# Relativity, Particles, Fields SS 2017

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https://www.t75.ph.tum.de/teaching/ss17-relativity-particles-fields/

**Sheet 7: Wick's Theorem, Feynman Diagrams** (20.6.2017)

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## 1 Scattering in $\phi^3$ theory

Consider the $g\phi^3$ theory,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{g}{3!}\phi^3\,,$$

where the coupling constant $g$ has the dimension of mass.

**a)** Use Dyson's formula and Wick's theorem to compute the amplitude for $\phi\phi \to \phi\phi$ scattering at $O(g^2)$. Proceed along the lines that we followed in class for the computation of nucleon-nucleon scattering in the scalar Yukawa theory.

**b)** Write down the Feynman diagrams for the $\phi\phi \to \phi\phi$ process, and use them to re-derive the expression of the amplitude. Notice that in the propagators appear the following Lorentz invariant quantities: $s = (p_1 + p_2)^2$, $t = (p_1 - p_4)^2$, $u = (p_1 - p_3)^2$, where $p_1, p_2$ are the four-momenta of the initial state particles, taken ingoing, while $p_3, p_4$ are the four-momenta of the final state particles, taken outgoing. The quantities $s, t, u$ are called *Mandelstam variables*. Show that they satisfy $s + t + u = 4m^2$.

## 2 Vacuum to vacuum amplitude in $\phi^4$ theory

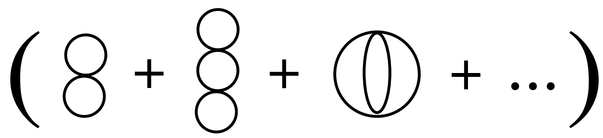
Consider the $\lambda\phi^4$ theory,

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4\,,$$

where the coupling constant $\lambda$ is dimensionless. Examine the quantity $\langle 0|\, S\, |0\rangle$, where $S$ indicates the $S$-matrix, to order $\lambda^2$.

**a)** Identify the different diagrams with the different contributions arising from an application of Wick's theorem.

**b)** Confirm that to order $\lambda^2$, the combinatoric factors work out so that the vacuum to vacuum amplitude is given by the exponential of the sum of distinct vacuum bubble types, that is

$\langle 0|\, S\, |0\rangle = \exp$ 

If you are unsure about the combinatoric factors, you can use the Dyson series instead of Feynman diagrams.