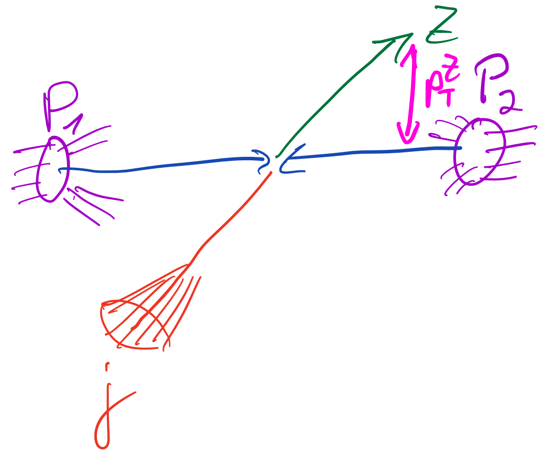
Resummation, parton showers & matching

Recap: Higher-order calculations



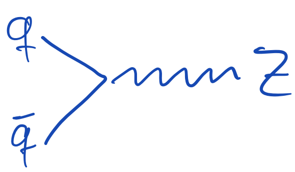
$p_T = \sqrt{p_x^2 + p_y^2}$

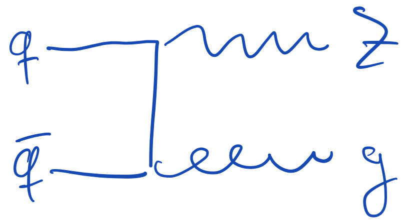
cross section:

$$d\sigma^{had.} = \sum_{i,j} dx_1\, dx_2\, f_i(x_1,\mu_F)\, f_j(x_2,\mu_F)\, \underbrace{d\sigma^1_{ij}(x_1P_1, x_2P_2, \mu_F)}_{= d\phi_n \frac{1}{2s} |\overline{\mathcal{M}}_{ij}|^2}$$

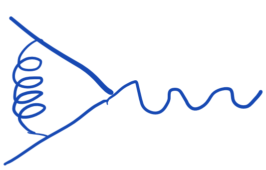
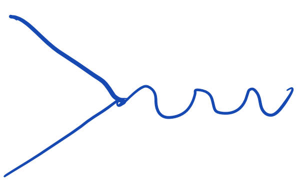
average/sum over spin & colour for incoming/outgoing particles

example pp → Z + X ← anything els

LO: $\Big|$  $\Big|^2$ → $d\sigma^B$ (Born)

NLO: $\Big|$  $\Big|^2$ → $d\sigma^R$ (Real)

PDF collinear counterterm let's assume to be implicit here

2 Re $\Big|$  $\otimes$  $\Big|$ → $d\sigma^{V(+PDF)}$ (Virtual)

2

NLO cross section

• (local) subtraction

$$\sigma^{NLO} = \int_1 d\sigma^B + \int_2 \left(d\sigma^R - d\sigma^{CT}\right) + \int_1 \left(d\sigma^V + \int_1 d\sigma^{CT}\right)$$

→ finite phase-space integration

→ finite in $d=4$, cancellation of $\frac{1}{\varepsilon}$ poles

- FKS [Frixione, Kunszt, Signer hep-ph/9512328]
- Dipole subtraction [Catani, Seymour hep-ph/9605323]

• slicing

- non-local subtraction (subtraction term not defined in full real phasespace)
- use slicing cut to regulate real
- simple and therefore powerful at higher orders (NNLO, N^3LO, ...)

e.g. q_T slicing [Catani, Grazzini hep-ph/0703012]

$$\sigma^{NLO} = \int_1 d\sigma^B + \int_2^{q_T > q_T^{cut}} d\sigma^R - \int_{1+q_T}^{q_T > q_T^{cut}} d\sigma^{CT} + \int_1 \left[d\sigma^V + \int_{q_T} d\sigma^{CT}(q_T^{cut})\right]$$

take limit $q_T^{cut} \to 0$ by computing several small q_T^{cut} values.

Resummation (in p_T)

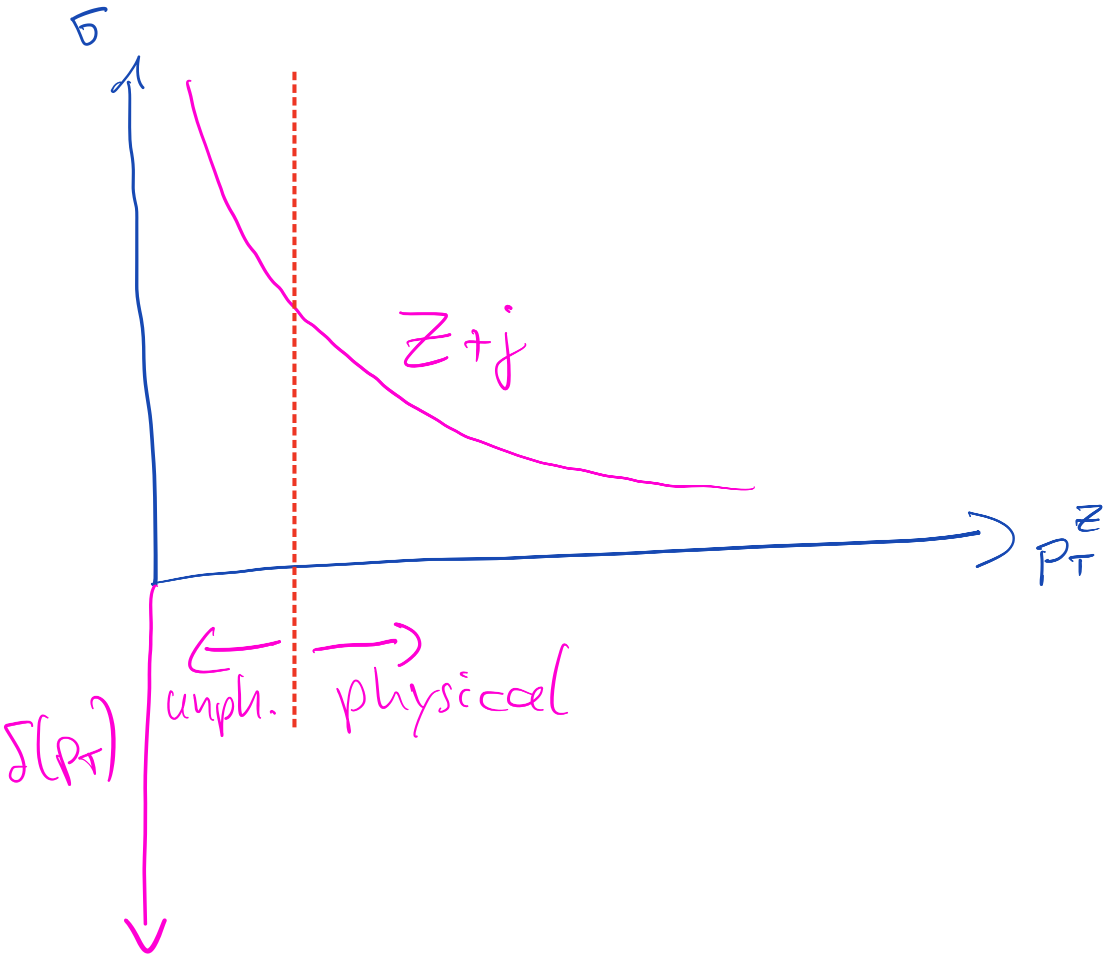
- limitation of fixed-order calculations

$\boxed{\text{cancellation of real \& virtual singularities is spoiled in certain phase-space regions (when observable sensitive to soft/coll. rad.)}}$

- gluon radiation produces $\log^2$ behavior (1 coll. & 1 soft log)

$$\sim \frac{\alpha_s}{\pi} \frac{d\Theta}{\Theta^2} \frac{dE}{E}$$

p_T distribution



4.1

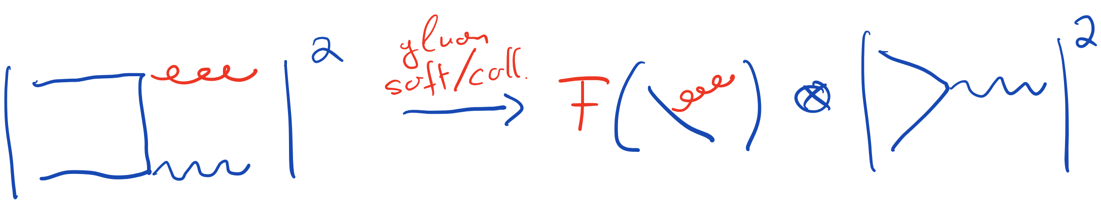
Why?

→ large logarithms $L = \ln\left(\frac{M^2}{p_T^2}\right)$ $\qquad M \sim \sqrt{s}$

$$\sigma^{\text{cum.}}(p_T^{\text{cut}}) \equiv \int_0^{p_T^{\text{cut}}} \frac{d\sigma}{dp_T}\, dp_T \quad \Rightarrow \quad \frac{d\sigma}{dp_T} = \frac{d}{dp_T}\,\sigma^{\text{cum.}}$$

	cumulant	differential
$\sim \alpha_s$:	L^2, L	$\frac{1}{p_T} L, \ \frac{1}{p_T}$
$\sim \alpha_s^2$:	L^4, L^3, L^2, L	$\frac{1}{p_T} L^3, \ \frac{1}{p_T} L^2, \ldots$
$\sim \alpha_s^3$:	$L^6, L^5, L^4, L \ldots$	$\frac{1}{p_T} L^5, \ \frac{1}{p_T} L^4, \ldots$
$\vdots$		
	$\alpha_s^n L^{2n-k} \quad k = 0, 1, \ldots, 2n$	$\alpha_s^n \frac{1}{p_T} L^{2n-k-1}$

· factorization of soft/collinear radiation



$$|M_{n+1}|^2 \longrightarrow F \otimes |M_n|^2$$

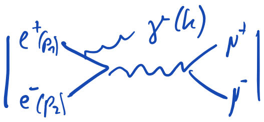
↳ eikonal in soft limit
splitting function in coll. limit
(& colour factors)

important concept, appearing throughout QFT calc. :
* subtraction
* resummation
* showers
⋮

4.2

example: (excercise)

determine $\frac{L}{p_T}(L^2)$ term at $\mathcal{O}(\alpha)$ in $e^+e^- \to \mu^+\mu^-$



$$\left| \cdots \right|^2 = |M(e^+e^- \to \mu^+\mu^-\gamma)|^2 \xrightarrow{k \to 0} \underbrace{e^2}_{=4\pi\alpha} \frac{2\, p_1 \cdot p_2}{(p_1 \cdot k)(p_2 \cdot k)} |M(e^+e^- \to \mu^+\mu^-)|^2$$

EIKONAL FAKTOR

$p_1 = E_1(1,0,0,1)$, $p_2 = (1,0,0,-1)$

$k = \omega(1, \sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$

$$\Rightarrow (p_1 k)(p_1 k) = E_1 E_2 \omega^2 \underbrace{(1-\cos\theta)(1+\cos\theta)}_{=\sin^2\theta} = E_1 E_2 k_T^2, \quad 2 p_1 \cdot p_2 = 4E_1E_2$$

$$\Rightarrow |M(e^+e^- \to \mu^+\mu^-\gamma)|^2 \to \frac{16\pi\alpha}{k_T^2} |M(e^+e^- \to \mu^+\mu^-)|^2$$

phase space: (neglect soft k in mom. cons. δ-function)

$$\frac{d^3k}{(2\pi)^3 2\omega} = \frac{d\cos\theta\, d\varphi\, \omega\, d\omega}{16\pi^3} \overset{\text{integrate over } \varphi}{=} \frac{d\cos\theta\, \omega\, d\omega}{8\pi^2}, \qquad \omega \sin\theta = k_T$$

$$= \frac{2k_T\, dk_T\, d\omega}{8\pi^2 \sqrt{\omega^2 - k_T^2}} \qquad \Rightarrow \cos\theta = \pm\sqrt{1 - \frac{k_T^2}{\omega^2}}$$

$$\Rightarrow d\cos\theta = \frac{2k_T\, dk_T}{\omega\sqrt{\omega^2 - k_T^2}}$$

$\& \; 0 \le k_T \le \omega$

$$\int \frac{d\omega}{\sqrt{\omega^2 - k_T^2}} = \int_{k_T}^{\sqrt{s}} \frac{d\omega}{\omega} = \ln\left(\frac{\sqrt{s}}{k_T}\right)$$

leadin log as $k_T \to 0$ $\quad$ $\omega \ge k_T$

δ function without γ momentum in soft limit

$$\boxed{\Rightarrow \frac{d\sigma(e^+e^- \to \mu^+\mu^-\gamma)}{dk_T} = \sigma_0(e^+e^- \to \mu^+\mu^-) \frac{16\pi\alpha}{k_T^2} \frac{2k_T}{8\pi^2} \frac{1}{2} \ln\left(\frac{s}{k_T^2}\right) = \sigma_0 \frac{\alpha}{\pi} \frac{2}{k_T} \ln\left(\frac{s}{k_T^2}\right)}$$

$\longrightarrow$ double-log divergence as $k_T \to 0$

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cumulant

$\sim \alpha_s$: L^2, L

$\sim \alpha_s^2$: L^4, L^3, L^2, L

$\sim \alpha_s^3$: $L^6, L^5, L^4, L\ldots$

$\vdots$

$\alpha_s^n L^{2n-k}$ $\quad k = 0, 1, \ldots, 2n$

differential

$\frac{1}{p_T} L, \frac{1}{p_T}$

$\frac{1}{p_T} L^3, \frac{1}{p_T} L^2, \ldots$

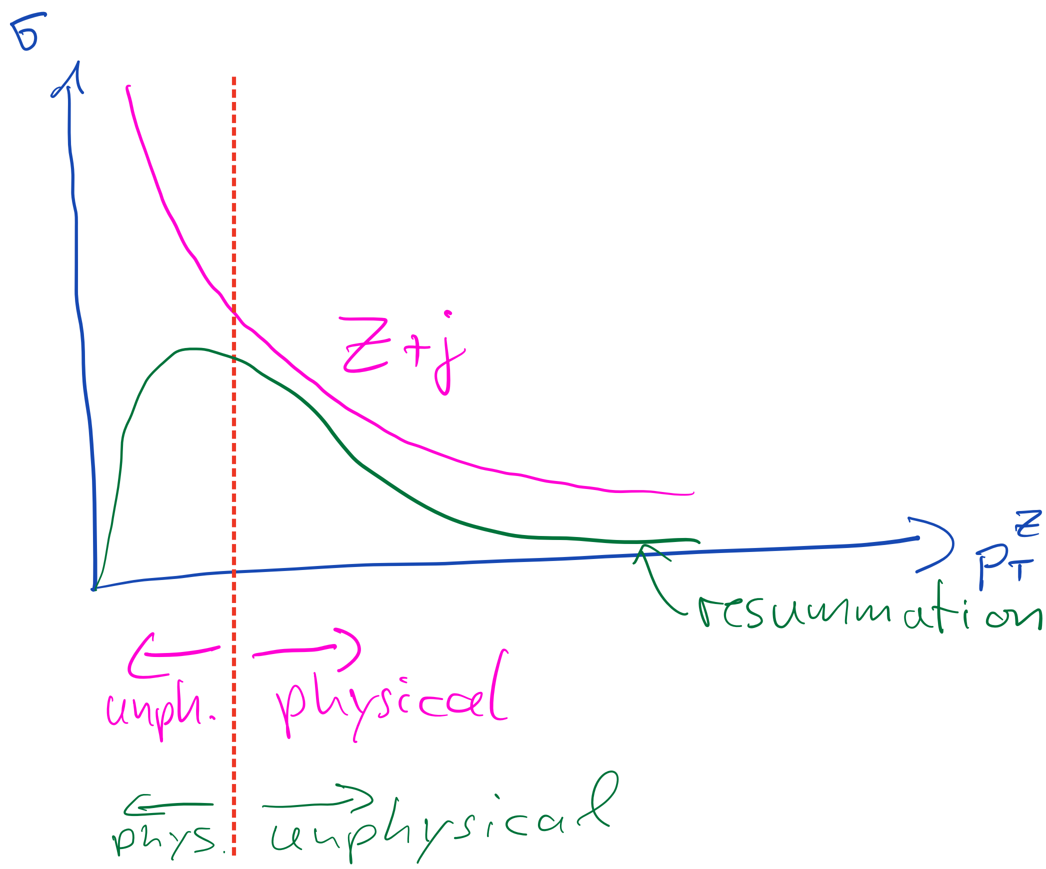
$\frac{1}{p_T} L^5, \frac{1}{p_T} L^4, \ldots$

$\alpha_s^n \frac{1}{p_T} L^{2n-k-1}$

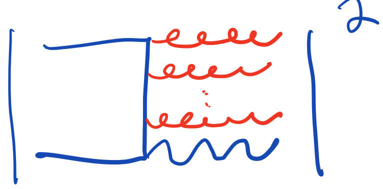
solution: resummation of L

$$e^{-\alpha_s L^2} = 1 - \alpha_s L^2 + \alpha_s^2 \frac{L^4}{2} - \alpha_s^3 \frac{L^6}{6} + \mathcal{O}(\alpha_s^4) \quad \text{(cumulant)}$$

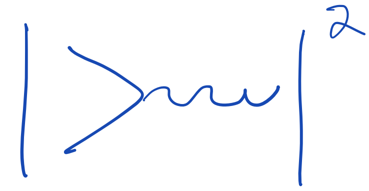
$$\alpha_s \frac{L}{p_T} e^{-\alpha_s L^2} = \alpha_s \frac{L}{p_T} - \alpha_s^2 \frac{L^3}{p_T} + \alpha_s^3 \frac{L^5}{2 p_T} + \mathcal{O}(\alpha_s^4) \quad \text{(differential)}$$



- recursive: multiple soft/coll. radiation



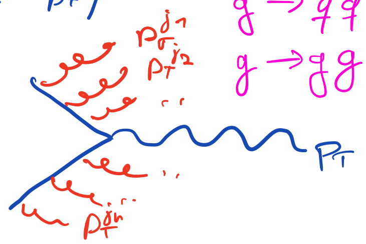
$$\longrightarrow \underbrace{F \otimes F \otimes \cdots \otimes F}_{\exp(-S)} \otimes$$



* of course these can be all kinds of splittigs $q \to gq$, $q \to qg$, $g \to q\bar{q}$, $g \to gg$

- factorize phase space using conjugate impact parameter b $\left(\sim \frac{1}{p_T}\right)$

$$\delta^{(2)}\left(\vec{p}_T - \sum_{i=1}^{n} \vec{p}_T^{\,j_i}\right) \longrightarrow \int \frac{d^2\vec{b}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{p}_T} \prod_{i=1}^{n} e^{i\vec{b}\cdot\vec{p}_T^{\,j_i}}$$



see e.g. [Parisi, Petronzio '79], [Collins, Soper, Sterman '84]

p_T resummation formula (no derivation)

$$\boxed{\frac{d\sigma^{res}_{cum}}{}(p_T) = p_T \sum_c \sigma_{c\bar{c}} \int db\, J_1(b \cdot p_T)\, e^{-S_c(b)} \sum_{i,j} H_{c\bar{c}} \left(C_{ci} \otimes f_i\right) \left(C_{\bar{c}j} \otimes f_j\right)}$$

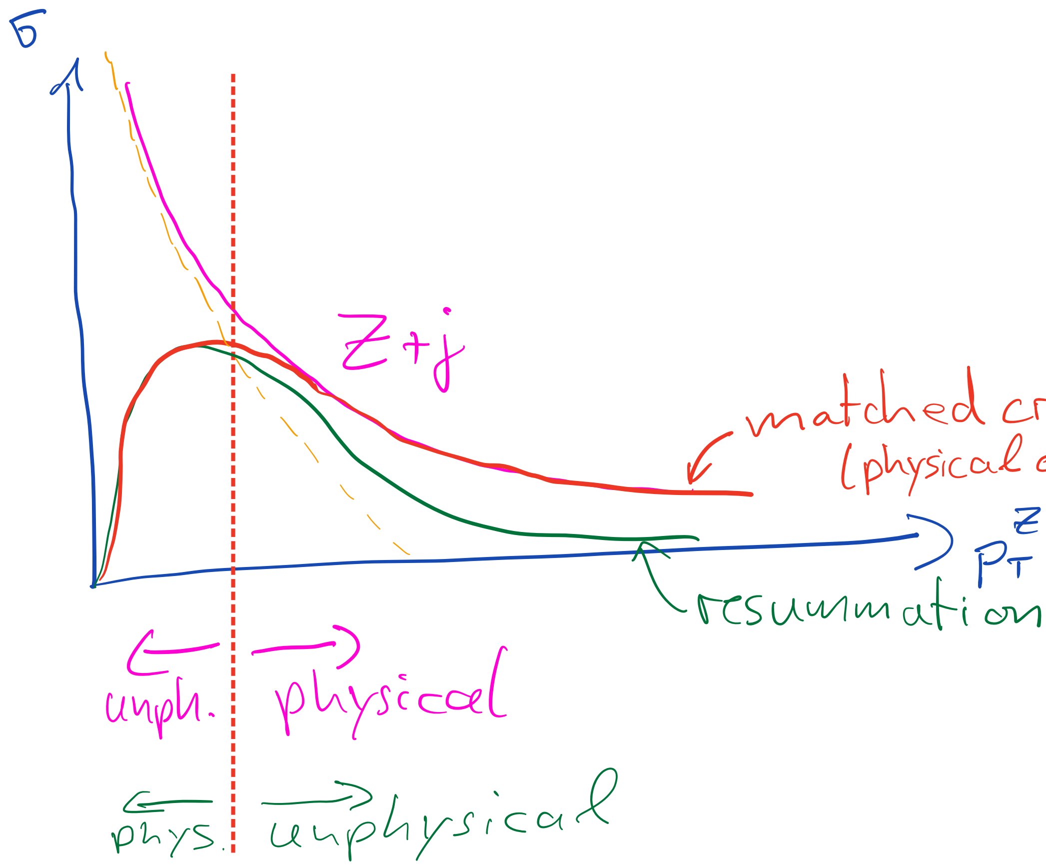
- Fourier transformation from b (impact parameter) to p_T space, J_1: bessel function (from $e^{-i\vec{b}\cdot\vec{p}_T}$)
- H: hard function containing virtual correction
- C: Collinear functions – convolution $(g \otimes h)(z) = \int_x^1 dx\, g(x)\, f\left(\frac{z}{x}\right)$
- e^{-S}: Sudakov form factor → determines logarithmic acc.

$$S = \underbrace{\underbrace{\underbrace{L\, g^{(1)}(\alpha_s L)}_{LL} + g^{(2)}(\alpha_s L)}_{NLL} + \alpha_s\, g^{(3)}(\alpha_s L)}_{NNLL} + \alpha_s^2 \cdots \quad \Big| \quad \underline{\underline{\alpha_s L \sim 1}}$$

7] Matching with fixed-order

- resummation provides "just" logarithmic (and constant) terms in p_T, but to all orders → physical at low p_T
- fixed-order provides all terms in p_T (including power corrections), but to finite order → physical at large p_T

→ matching: combine both consistently



$$\frac{d\sigma^{\text{matched}}}{dp_T} = \underbrace{\frac{d\sigma^{f.o.}_{Z+j}}{dp_T} - \left[\frac{d\sigma^{res}}{dp_T}\right]_{f.o.}}_{\Rightarrow \text{ finite}} + \frac{d\sigma^{res}}{dp_T} \qquad \text{(additive)}$$

→ removes double counting & makes $d\sigma^{f.o.}$ finite

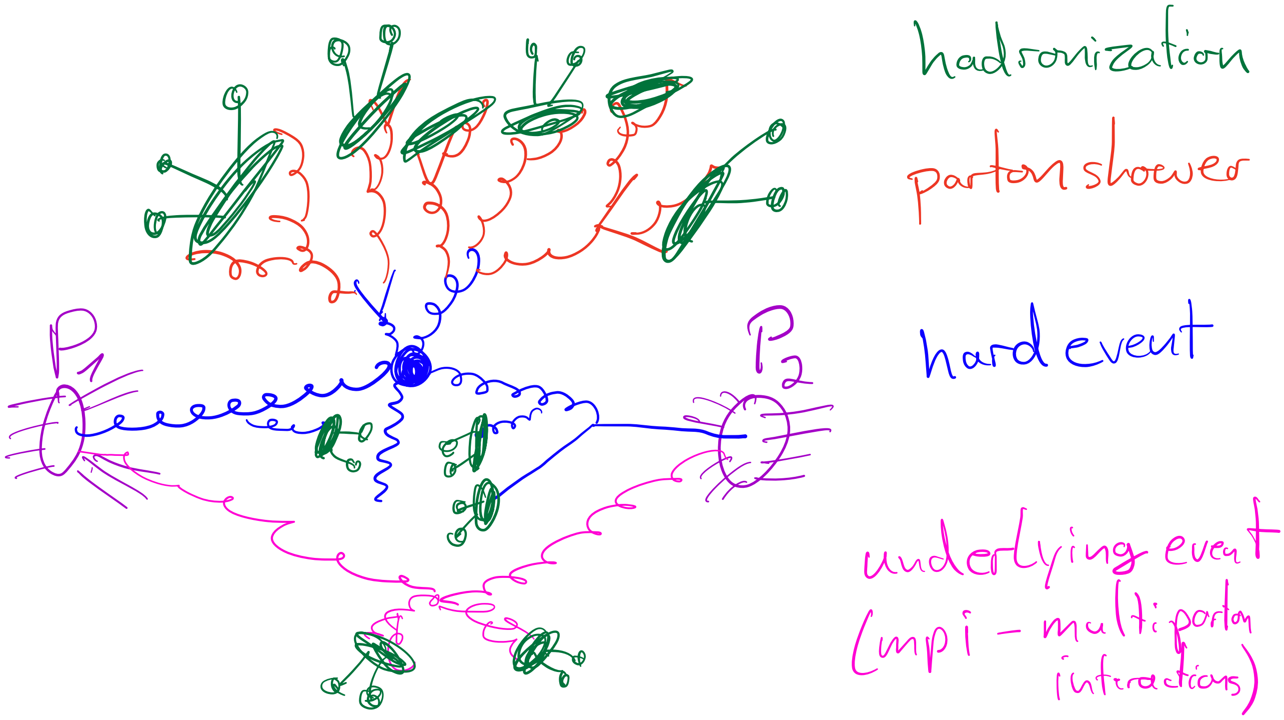
* integrated over p_T ⇒ f.o. cross section

* starting point for fixed-order method → q_T slicing (non local) [Catani, Grazzini arXiv: ...]

Parton Showers (PS)

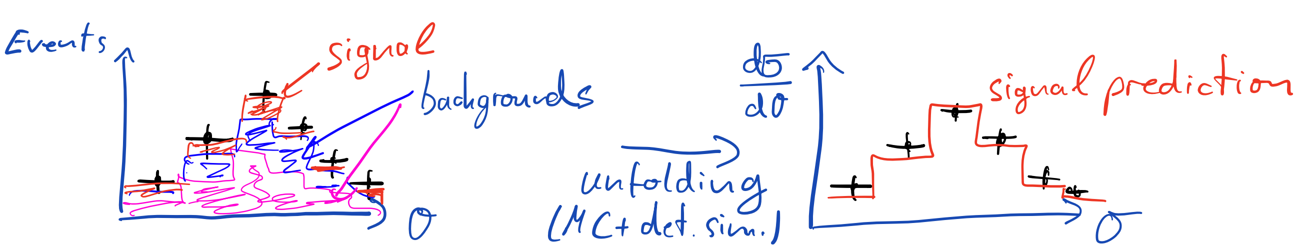
- based on recursive algorithm of soft/collinear emissions
- essentially a numerical approach to resummation
- BUT low accuracy
- INSTEAD fully exclusive description of partonic events in soft/coll. approximation ⇒ resummation of several observables

+ typically combined with hadronization/underlying event models (Rhorry's lecture)



PS event generators simulate the entire hadronic event

- theoretical foundation of exp. analyses
- most used theoretical tools (by far)
- connect detector-level events (what is measured) with fiducial cross sections (what can be predicted in QFTs)



9.1

PS algorithm

simple example: consider isotope with average lifetime τ

$\rightarrow$ propability of decay is always related to propability of no decay

propability of no decay up to t:

$$P_{no-dec} = e^{-t/\tau} = e^{-\Gamma t}$$ decay rate

decay propability:

$$P_{dec} = 1 - P_{no-dec} = 1 - e^{-\Gamma t}$$

$$\frac{d P_{dec}(t)}{dt} = \Gamma P_{nodec}(t)$$

$\Gamma \rightarrow \Gamma(t)$:

$$P_{no-dec}(t) = e^{-\int_0^t dt' \Gamma(t')} \quad , \quad \frac{d P_{dec}(t)}{dt} = \Gamma(t) P_{no-dec}(t)$$

$$P_{dec}(t) = \int_0^t dt' \Gamma(t') P_{no-dec}(t')$$

$\rightarrow$ PS follows same idea:

approx. of coll. rad.

splitting function

$$dt\, \Gamma(t) \rightarrow d\phi\, \mathcal{P}(\phi) \doteq dv\, dz\, d\varphi \frac{\alpha_s}{4\pi^2} \frac{1}{v} P(z)$$

in principle: several possible parametrizations (should cover coll./soft limits)

$$= dv\, dz \frac{\alpha_s}{2\pi} \frac{1}{v} P(z)$$

$\Rightarrow$ propability for no emission between v_i and $v_j < v_i$:

$$\Delta(v_i, v_j) = \exp\left(-\int_{v_j}^{v_i} d\phi\, \mathcal{P}(\phi)\right)$$

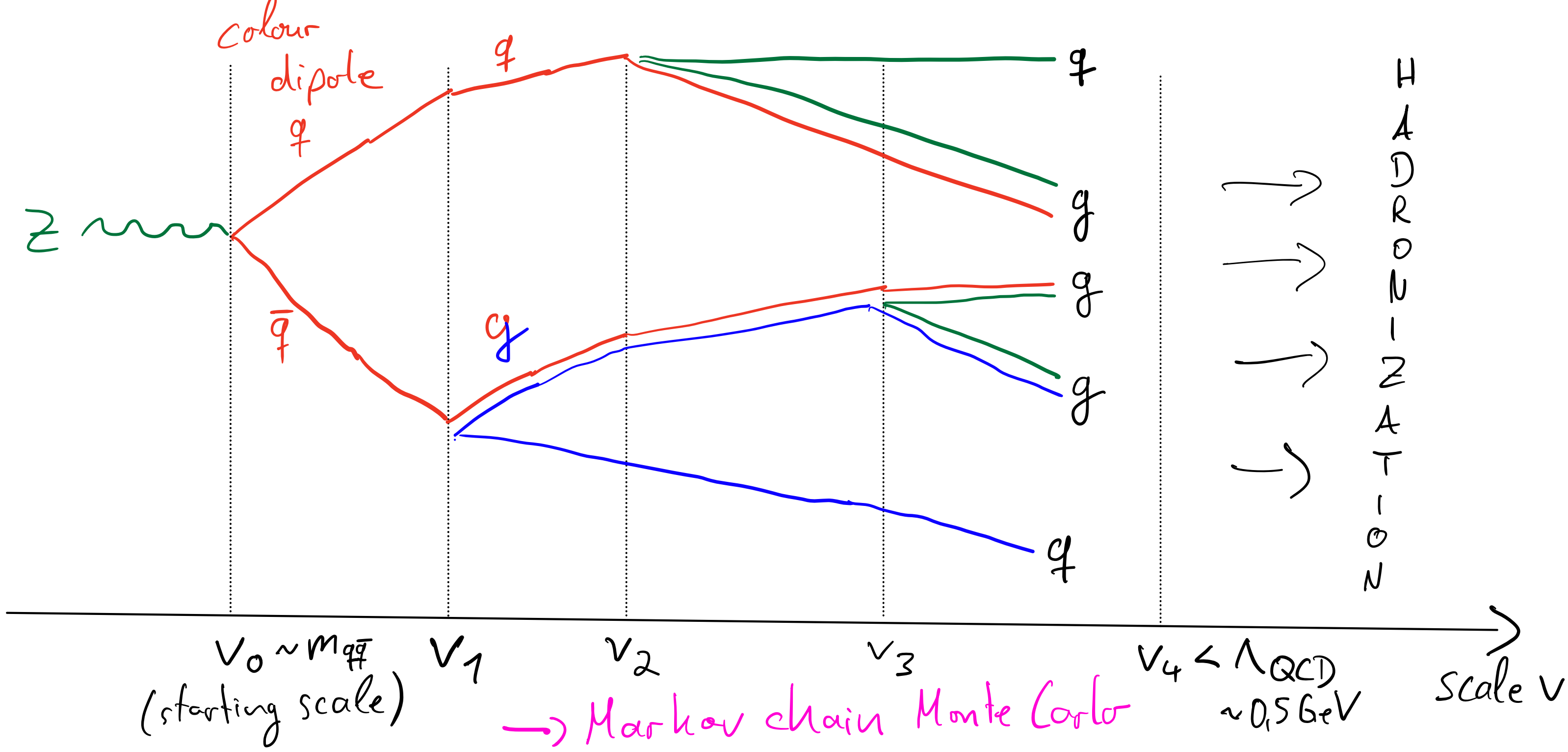
$\Rightarrow$ propability for one emission at scale v_1 starting from v_0:

IMPORTANT that directly related

$$\int d\phi_1 \mathcal{P}(\phi_1) \Delta(v_0, v_1) \quad , \phi_1 \equiv \phi(v_1)$$

PS algorithm

- dipole shower [Gustafson, Pettersson '87]



→ Markov chain Monte Carlo

propability for no emission between v_i and $v_j < v_i$:

$$\Delta(v_i, v_j) = \exp\left(-\int_{v_j}^{v_i} d\phi\, \mathcal{P}_{q\bar{q}}(\phi)\right), \quad \phi = \{v, z, \varphi\}$$

Dipole function alla Catani-Seymour (covering soft & coll. limit)

→ throw random number $r_1 \in [0,1]$

and solve for scale v_1: $\Delta(v_0, v_1) = r_1$

→ determines to what scale $q\bar{q}$ system persists

→ at v_1 dipole splits ($2 \to 3$), emitting a gluon

determine kinematics ($v_1 + z, \varphi$) random, random $\; r_2 \cdot \int_{\frac{\Lambda^2_{QCD}}{v_1^2}}^{1-\frac{\Lambda^2_{QCD}}{v_1^2}} dz'\, P_{q\bar{q}}(z') = \int_{z_{min}}^{z} dz'\, P_{q\bar{q}}(z')$

& momentum conservation by absorbing recoil into $q, \bar{q}$

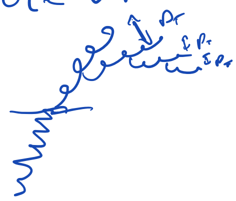
→ now we have two dipoles (qg), $(g\bar{q})$

→ ITERATE independently with v_1 as starting scale

break condition: $v_{n+1} < \Lambda_{QCD} \sim 0{,}5\,\text{GeV}$

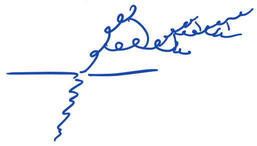
ordering variable v:

- $v \sim p_T$



  - Pythia
  - Sherpa

- $v \sim$ angle



  - Herwig

PS formula

$B \hat{=} \left|\sum \mathcal{M}\right|^2 \qquad d\sigma^0 = d\phi_B\, B$

$$d\sigma_{PS} = d\phi_B\, B \times \underbrace{\Big\{ \underbrace{\Delta(v_0, \Lambda_{QCD})}_{\text{no emission}} + \underbrace{d\phi_1\, \Delta(v_0, v_1)\, \mathcal{P}(d\phi_1)}_{\text{emmission at } v_1} \Big\}}_{\text{integrates to } 1 \;\Rightarrow\; \text{"unitarity" of PS}}$$

$\longrightarrow$ ITERATE!

$$d\sigma_{PS} = d\phi_B\, B \times \underbrace{\Big\{ \underbrace{\Delta(v_0, \Lambda_{QCD})}_{0\text{ emissions}} + \underbrace{d\phi_1\, \Delta(v_0, v_1)\, \mathcal{P}(d\phi_1) \times \Big\{ \Delta(v_1, \Lambda_{QCD})}_{1\text{ emission}} + \underbrace{d\phi_2\, \Delta(v_1, v_2)\, \mathcal{P}(d\phi_2) \times \Big\{ \Delta(v_2, \Lambda_{QCD})}_{2\text{ emissions}} + \underbrace{d\phi_3\, \Delta(v_2, v_3)\, \mathcal{P}(d\phi_3)}_{3\text{ emissions}} \underbrace{\times \{ \ldots \} \ldots}_{\ldots} \Big\}\Big\}\Big\}}_{=:\, I_{PS}^{(n)} \text{ shower emissions from } n\text{-parton final state}}$$

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Matching PS & fixed order

$$d\sigma^V = d\phi_B\, V\,, \quad d\sigma^R = d\phi_R\, R\,, \quad d\phi_R = d\phi_B \cdot d\phi_{rad}$$

$$d\sigma^{NLO} = d\phi_B\,(B + V + d\phi_{rad}\, R)$$

MC@NLO (additive matching) [Frixione, Webber, hep-ph/0204244]

shower formula: $d\sigma_{PS} = d\phi_B\, B \times \{\Delta(v_0, \Lambda_{QCD}) + d\phi_1\, \Delta(v_0, v_1)\, \mathcal{P}(d\phi_1)\}$

$$d\sigma_{MC@NLO}^{\cancel{naive}} = \left[d\phi_B\,(B + V + \textstyle\int d\phi_{rad}\, CT)\right] \times \mathcal{I}_{PS}^{(n)} + \left[d\phi_B\, d\phi_{rad}\,(R - CT)\right] \times \mathcal{I}_{PS}^{(n+1)}$$

→ double counting! 1st radiation in both $(B \times \mathcal{I}_{B}^{(n)})$ & (R)

$$CT^{PS} := B \times \left[\frac{d\phi_1}{d\phi_{rad}}\, \mathcal{P}(d\phi_1)\right]$$

PS counterterm

NOTE: unlike local subtraction, we are not adding a "0"

NLO accuracy ? → expand to NLO

expansion of $\left[\mathcal{I}_{PS}^{(n)}\right]_{NLO} = \left\{\left(1 - \int d\phi_1\, \mathcal{P}(d\phi_1)\right) + d\phi_1\, \mathcal{P}(d\phi_1)\right\}$

$$= \left\{1 - \int d\phi_{rad}\, \frac{CT}{B} + d\phi_{rad}\, \frac{CT}{B}\right\}$$

$$\left[d\sigma_{MC@NLO}\right]_{NLO} = d\phi_B\,\Big(\underbrace{B \times \left[\mathcal{I}_{PS}^{(n)}\right]_{NLO}} + V + \cancel{\textstyle\int d\phi_{rad}\, CT}\Big) + d\phi_B\, d\phi_{rad}\,(R - \cancel{CT})$$

$$= B - \cancel{\textstyle\int d\phi_{rad}\, CT} + \cancel{d\phi_{rad}\, CT}$$

$$= d\sigma^{NLO} \qquad \text{q.e.d}$$

$$\boxed{d\sigma_{MC@NLO} = \left[d\phi_B\,(B + V + \textstyle\int d\phi_{rad}\, CT)\right] \times \mathcal{I}_{PS}^{(n)} + \left[d\phi_B\, d\phi_{rad}\,(R - CT)\right] \times \mathcal{I}_{PS}^{(n+1)}}$$

"S" events (soft)   "H" events (hard)

NOTE: S & H events can be negative → only sum positive definite + physical

POWHEG [Nason, hep-ph/0305252]

shower formula: $d\sigma_{PS} = d\phi_B\, B \times \left\{ \Delta(v_0, \Lambda_{QCD}) + d\phi_1\, \Delta(v_0, v_1)\, P(d\phi_1) \right\}$

* replace first radiation of shower

subtracted virtual & real (with any local subtraction scheme)

$$\boxed{d\sigma_{POWHEG} = \underbrace{d\phi_B \left(B + V^{fin.} + \int d\phi_{rad}\, R^{fin.} \right)}_{=:\, \tilde{B}} \times \underbrace{\left\{ \Delta_{pwg}(\Lambda_{pwg}) + d\phi_{rad}\, \Delta_{pwg}(p_{T,rad}) \frac{R}{B} \times I_{PS}^{(n+1)} \right\}}_{\text{no emission + 1st emission}}}$$

→ inclusive NLO cross section

→ unitar & exlusive real radiation

$$\Delta_{pwg}(k_T) = \exp\left(-\int_{k_T}^{\hat{s}} d\phi_{rad} \frac{R}{B} \right),$$

$p_{T,rad}$: p_T of the radiation

Λ_{pwg}: POWHEG cutoff

NLO accuracy?

→ by construction

$+ V^{fin.} + \int d\phi_{rad}\, R^{fin.}$

expansion to NLO:

$$\left[d\sigma_{POWHEG} \right]_{NLO} = d\phi_B \left(B \times \left\{ 1 - \cancel{\int d\phi_{rad} \frac{R}{B}} + d\phi_{rad} \frac{R^{fin.}}{B} \right\} + V^{fin.} + \cancel{\int d\phi_{rad}\, R^{fin.}} \right)$$

$$= d\sigma^{NLO}$$

q.e.d.

MC@NLO vs. POWHEG

$\longrightarrow$ due to exponentiation of full $\frac{R}{B}$

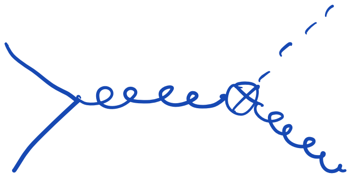
$\longrightarrow$ reduce effect by redefining $R \to R^S + R^f$

$$d\sigma_{\text{POWHEG}} = d\Phi_B \left(B + V^{\text{fin.}} + \int d\Phi_{\text{rad}} R^{\text{fin.}}\right) \times \left\{ \Delta^S_{\text{pwg}}(\Lambda_{\text{pwg}}) + d\Phi_{\text{rad}}\, \Delta^S_{\text{pwg}}(p_{T,\text{rad}}) \frac{R^S}{B} \times \mathcal{I}_{PS}^{(n+1)} \right\}$$

$$\underbrace{+\, d\Phi_B\, d\Phi_{\text{rad}}\, R^f \times \mathcal{I}_{PS}^{(n+1)}}_{\text{"remnant"}}$$

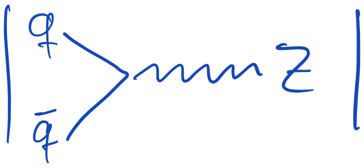
NOTE: There can also be regular real contributions that can just be added separately, since they are finite

e.g.

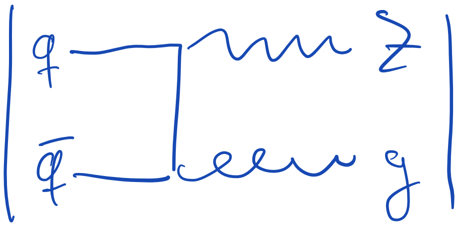


$\longrightarrow$ no divergence, treat as "regular" contribution

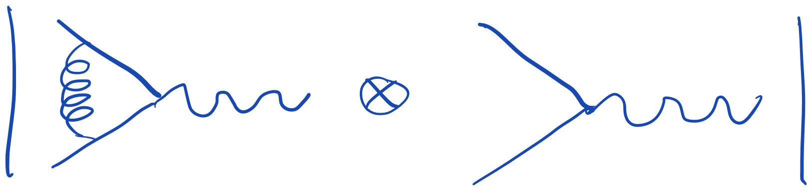
NNLO+PS matching

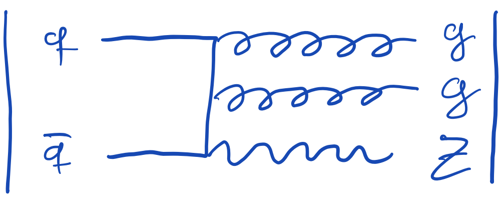
LO: 

$^2 \longrightarrow d\sigma^B$ (Born)

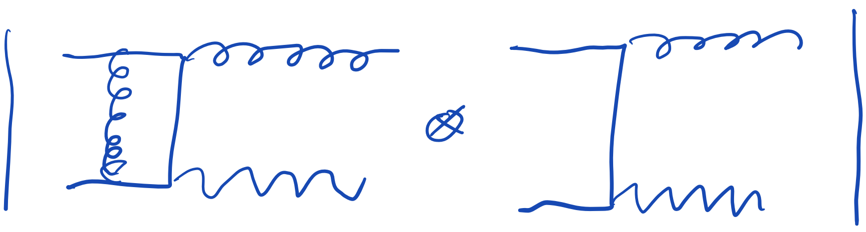
NLO: 

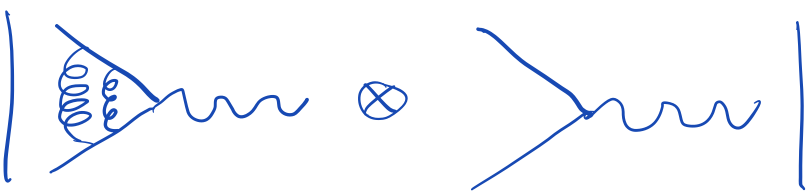
$^2 \longrightarrow d\sigma^R$ (Real)

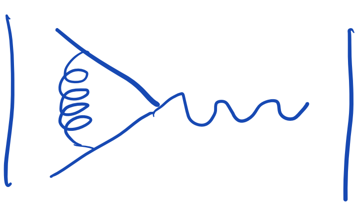
2 Re  $\longrightarrow d\sigma^V$ (Virtual)

NNLO: 

$^2 \longrightarrow d\sigma^{RR}$ (Double Real)

2 Re  $\longrightarrow d\sigma^{RV}$ (Real-Virtual)

2 Re 

\+  ^{2} $\longrightarrow d\sigma^{VV}$ (Double Virtual)

NNLO+PS problems:

- extension of NLO+PS methods non-trivial
  - → MC@NLO: expansion of shower does not reproduce full RR divergence (2 unresolved) → requires NNLL @ full colour
    - under development
    - already difficult @LL
  - → Powheg: generating 1st & 2nd radiation with $\Delta_{\text{pwg}}\left(\frac{R}{B}, \frac{RR}{R}\right)$ leads to problems
- numerically very challenging
- preserving shower accuracy @LL (NLL, NNLL in future) & NNLO accuracy very challenging

⇒ 2 widely used solutions (among other ideas):

- Geneva → uses differential implementation of resummation to create NNLO events up to 2 extra partons, then showers beyond
- MiNNLO$_{PS}$ → focus here

$\text{MiNNLO}_{\text{PS}}$

- based on POWHEG, start from Z+jet NLO:

$$\boxed{d\sigma^{Z+j}_{\text{POWHEG}} = d\Phi_R\left(R+RV^{\text{fin.}}+\int d\Phi_{\text{rad}}RR^{\text{fin.}}\right)\times\left\{\Delta_{\text{pwg}}(\Lambda_{\text{pwg}})+d\Phi_{\text{rad}}\,\Delta_{\text{pwg}}(p_{T,\text{rad}})\,\frac{RR}{R}\times I_{\text{PS}}^{(n+1)}\right\}}$$

turn $\tilde{B}$ NNLO accurat while respect scaling of resummation

→ use analytic resummation formula

- start from resummation formula (cumulant)

$$d\sigma^{\text{cum.}} = e^{-S}\mathcal{L}\,, \qquad \mathcal{L}=\sum_c \sigma^{(0)}_{c\bar{c}}\sum_{i,j}H_{c\bar{c}}\,(C_{ic}\otimes f_i)(C_{j\bar{c}}\otimes f_j)$$

(transformed from b-space)

- distribution through derivative

$$\frac{d\sigma^{\text{res}}}{dp_T}=\frac{d}{dp_T}\left(e^{-S}\mathcal{L}\right)=e^{-S}\underbrace{\left(-S'\mathcal{L}+\mathcal{L}'\right)}_{\equiv D}$$

- matching with FO (NLO Z+jet):

$$d\sigma^{\text{MiNNLO}} = \underbrace{d\Phi_B\,dp_T\,e^{-S}D}_{\text{resummation}} + \underbrace{d\Phi_R\left(R+RV^{\text{fin.}}+\int d\Phi_{\text{rad}}RR^{\text{fin}}\right)}_{\text{NLO Z+jet}} - \underbrace{d\Phi_B\,dp_T\left[e^{-S}D\right]_{\text{NLO}\,(\alpha_s^2)}}_{\text{exp. res.}}$$

→ removes double counting

$$= d\Phi_R\,e^{-S}\left\{\frac{d\Phi_B\,dp_T}{d\Phi_R}D+\frac{R+RV^{\text{fin.}}+\int d\Phi_{\text{rad}}RR^{\text{fin.}}}{e^{-S}}-\frac{d\Phi_B\,dp_T}{d\Phi_R}\underbrace{\frac{\left[\cancel{e^{-S}}D\right]_{\text{NLO}}}{\cancel{e^{-S}}}}_{=D^{(1)}+D^{(2)}}\right\}$$

to keep cancellation needs to be treated the same

expand & reorder in α_s

$$= d\Phi_R\,e^{-S}\underbrace{\left\{\underbrace{\underbrace{\underbrace{R(1+S^{(1)})}_{\sim\alpha_s}+\underbrace{RV^{\text{fin.}}+\int d\Phi_{\text{rad}}RR^{\text{fin.}}}_{\sim\alpha_s^2}}_{\text{MiNLO}'}+\frac{d\Phi_B\,dp_T}{d\Phi_R}\underbrace{\left(D-D^{(1)}-D^{(2)}\right)}_{\sim\alpha_s^{\geq 3}}}_{\text{MiNNLO}}\right\}}_{=:\tilde{B}^{\text{MiNNLO}}}$$

projection + spreading

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* regular terms beyond α_s^2 do not contribute at NNLO (expansion up to $S^{(1)}$ only)
* singular terms of $\mathcal{O}(\alpha_s^3)$ (and beyond) do contribute, counting:

$$\int_\Lambda^M dp_T \frac{1}{p_T} \alpha_s^m(p_T) \ln^n\left(\frac{Q}{p_T}\right) e^{-S(p_T)} \approx \mathcal{O}\left(\alpha_s^{m-\frac{n+1}{2}}(Q)\right)$$

$\to$ depending on n, α_s^m terms in the p_T distrubution contribute at lower orders $\alpha_s^{m-\frac{n+1}{2}}$ in the cumulant
$\Rightarrow$ α_s^3 (and beyond) are relevant for NNLO accuracy upon integration over p_T

* spreading of function $F(\phi_B, p_T)$ in ϕ_R phase space:

$$\int d\phi_B \, dp_T \, F(\phi_B, p_T) \overset{!}{=} \int d\phi_R \, f^{\text{spread}} \times F(\phi_B, p_T)$$

$\to$ simplest solution: $f^{\text{spread}} = \dfrac{1}{\int d\phi_R' \, \delta(\phi_B - \phi_B') \, \delta(p_T - p_T')}$

$\to$ general solution: $f^{\text{spread}} = \dfrac{\mathcal{J}^{\text{spread}}(\phi_R)}{\int d\phi_R' \, \delta(\phi_B - \phi_B') \, \delta(p_T - p_T') \, \mathcal{J}^{\text{spread}}(\phi_R)}$

$\mathcal{J}^{\text{spread}}$ determines spreading

e.g. $\mathcal{J}^{\text{spread}} = |M_R|^2 f_i f_j$ $\to$ full real cross section

BUT bad scaling with complexity of process

BETTER: collinear limit

$$\mathcal{J}^{\text{spread}} = \mathcal{P} \otimes \cancel{|M_B|^2} f_i f_j$$

splitting function

drops out in ratio

$\Rightarrow$ process independent ! (doesn't scale with complexity of process)

MiNNLO$_{PS}$ master formula

$$d\sigma^{Z+j}_{\mathrm{POWHEG}} = d\Phi_R\, \tilde{B}^{\mathrm{MiNNLO_{PS}}} \times \left\{ \Delta_{pwg}(\Lambda_{pwg}) + d\Phi_{rad}\, \Delta_{pwg}(p_{T,rad}) \frac{RR}{R} \times \mathcal{I}_{PS}^{(n+1)} \right\}$$

$$\tilde{B}^{\mathrm{MiNNLO_{PS}}} = d\Phi_R\, e^{-S} \left\{ R\left(1+S^{(1)}\right) + RV^{fin.} + \int d\Phi_{rad}\, RR^{fin.} + f^{\,\mathrm{spread}} \times \left(D - D^{(1)} - D^{(2)}\right) \right\}$$