativistic particle;

Consider now the case of the decay of a particle of mass M into two particles of masses m_1 and m_2 . The detail of the interaction hamiltonian that causes the decay are not important.

- 3. How would you generalise Fermi Golden rule for multi particle final states?
- 4. Compute $\rho(E)d\Omega$ in the decay situation outlined above, where E is the energy related to the single independent final state momentum.

2 Time dependent perturbation to the Hydrogen atom

Consider an Hydrogen atom in the ground state. At $t = -\infty$ the time dependent electric field

$$\vec{E}(t) = \vec{E}_0 e^{-t^2/\tau^2}.$$
(3)

- 1. Determine the time dependent perturbation.
- 2. Use first order perturbation theory to determine the probability to find the atom in any of the n = 2 states.



3 Sudden Approximation: the box

Consider a particle in the ground state of a box of length L. Argue on semiclassical grounds that the natural time period associated with it is $T \approx \frac{mL^2}{\hbar\pi}$. If the box expands symmetrically to double its size in time $\tau \ll T$, what is the probability of catching the particle in the ground state of the new box?

4 Sudden Approximation: β decay of H^3

In the β decay of tritium $H^3 \longrightarrow (He^3)^+ + e^- + \nu$, the emitted electron has a kinetic energy of 16 keV. Argue that the sudden approximation may be used to describe the response of an electron that is initially in the ground state of H^3 .

- 1. Compute the probability to find the electron in the ground state of $(He^3)^+$.
- 2. What is the probability for it to be in the state $|n = 16, l = 3, m = 0\rangle$ in this approximation?

5 A first look at scattering: potential well

A particle in one dimension travelling with momentum $p = \hbar k > 0$ from $x = -\infty$ encounters the steep-sided potential well $V(x) = -V_0 < 0$ for |x| < a. Use the Fermi golden rule to show that the probability that the particle will be reflected by the well is

$$P_{reflect} \approx \frac{V_0^2}{4E^2} \sin^2(2ka) \tag{4}$$

and compare it with the limit $E = \frac{p^2}{2m} \gg V_0$ of the exact reflection probability. Hint: adopt periodic boundary conditions so that the wave-functions of the in-state and out-state can be normalised and remember that Fermi golden rule gives you the probability rate.

6 Appendix

The first eigenfunctions of the hydrogen atom are

$$\psi_{1,0,0}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}} \exp\left(-\frac{r}{a_0}\right)$$

$$\psi_{2,0,0}(r,\theta,\phi) = \frac{1}{4} \left(2 - \frac{r}{a_0}\right) \frac{1}{\sqrt{2\pi a_0^3}} \exp\left(-\frac{r}{2a_0}\right)$$

$$\psi_{2,1,0}(r,\theta,\phi) = \frac{1}{4} \left(\frac{r}{a_0}\right) \frac{1}{\sqrt{2\pi a_0^3}} \exp\left(-\frac{r}{2a_0}\right) \cos(\theta)$$

$$\psi_{2,1,\pm 1}(r,\theta,\phi) = \mp \frac{1}{8} \left(\frac{r}{a_0}\right) \frac{1}{\sqrt{2\pi a_0^3}} \exp\left(-\frac{r}{2a_0}\right) \sin(\theta) e^{(\pm i\pi)\theta}$$