Advanced Quantum Field Theory SS 2019

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https://www.t75.ph.tum.de/teaching/ss19-quantum-field-theory-ii/

Sheet 7: Anomalies (05.07.2019)

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1 Another look at the Abelian anomaly

In this exercise we analyze what happens if we give up conservation of the vector current, insisting instead that the axial current be conserved.

a) Given the momentum-space expression of the amplitude $\langle 0| T\left\{j_5^\lambda(0) j^\mu(x_1) j^\nu(x_2)\right\}|0\rangle$, where $j_5^\lambda = \overline{\psi}\gamma^\lambda\gamma_5\psi$ and $j^\mu = \overline{\psi}\gamma^\mu\psi$:

$$\Delta^{\lambda\mu\nu}(a,k_1,k_2) = -i\int \frac{d^4p}{(2\pi)^4}\,\mathrm{Tr}\left[\gamma^\lambda\gamma_5 \frac{1}{\not p + \not a - \not q}\,\gamma^\nu \frac{1}{\not p + \not a - \not k_1}\,\gamma^\mu \frac{1}{\not p + \not a}\right] + \{\mu \leftrightarrow \nu, k_1 \leftrightarrow k_2\} \tag{1}$$

where $q = k_1 + k_2$ and a is an arbitrary momentum shift, by writing $a = \alpha(k_1+k_2) + \beta(k_1 - k_2)$ one can show that

$$\Delta^{\lambda\mu\nu}(a,k_1,k_2) = \Delta^{\lambda\mu\nu}(k_1,k_2) + \frac{i\beta}{4\pi^2}\,\epsilon^{\lambda\mu\nu\sigma}(k_1-k_2)_\sigma\,, \qquad \Delta^{\lambda\mu\nu}(k_1,k_2) \equiv \Delta^{\lambda\mu\nu}(0,k_1,k_2). \tag{2}$$

Notice that α drops out. In this notation, the choice $\beta = -1/2$ ensures that $k_{1\mu}\Delta^{\lambda\mu\nu}(a,k_1,k_2) = 0$, namely that the vector current is conserved. Setting $\beta = +1/2$ instead guarantees that $q_\lambda \Delta^{\lambda\mu\nu}(a,k_1,k_2) = 0$.

Consider the amplitude $\langle 0| T\left\{j_5^\lambda(0) j_5^\mu(x_1) j_5^\nu(x_2)\right\}|0\rangle$, given at lowest order by the triangle diagrams with axial currents at *all* vertices. By using $(\gamma_5)^2 = 1$ and Bose symmetry, show that the momentum-space amplitude reads

$$\Delta_5^{\lambda\mu\nu}(k_1,k_2) = \frac{1}{3}\left[\Delta^{\lambda\mu\nu}(a,k_1,k_2) + \Delta^{\mu\nu\lambda}(a,k_2,-q) + \Delta^{\nu\lambda\mu}(a,-q,k_1)\right]. \tag{3}$$

b) Calculate $q_\lambda \Delta_5^{\lambda\mu\nu}(k_1,k_2)$. If we insisted on setting $\beta = +1/2$, what would this result tell us about our hope to retain conservation of the axial current?

2 Anomaly cancellation

Consider a $U(1)_1 \times U(1)_2$ gauge theory with two left-handed Weyl fermions $\psi^{(1,2)}$ charged under both groups. Find the conditions that the charges $Q_a^{(i)}$ have to satisfy in order for the theory to be anomaly free.

3 Neutrinos

In the Standard Model, for which types of neutrino masses (Majorana, Dirac, or both) is lepton number L anomalous? For which types of masses is baryon-minus-lepton number $B - L$ anomalous?

Hint: see Chapter 13 of QFT1 and e.g. Sec. 29.3.4 in Schwartz, *Quantum Field Theory and the Standard Model*, or another textbook of your choice that discusses neutrino masses in the Standard Model.