Renormalization 1 :

Divergences and counterterms

In previous lectures we have established a technique to compute perturbative corrections to GREEN FUNCTIONS & we have shown how to relate them to OBSERVABLES in scattering theory $\Rightarrow$ Decay Rates and Cross Sections.

We also encountered a problem $\Rightarrow$ PERTURBATIVE CALCULATIONS PRODUCE DIVERGENT RESULTS

$\Rightarrow$ We have seen how to REGULARIZE these divergences with a method called DIM. REG. $\Rightarrow$ still problem remains: WHAT TO DO WITH THESE $\frac{1}{\varepsilon}$ POLES?

$\Rightarrow$ RENORMALIZATION will allow us, among the other things, to consistently remove these divergences and make sense of our theory $\Rightarrow$

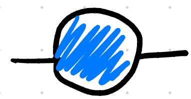
NOTE: renormalization is NOT ONLY needed in DIVERGENT THEORIES!

Let us stay with our ϕ^4 theory

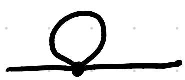
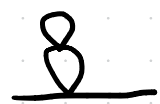
$$\boxed{\begin{array}{l}\text{rename} \\ m = m_0,\ \lambda = \lambda_0 \\ \phi = \phi_0\end{array}}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0 - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0}{4!}\phi_0^4$$

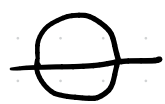
and consider one-loop corrections to two-point green function:

$$\int d^4x \, \langle \Omega | T\{\phi(x)\phi(0)\} | \Omega \rangle \, e^{ip\cdot x} =$$ 

used translation invariance $x = x - y$

$$= \frac{i}{p^2 - m_0^2 + i\delta} +$$  $+$  $+$ 

$O(\lambda_0)$

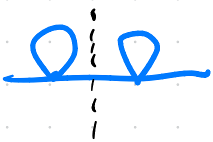
$+$  $+ \cdots$

$O(\lambda_0^2)$

note part of graphs $O(\lambda_0^2)$ is "repetition" of $O(\lambda_0)$

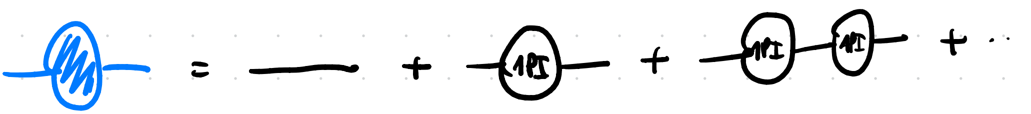
[1PI]

=> we call **1-PARTICLE IRREDUCIBLE** those graphs that CANNOT BE SEPARATED in two pieces by cutting ONE SINGLE LINE (PARTICLE)



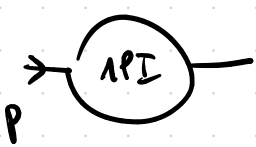
= this is 1 PARTICLE REDUCIBLE !

with this, we can write



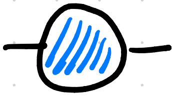
sum of all 1PI graphs to all λ orders!

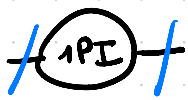
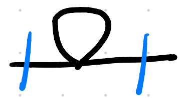
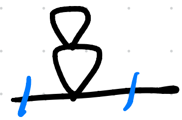
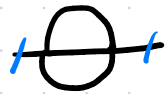
Now let's write



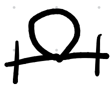
$$= \frac{i}{p^2 - m_0^2 + i\delta} \underbrace{\left[-i\,\Pi(p^2, m_0^2) \right]}_{\text{AMPUTATED (but OFF SHELL!)}} \frac{i}{p^2 - m_0^2 + i\delta}$$

then by formally resumming the series we clearly get

 $= \dfrac{i}{p^2 - m_0^2 - \Pi(p^2, m_0^2) + i\delta}$

$-i\Pi =$  $=$  $+$  $+$ 

$O(\lambda)$ $\qquad$ $O(\lambda^2)$

 $= \underbrace{\tfrac{1}{2}}_{\text{symm.}} \underbrace{(-i\lambda_0)}_{\text{vertex}} \mu^{2\varepsilon} \int \dfrac{d^D k}{(2\pi)^D} \dfrac{i}{k^2 - m_0^2 + i\delta}$ $\leftarrow$ prop

such that $O(\lambda)$ is exactly the todpole that we computed in Lecture 15

$-k_E^2 - m^2 = -(k_E^2 + m^2)$ $\qquad$ WICK ROT

$$T(D=4-2\varepsilon, m_0^2) = -iC(\varepsilon)\left[\frac{m_0^2}{16\pi^2}\right]\left(-\frac{1}{\varepsilon} - 1 - O(\varepsilon)\right)$$

$$C(\varepsilon) = \left[(4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon)\right]\left(\frac{m_0^2}{\mu^2}\right)^{-\varepsilon}$$

so we get [use $-i \cdot i = +1$, one from PROP, one from WICK]

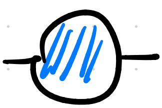
$$-i\Pi = +\left(\frac{1}{2}\right)(-i\lambda_0)\left[\frac{m_0^2}{16\pi^2}\right]\left(-\frac{1}{\varepsilon}-1+O(\varepsilon)\right)C(\varepsilon)$$

Symmetry factor

vertex factor

by CHANCE this is p^2 INDEPENDENT! in general it's not!

$$\Pi = \frac{\lambda_0}{32\pi^2}\, m_0^2\left(-\frac{1}{\varepsilon}-1+O(\varepsilon)\right)C(\varepsilon)$$ such that



$$= \frac{i}{p^2 - m_0^2\left[1-\frac{\lambda_0}{32\pi^2}\left(\frac{1}{\varepsilon}+1\right)\right]C(\varepsilon)+i\delta}$$

one-loop corrections redefine the mass and "normalization" of the two point function!

now remember K.L. representation

$$\Pi(p^2) = \frac{iZ}{p^2 - m^2_{phys} + i\varepsilon} + i\int_{M^2}^{\infty} dq^2 \, \frac{\rho_{n\neq 0,1}(q^2)}{p^2 - q^2 + i\varepsilon}$$

this should be a physical mass => something we can measure => the position of POLE has physical meaning

One-loop corrections have moved the POLE => the new "residue" is at $p^2 = m^2_{phys}$ such that

$$\boxed{m^2_{phys} - m_0^2 - \Pi(p^2 = m^2_{phys}, m_0^2) = 0}$$

this equation "DEFINES" the renormalized mass m

in the on-shell scheme (OS) $m_{phys} = m_{OS} \underline{\underline{= m}}$

From now on!

$$m^2 = m_0^2 \left(1 - \frac{\lambda_0}{32\pi^2}\left(\frac{1}{\varepsilon} + 1\right)\right) c(\varepsilon)$$

on-shell

so we can match the two formulas by defining

a renormalized mass = bare mass + UV pole

$\Rightarrow$ Bare mass not observable, it must itself be DIVERGENT to compensate for this and give a FINITE m !

What about Z ? We can compute it :

$$\frac{i}{p^2 - m_0^2 - \Pi(p^2, m_0^2) + i\delta} =$$

$$\frac{i}{p^2 - \underbrace{[m_0^2 + \Pi(m^2, m_0^2)]}_{m^2} - \Pi(p^2, m_0^2) + \Pi(m^2, m_0^2) + i\delta}$$

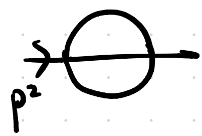
Compute residue

$$\lim_{p^2 \to m^2} (p^2 - m^2) \left[\frac{i}{p^2 - m^2 - (\Pi(p^2, m_0^2) - \Pi(m^2, m_0^2))} \right] =$$

$$= \frac{i}{1 - \frac{\partial \Pi}{\partial p^2}\Big|_{p^2=m^2}} \overset{!}{=} i \quad \Rightarrow \quad \boxed{Z = \frac{1}{1 - \frac{\partial \Pi}{\partial p^2}\Big|_{p^2=m^2}}}$$

In our case, Π is independent of p^2 so $Z = 1$

to be precise, $Z = 1 + O(\lambda^2)$

it will get contributions from 

$\sim \underline{\underline{\lambda^2}}$

In general, in ON-SHELL RENORMALIZATION SCHEME we want residue $= 1$

We do this by rescaling fields by $\underline{Z_{os}^{1/2} \phi(x)}$

with $$Z_{os} = \frac{1}{1 - \frac{\partial \Pi}{\partial p^2}\Big|_{p^2=m^2}}$$

CALL $m_{os} = m$ physical MASS, then also

$Z_{os} = Z$ on-shell field strength

⇒ So, we have seen that at $\underline{\underline{O(\lambda)}}$ we can define a new m and rescaling fields by $Z^{1/2}$

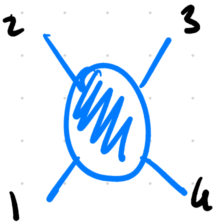
($Z = 1 + O(\lambda^2)$ in special case of ϕ^4 theory!)

is it ENOUGH ? ⟶

Let's consider a different process involving more fields

next: 2 → 2 Scattering : $p_1 + p_2 \to p_3 + p_4$

$$s = (p_1+p_2)^2 \, ; \quad t = (p_1 - p_3)^2 \, ; \quad u = (p_2 - p_3)^2$$



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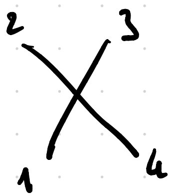
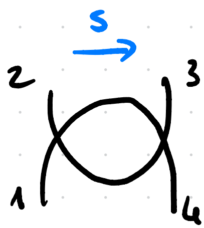


$O(\lambda)$

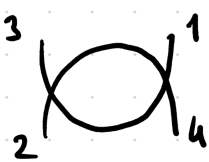
+ [



+ 1↔3 + 2↔3]

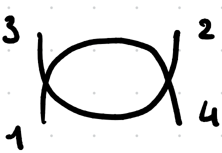
$O(\lambda^2)$

1↔3 =

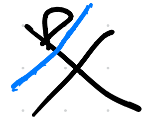


= function of $u = (p_2 - p_3)^2$

2↔3 =



= function of $t = (p_1 - p_3)^2$

note that we have **NOT** included  etc

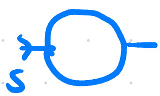
since we are interested in S-matrix

$\Rightarrow$ Amputated, ON-SHELL GREEN FUNCTION!

due to $(i)^2$ in two propagators!

$$\tilde{G}^{(4)}_{AMP} = \underbrace{-i\lambda_0}_{\text{"BARE COUPLING"}} - (-i\lambda_0)^2 \cdot \underbrace{\frac{1}{2}}_{\text{symm. factor}} \cdot \Big[\mathrm{Bub}(s) + \mathrm{Bub}(t) + \mathrm{Bub}(u) \Big]$$

Bubble integral evaluated in Lecture 15!

$\mathrm{Bub}(s) =$  $= (\mu^2)^\varepsilon \displaystyle\int \frac{d^D k}{(2\pi)^D} \frac{1}{(k^2 - m_0^2)\left((k-(p_1+p_2))^2 - m_0^2\right)}$

Depends only on $(p_1+p_2)^2$!

$$= \underbrace{i}_{\text{WICK ROT!}} \overbrace{\frac{(4\pi)^\varepsilon \Gamma(1+\varepsilon)}{16\pi^2}}^{S_\varepsilon} \left(\frac{\mu^2}{m_0^2}\right)^\varepsilon \left[\frac{1}{\varepsilon} - \underbrace{A(s, m_0^2)}_{\text{irrelevant here} \Rightarrow \text{see L. 15}} \right]$$

EXPLICIT

Now notice important thing :

In DIM REG dimensions of λ_0 change !

$$[\mathcal{L}] = D \; ; \quad [\partial_\mu] = 1 \; ; \quad [m_0] = 1$$

$$[\partial_\mu \phi_0 \partial^\mu \phi_0] = D \implies [\phi_0] = \frac{D}{2} - 1$$

$$[\lambda_0 \phi_0^4] = D \implies [\lambda_0] = 4 - D = +2\varepsilon$$

so $[\mu^{-2\varepsilon} \lambda_0] = 0$ Dimensionless !

We then introduce a DIMENSIONLESS COUPLING $\bar{\lambda}_0$

$\bar{\lambda}_0 \mu^{2\varepsilon} = \lambda_0 \rightarrow \bar{\lambda}_0 = \mu^{-2\varepsilon} \lambda_0$ in LAGRANGIAN

this justifies "physically" the $\mu^{2\varepsilon}$ added to integration measure ! In fact, imagine you had

no $\mu \implies$

$$\tilde{G}^{(4)}_{1\,\mathrm{AMP}} = -i\lambda_0\left[1 - \frac{\lambda_0}{32\pi^2}\left(\frac{3}{\varepsilon} - 3\ln m_0^2 - A(s) - A(t) - A(u)\right) S_\varepsilon\right]$$

$\uparrow$ no μ !

In Dim reg I introduce $\lambda_0 = \mu^{2\varepsilon}\bar{\lambda}_0$, $\bar{\lambda}_0$ DIMENSIONLESS

which produces the $\mu^{2\varepsilon}$ which goes with $\int \frac{d^D k}{(2\pi)^D}$

In brackets, $\lambda_0 = \mu^{2\varepsilon}\bar{\lambda}_0 = \mu^{2\varepsilon}\left[\mu^{-2\varepsilon}\lambda_0\right]$

$$\tilde{G}^{(4)}_{1\,\mathrm{AMP}} = -i\lambda_0\left[1 - \frac{\lambda_0 \mu^{-2\varepsilon}}{32\pi^2}\left(\frac{3}{\varepsilon} - 3\ln \frac{m_0^2}{\mu^2} - A(s) - A(t) - A(u)\right) S_\varepsilon\right]$$

So adding $\mu^{2\varepsilon}$ to measure, corresponds to

having changed $\mathcal{L} \to \mathcal{L}_D$

$$\mathcal{L} \to \mathcal{L}_D = \frac{1}{2}\left[\partial_\mu \phi_0 \partial^\mu \phi_0 - m^2 \phi_0^2\right] - \frac{\mu^{2\varepsilon}\bar{\lambda}_0}{4!}\phi_0^4$$

& writing loop corrections in terms of $\underline{\underline{\bar{\lambda}_0}}$

So we see that :

- $m_0 \to m$, renormalizing the mass, cannot remove this $\frac{1}{\varepsilon}$ pole $\Rightarrow$ effect at $O(\lambda^3)$

- Same is true for $Z_{OS} = 1 + O(\lambda^2)$

BUT :

Physically, we cannot measure absolute quantities

$\Rightarrow$ s, t, u defined w.r.t. a scale [energy scale!]

IN FACT $$\boxed{\tilde{G}^{(4)}_{AMP}(s,t,u) - G^{(4)}_{AMP}(\bar{s},\bar{t},\bar{u}) = \text{Finite}}$$

because $\frac{1}{\varepsilon}$ part is independent of s, t, u !

only the difference has a PHYSICAL MEANING $\Rightarrow$ we

We can turn this around : <u>we actually don't know λ_0 !</u>

We can use $\tilde{G}^{(4)}_{1AMP}$ to MEASURE the interaction strength at a given "scale" $\Rightarrow$ value of "s, t, u"

Say $s = 4m_0^2$ (threshold) $t = 0$, $u = 0$

Convenient because, from Lecture 15

$$-\!\bigcirc\!- = i\,C(\varepsilon)\,\frac{1}{16\pi^2\varepsilon}\left[\int_0^1 dx - \varepsilon\int_0^1 dx\,\ln\left(1 + \underbrace{\frac{p^2}{m_0^2}x(1-x)}\right)\right]$$

$$\xrightarrow{p^2\to 0} = i\,C(\varepsilon)\,\frac{1}{16\pi^2\varepsilon} = \frac{i}{16\pi^2}\left[\frac{1}{\varepsilon} - \ln\left(\frac{m_0^2}{\mu^2}\right)\right]\underbrace{(4\pi)^{\varepsilon}\,\Gamma(1+\varepsilon)}_{S_\varepsilon}$$

so in this limit

$$\tilde{G}^{(4)}_{AMP}(4m_0^2, 0, 0) = -i\lambda_0\left[1 - \frac{\lambda_0\mu^{-2\varepsilon}}{32\pi^2}\left(\frac{3}{\varepsilon} - 3\ln\left(\frac{m_0^2}{\mu^2}\right) - A(4m_0^2)\right)S_\varepsilon\right]$$

$[\sqrt{Z}]^4$

$$\stackrel{\text{DEFINE}}{\equiv} -i\lambda_R\,\mu^{2\varepsilon} \Rightarrow \text{we want } \lambda_R \text{ to be } \underline{\text{DIMENSIONLESS}}!$$

we forget about Z because we have seen it does not contribute at this order $\Rightarrow$ then

$$\lambda_R \mu^{2\varepsilon} = \lambda_0 \left[1 - \frac{\lambda_0 \mu^{-2\varepsilon}}{32\pi^2} \left(\frac{3}{\varepsilon} - 3 \ln \frac{m_0^2}{\mu^2} - A(4m_0^2) \right) S_\varepsilon \right]$$

which allows us to express $\lambda_0 \to \lambda_R$: renormalize the coupling !

$\Rightarrow$ IMPORTANT $\lambda_R(\lambda_0, m_0)$ depends on "ren scheme"

we could have defined it at a different energy

$$\lambda_0 = \lambda_R \mu^{2\varepsilon} \left[1 + \frac{\lambda_R}{32\pi^2} \left(\frac{3}{\varepsilon} - 3 \ln \frac{m^2}{\mu^2} - A(4m^2) \right) S_\varepsilon \right]$$

$+ O(\lambda_R^2)$

here, $\lambda_0 = \lambda_R \mu^{2\varepsilon}$ at this order

cancels $\mu^{2\varepsilon}$!

I also changed $m_0 \to m$ since this is the same at this λ order so take now result for $\tilde{G}^4_{1\,AMP}$ and express it

in terms of $\underbrace{m, (Z^{1/2}\phi)}_{\text{make no difference at this order}}$ and λ_R ← this makes amplitude FINITE

$$\left[\sqrt{Z}^4 \tilde{G}^{(4)}_{1\,AMP}\right] = -i\lambda_R \mu^{2\varepsilon}\left[1 + \frac{\lambda_R}{32\pi}\left(\frac{3}{\varepsilon} - 3\ln\frac{m^2}{\mu^2} - A(4m^2)\right)S_\varepsilon\right]$$

$$\times\left(1 - \frac{\lambda_R}{32\pi^2}\left(\frac{3}{\varepsilon} - 3\ln\frac{m^2}{\mu^2} - A(s) - A(t) - A(u)\right)S_\varepsilon\right)$$

$$= -i\lambda_R\mu^{-2\varepsilon}\left(1 - \frac{\lambda_R}{32\pi^2}\left[-A(s) - A(t) - A(u) + A(4m^2)\right]S_\varepsilon\right)$$

Poles cancelled ; $\varepsilon \to 0$; $S_\varepsilon \to 1$, $\mu^{-2\varepsilon} \to 1$

$$= -i\lambda_R\left[1 + \frac{\lambda_R}{32\pi^2}\left(A(s) + A(t) + A(u) - \underset{\text{scheme!}}{\underline{A(4m^2)}}\right)\right]$$

renormalized result must be INDEPENDENT of REGULARIZATION SCHEME USED ⇒ (Dim reg, or cut off or others)

But it still depends on RENORMALIZATION SCHEME through $A(4m^2)$ in its FUNCTIONAL FORM.

What if we used different definition? for

$$-i\lambda_R' \mu^{2\varepsilon} = [\sqrt{Z}]^4 \tilde{G}^{(4)}_{1\,AMP} \Big|_{s=t=u=-q^2}$$

arbitrary value

then, following same steps:

$$\left[\sqrt{Z}^4 \tilde{G}^{(4)}_{AMP}\right] = -i\lambda_R'\left(1 + \frac{\lambda_R'}{32\pi^2}\left[A(s)+A(t)+A(u)-3A(-q^2)\right]\right)$$

looks different but λ_R' is also different!

$$\lambda_R' = \lambda_R\left(1 + \frac{\lambda_R}{32\pi^2}\left[A(4m^2) - 3A(-q^2)\right]\right)$$

coupling DEPENDS on energy...

such that NUMERICALLY I get the same result for $\tilde{G}^4_{1\,AMP}$ at this order in λ_R if I consistently account for relation between λ_R & λ_R'

In conclusion, we have made 2- & 4-point G FINITE by renormalizing available parameters

$\Rightarrow m_0, \lambda_0$ [& rescaling $\phi_0 \to \sqrt{Z}\,\phi$]

We did this in <u>ON-SHELL SCHEME</u>

CONJECTURE [to be proved in NEXT LECTURE] :

we can make all $G^{(n)}$ finite as follows :

1. COMPUTE $G_0^{(n)}$ in terms of λ_0, m_0, ϕ_0

2. Eliminate $\lambda_0, m_0 \longrightarrow \lambda_R, m_R$ (of "on-shell" λ, m)
   and multiply by $[\sqrt{Z}]^{-n}$; $Z = Z[m_0(m,\lambda), \lambda_0(m,\lambda)]$

$$G^{(n)}(x_1 \ldots x_n) = \langle \Omega | T\{\phi(x_1) \ldots \phi(x_n)\} | \Omega \rangle =$$

$$= \left(\sqrt{Z}\right)^{-n} G_0^{(n)}(x_1, \ldots, x_n) \quad \left(\text{because } \phi_0 = \sqrt{Z}\,\phi\right)$$

Resulting $G^{(n)}$ should be free of UV divergences

3. when I AMPUTATE using LSZ I get (L.17)

$$iT = i\mathcal{M}_{fi} = (\sqrt{Z})^n \, \tilde{G}^{(n)}_{0,AMP}(p_1 \dots p_n)$$

$\uparrow$ in mom. space

so we can leave out Z if we use on-shell ren $\tilde{G}_{AMP}$!

ALTERNATIVE: RENORMALIZED PERTURBATION THEORY

in $\mathcal{L}$ do:

$$\phi_0 \to \sqrt{Z_R}\,\phi_R \qquad \lambda_0 \to Z_\lambda^R \lambda_R \mu^{2\varepsilon}$$

$$m_0^2 \to m_R^2 + \delta m_R^2 = \left[Z_m^R m_R\right]^2$$

and using

$$Z_R - 1 = O(\lambda) \qquad \& \qquad \delta m_R^2 = O(\lambda)$$

$$Z_\lambda^R - 1 = O(\lambda)$$

we can write

$$\mathcal{L} = \frac{1}{2}\left[(\partial_\mu\phi_0)(\partial^\mu\phi_0) - m_0^2\phi_0^2\right] - \frac{\lambda_0}{4!}\phi_0^4$$

$$\to \frac{1}{2} Z_R \,\partial_\mu\phi_R\partial^\mu\phi_R - \frac{1}{2} Z_R (m_R^2 + \delta m_R^2)\phi_R^2 - Z_\lambda^R Z_R^2 \mu^{2\varepsilon}\frac{\lambda_R}{4!}\phi_R^4$$

$$= \frac{1}{2}\left[\partial_\mu \phi_R \partial^\mu \phi_R - m_R^2 \phi_R^2\right] - \frac{\lambda_R}{4!}\phi_R^4 \quad \Big\} \; \mathcal{L}_R$$

RENORMALIZED LAGRANGIAN

$$+ \frac{1}{2}\underbrace{(Z_R - 1)}_{\delta_Z^R}\partial_\mu \phi_R \partial^\mu \phi_R - \frac{1}{2}\Big(\underbrace{[Z_R - 1]m_R^2 + Z_R\,\delta m_R^2}_{\delta_m^R}\Big)\phi_R^2$$

$$- \underbrace{(Z_1^R Z_R^2 - 1)}_{\delta_1^R}\frac{\lambda_R \mu^{2\varepsilon}}{4!}\phi_R^4 \quad \Big\} \; \mathcal{L}_{C.T.}$$

COUNTER-TERM LAGRANGIAN !

We then compute all green functions directly in terms of $\mathcal{L}_R + \mathcal{L}_{C.T.}$ $\Rightarrow$ BARE QUANTITIES never show up! $\mathcal{L}_{C.T.}$ treated as "new interactions"

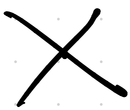
$$\begin{cases} \delta_Z^R = Z_R - 1 \\ \delta_m^R = (Z_R - 1)m_R^2 + Z_R\,\delta m_R^2 \\ \delta_1^R = Z_1^R Z_R^2 - 1 \end{cases}$$

Counter terms must be determined using 3 renormalization conditions !

WHAT ARE THE FEYNMAN RULES OF THIS THEORY?

 $= \dfrac{i}{p^2 - m_R^2 + i\delta}$

$m_R^2 =$ ren mass

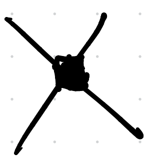
 $= -i\lambda_R$

Vertex λ ren !

 $= -i\left(-\delta_Z^R p^2 + \delta_m^R\right)$

from $\mathcal{L}_{ct}$

$$\frac{1}{2}\delta_Z \partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\delta_m \phi^2$$

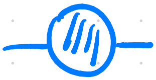
 $= -i\delta_\lambda^R \lambda_R \mu^{2\varepsilon}$

from $\mathcal{L}_{c.t}$

$$-\delta_\lambda^R \frac{\lambda_R \mu^{2\varepsilon}}{4!}\phi_R^4$$

⇒ Different conditions ≡ different schemes

ON-SHELL SCHEME corresponds to $\left[m_{os}^2 = m^2 \; ! \right]$

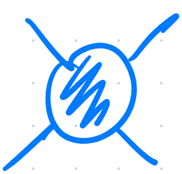
 $= \dfrac{i}{p^2-m^2-\Pi(p^2,m^2)+i\delta} \longrightarrow \dfrac{i}{p^2-m^2+i\delta}$ + rest

⇒ POLE at m^2, residue $= 1$ AT ALL ORDERS

this implies two conditions :

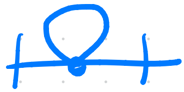
$$\Pi(p^2=m^2, m^2) = 0 \quad ; \quad \left.\frac{\partial\Pi}{\partial p^2}\right|_{p^2=m^2} = 0$$

THIRD one from 4-point

 $\Big|_{\substack{\text{AMP} \\ s=4m^2 \\ t=u=0}} = -i\lambda\mu^{2\varepsilon}$

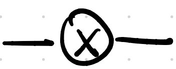
$\lambda_{os} = \lambda$

The calculation to δ_x counterterms is as the one we already performed BUT looks a bit different:

$-i\Pi(p^2, m^2) =$  $+$  $+ \cdots$

for OS SCHEME DROP "R", using Feynman rules for c.t

we get

 $= -i(-\delta_Z p^2 + \delta_m)$ and full calculation gives

use $$\Pi = \frac{\lambda}{32\pi^2} m^2 \left(-\frac{1}{\varepsilon} - 1 + O(\varepsilon^2)\right) C(\varepsilon)$$

$$-i\Pi(p^2, m^2) = -\frac{i\lambda}{32\pi^2} m^2 \left(-\frac{1}{\varepsilon} - 1\right) C(\varepsilon) - i(\delta_Z p^2 + \delta_m)$$

with $p^2 = m^2$ we then get

$$\delta_Z m^2 + \delta_m + \frac{\lambda}{32\pi^2} m^2 \left(-\frac{1}{\varepsilon} - 1\right) C(\varepsilon) = 0$$

SOLUTION

$$\delta_Z = 0 \quad ; \quad \delta_m = -\frac{1}{32\pi^2} m^2 \left[-\frac{1}{\varepsilon} - 1 + \ln\left(\frac{m^2}{\mu^2}\right)\right] S_\varepsilon$$

remember now relation between δ_m & δm

$$m_0^2 \to m_R^2 + \delta m_R \quad \& \quad \delta_m^R = (Z_R - 1)\, m_R^2 + Z_R\, \delta m_R^2$$

$Z = 1 + O(\lambda^2)$ then

$$\delta_m = \delta m + O(\lambda^2)$$

so they are the same at $O(\lambda)$!

in fact on page 7 we found:

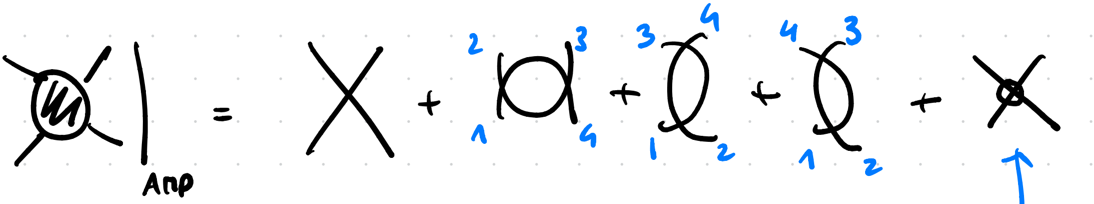
$$m^2 = m_0^2 \left(1 - \frac{\lambda}{32\pi^2}\left(\frac{1}{\varepsilon} + 1\right)\right) c(\varepsilon)$$

$$= m_0^2 - \frac{\lambda}{32\pi^2} m_0^2 \left(\frac{1}{\varepsilon} + 1 - \ln\left(\frac{m^2}{\mu^2}\right)\right) S_\varepsilon$$

$$\delta m = \frac{\lambda}{32\pi^2} m^2 \left(\frac{1}{\varepsilon} + 1 - \ln\left(\frac{m^2}{\mu^2}\right)\right) S_\varepsilon$$

as ABOVE!

Similarly, we compute AMP. 4-point Green function



counterterm $-i\delta_\lambda \lambda \mu^{2\varepsilon}$

by using results obtained before and imposing

that 

$= -i\lambda\mu^{2\varepsilon}$

we fix δ_λ at $O(\lambda)$

gives $$\delta_\lambda^{OS} = \delta_\lambda = \frac{\lambda}{32\pi^2}\left[+\frac{3}{\varepsilon} - 3\ln\left(\frac{m^2}{\mu^2}\right) - A(4m^2)\right] S_\varepsilon$$

- What happens if we change scheme?
- what happens at higher orders?