Advanced Quantum Field Theory SS 2023

Lecturer: Prof. Lorenzo Tancredi
Assistants: Philipp Alexander Kreer, Cesare Mella, Nikolaos Syrrakos, Fabian Wagner
https://www.groups.ph.tum.de/ttpmath/teaching/ss-2023/

Sheet 8: QCD β-function.

In this exercise, we study the β-function in a Yang-Mills theory coupled to different types of matter fields. Throughout the exercise sheet, we work in dim reg and in the Feynman gauge $\xi = 1$.

1 QCD β-Function

In the last sheet, you computed all the pieces to obtain the QCD β-function. It is defined through

$$\beta(g_s) = g_s \mu \frac{\partial}{\partial \mu}(-\delta_1 + \delta_2 + \frac{1}{2}\delta_3), \tag{1}$$

where μ is the renormalization scale, $\delta_{1,2,3}$ are the counter terms for the vertex-correction, fermion self-energy and the gluon self-energy. They remove the divergences from the last sheet. Put the pieces together and compute the β-function in a non-abelian YM theory coupled to n_f fermionic fields.

2 Ghost counter term

In the lecture and in the previous exercise sheet, we have computed almost all one-loop counter terms required to renormalize a Yang-Mills gauge theory based on the group SU(N) and coupled to a fermionic matter field in the fundamental representation. The only missing counter term is the ghost counter term, which we called $\bar{\delta}_3^c = Z_3^c - 1$ in the lectures. By evaluating the ghost two-point function, calculate this counter term in dimensional regularization in the $\overline{\text{MS}}$ scheme.

3 Scalar Field Extension to QCD β-Function

Consider a SU(3) gauge theory with two types of matter fields:

1. n_f massless Dirac fermions
2. n_s massless, complex scalars that transform in the fundamental representation of SU(3)

In this exercise we will compute the counter-terms $\widetilde{\delta_i}$ in this theory.

Step 1: Derive the new Feynman rules in this theory. You should find a simple modification of the Feynman rules of scalar QED.
Step 2: How does $\tilde{\delta}_1 - \tilde{\delta}_2$ change with respect to $\delta_1 - \delta_2$ from the previous exercise? Hint: Remember the renormalization conditions imposed by gauge symmetry.
Step 3: Compute the counter term $\tilde{\delta}_3$ in the new theory. Note that in comparison to δ_3 from above, you only have one additional diagram. Which one?
Step 4: Putting all the pieces together compute the β function in this theory. You should find

$$\beta = -\frac{g^3}{(4\pi)^2}\left(\frac{11}{3}C_F - \frac{1}{3}n_s C_A - \frac{4}{3}n_f C_A\right). \tag{2}$$