Quantum Field Theory WS 2018/2019

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http://www.t75.ph.tum.de/teaching/ws18-quantum-field-theory/

Sheet 8: scattering cross section
(12.12.2018; solution due by 19.12 at 16:00, the parts required for hand-in will be announced on 18.12 at 8:00am)
Discussed at tutorials of **19.12:**
Group 1: usual time and place
Groups 2 and 3: 16:00 – 18:00, PH II 227 (Seminarraum E20)

1*[1] $2 \to 2$ scattering via massive vector boson

Consider the theory of a complex scalar field ϕ which interacts with a real field $A_\mu(x)$ of mass m_A as

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) - m^2 \phi^\dagger \phi \,, \tag{1}$$

where $D^\mu = \partial^\mu - igA^\mu$.

a) Work out the Feynman rules for the interaction vertices.

b) Derive the scattering amplitude for $2 \to 2$ particle-particle scattering, $\phi\phi \to \phi\phi$, at $O(g^2)$ and express it in terms of the Mandelstam variables. You may assume that the Feynman propagator of the A_μ field is the same as the one for a massive scalar field of mass m_A but multiplied by $(-\eta_{\mu\nu} + k_\mu k_\nu / m_A^2)$, where k is the momentum of the A-field propagator.

c) Calculate the differential scattering cross section $d\sigma/d\cos\theta$ as a function of the energy E of the scalar particle in the center-of-mass frame of the scattering. Plot the distribution in the interval $\cos\theta \in [-1, 1]$ for $E \to m$ and for $E \gg m, m_A$.

d) Calculate the total cross section and plot its energy dependence. Provide analytic expressions for the limiting behaviours $E \to m$ and $E \gg m, m_A$.

e) Describe how the above results should be modified if one considers particle-antiparticle instead of particle-particle scattering.

f) Derive the scattering amplitude for $\phi\phi \to \phi\phi$ from the following Lagrangian at $O(c)$ and $O(\lambda)$,

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) + c\,(\partial_\mu \phi)^\dagger (\partial^\mu \phi) \phi^\dagger \phi - m^2 \phi^\dagger \phi - \lambda\,(\phi^\dagger \phi)^2 \,. \tag{2}$$

Compare it with the amplitude you derived in **b)** after expanding it in E^2/m_A^2 and m^2/m_A^2. Extract the values of c and λ as a function of g and m_A such that the leading terms in such an expansion coincide.

[1] The * means to be handed in.