Quantum Field Theory WS 2018/2019

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http://www.t75.ph.tum.de/teaching/ws18-quantum-field-theory/

Sheet 4: Feynman rules
(14.11.2018; solution due by 21.11 at 16:00, the parts required for hand-in will be announced on 20.11 at 8:00am; discussed at tutorials of 21.11, 22.11 and 26.11)

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1 Fourier transformation and translational invariance

Show that the position-space Green functions are translational invariant:

$$G(x_1 - y, \ldots, x_n - y) = G(x_1, \ldots, x_n)\,. \tag{1}$$

Show that this implies that the Fourier transform of $G(x_1, \ldots, x_n)$ with respect to all its arguments can always be written in the form

$$(2\pi)^4 \delta^{(4)}(p_1 + \cdots + p_n)\tilde{G}(p_1, \ldots, p_n)\,. \tag{2}$$

2 Feynman diagrams

a) Draw all diagrams that contribute to

1)* [1] the two-point function at order $\mathcal{O}(\lambda^2)$ in the theory of a complex scalar field with interaction

$$-\frac{\lambda}{4}(\phi^\dagger\phi)^2\,, \tag{3}$$

2) the three-point function at $\mathcal{O}(g^3)$ in the theory of a real scalar field with interaction

$$-\frac{g}{3!}\phi^3\,. \tag{4}$$

In both cases draw also the disconnected vacuum diagrams (although they do not contribute to the Green functions). For the connected diagrams, determine also the symmetry factor, i.e. the number of contractions resulting in identical diagrams, multiplied by $1/n!$ from the expansion of the exponential of the interaction Lagrangian to order n, and by the factors $1/4$ in case 1) or $1/3!$ in case 2) from the vertices.

b)* Choose one of the $\mathcal{O}(\lambda^2)$ diagrams from case 1) above and write down the complete mathematical expression for this diagram using the momentum-space Feynman rules.

c)* Determine the momentum-space vertex factor Feynman rule for the following terms in an interaction Lagrangian

$$1)\ \lambda\,\phi_1\phi_2\phi_1\phi_2 \tag{5}$$
$$2)\ \lambda\,\phi\phi(\partial^\mu\phi)(\partial_\mu\phi) \tag{6}$$
$$3)\ g\,f^{ABC}(\partial_\mu A^A_\nu)A^{\mu B}A^{\nu C} \tag{7}$$

[1] The * means to be handed in.

Here ϕ, ϕ_1, ϕ_2 denote real scalar fields, A_μ^A a real field that carries a Lorentz vector index μ and an internal index A, λ, g are coupling constants specifying the strength of the interaction, and f^{ABC} is totally antisymmetric in its indices. The summation convention is implied for all index types.
Hint: Before you start with 3), think about the possible form such a vertex can take, and write down the (correct!) result without explicit computation.

3* Mass term as interaction

In the lecture we made a distinction between kinetic terms, which are bilinear in fields, and interactions, which have three or more fields. Time evolution with the kinetic terms is solved exactly as part of the free Hamiltonian. Suppose, instead, we only put the derivative terms in the free Hamiltonian and treated the mass as an interaction,

$$\mathcal{H}_0 = \frac{1}{2}\phi\Box\phi\,, \qquad \mathcal{H}_{\rm int} = \frac{1}{2}m^2\phi^2\,. \tag{8}$$

a) Draw the (somewhat degenerate looking) Feynman graphs that contribute to the two-point function $\langle 0|T\{\phi(x)\phi(y)\}|0\rangle$ using only this interaction, up to $\mathcal{O}(m^6)$.

b) Evaluate the graphs.

c) Sum the series to all orders in m^2 and show you reproduce the propagator that would have come from taking $\mathcal{H}_0 = \frac{1}{2}\phi(\Box + m^2)\phi$.

d) Repeat the exercise classically: Solve for the massless propagator using an external current, perturb with the mass, sum the series, and show that you get the same answer as if you included the mass to begin with.
Hint: The solution of the equation of motion for a massless scalar sourced by an external current J, $\mathcal{L} = -\frac{1}{2}\phi\Box\phi + \phi J$, is $\phi_0 = (1/\Box)J$. Upon adding the perturbation $\Delta\mathcal{L} = -\frac{1}{2}m^2\phi^2$, solve the equation of motion for the perturbed scalar $\phi = \phi_0 + \Delta\phi$ order by order in m^2.