Quantum Field Theory WS 2018/2019

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http://www.t75.ph.tum.de/teaching/ws18-quantum-field-theory/

Sheet 13: Dirac and Majorana fermions
(30.01.2019; solution due by 06.02.2019 at 16:00, the parts required for hand-in will be announced on 05.02.2019 at 8:00am)
Discussed at tutorials of **06.02:**
Group 1: usual time and place
Groups 2 and 3: 16:00 – 18:00, PH II 227 (Seminarraum E20)

1* [1] Dirac spinors

a) Starting from the Lagrangian

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi \tag{1}$$

for a Dirac spinor field ψ, show that the free Hamiltonian $H_0 = \int \mathrm{d}^3x\,(\pi\dot{\psi} - \mathcal{L})$ with $\pi = \frac{\partial\mathcal{L}}{\partial\dot{\psi}}$ is given by

$$H_0 = \int \mathrm{d}^3x\,\bar{\psi}(x)\left(\gamma^i\frac{1}{i}\nabla^i + m\right)\psi(x) \tag{2}$$

$$= \sum_{s=\pm\frac{1}{2}}\int\frac{\mathrm{d}^3p}{(2\pi)^3 2E_p}E_p\left(a^\dagger(p,s)a(p,s) + b^\dagger(p,s)b(p,s)\right) \tag{3}$$

up to the zero-point energy for every degree of freedom. Both forms of the Hamiltonian should be derived, and $E_p = \sqrt{\vec{p}^{\,2} + m^2}$.

b) Verify the completeness relation for the fermionic coordinate eigenstates:

$$\int\prod_n \mathrm{d}\xi_n^*\,\mathrm{d}\xi_n\,|\xi\rangle e^{-\sum_m \xi_m^*\xi_m}\langle\xi| = 1\,. \tag{4}$$

The result $\langle\xi|\eta\rangle = \exp(\sum_n \xi_n^*\eta_n)$ may be used.

2* Majorana spinors

Compute the free generating functional and derive the non-vanishing contractions for a left-handed Majorana fermion field with Lagrangian

$$\mathcal{L} = \psi^\dagger_\alpha i\bar{\sigma}^\mu_{\alpha\beta}\partial_\mu\psi_\beta - \frac{m}{2}\left(\psi_\alpha\epsilon_{\alpha\beta}\psi_\beta - \psi^\dagger_\alpha\epsilon_{\alpha\beta}\psi^\dagger_\beta\right)\,. \tag{5}$$

3* Compton scattering of a scalar particle on an electron

a) Compute the scattering of a real massless scalar ϕ with a massive electron ψ, i.e. $\phi\psi \to \phi\psi$, with interaction

$$\mathcal{L} = g\bar{\psi}\psi\phi\,. \tag{6}$$

[1] The * means to be handed in.

Specifically, derive the differential scattering cross-section in the rest frame of the incoming electron,

$$\frac{d\sigma}{d\cos\theta} = \frac{g^4}{32\pi m^2}\left(\frac{E'}{E}\right)^2\left(\frac{E}{E'}+\frac{E'}{E}+2\cos 2\theta\right). \tag{7}$$

θ is the scattering angle between the incoming and outgoing scalars and E, E' their respective energies. Note you should average and sum over incoming and outgoing spins of the electron, respectively.

b) Repeat the calculation but with the interaction

$$\mathcal{L} = ig\bar{\psi}\gamma_5\psi\phi\,. \tag{8}$$

Explain, why we have written an i here.