Advanced Quantum Field Theory SS 2019

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https://www.t75.ph.tum.de/teaching/ss19-quantum-field-theory-ii/

Sheet 3: Yang-Mills renormalization at 1-loop (24.05.2019)

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1 Feynman rules

The $SU(N)$-invariant Lagrangian of non-Abelian gauge fields coupled to a fermion and a scalar multiplet in the fundamental representation of $SU(N)$ is given, in a covariant gauge, by

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2\xi}(\partial_\mu A_\nu)^2 + (\partial_\mu \bar{c}^a) D^{ac}_\mu c^c \\ &+\bar{\psi}_i \gamma^\mu (i D^\mu_{ij} - m\delta_{ij})\psi_j \\ &+(D^\mu_{ik}\phi_k)^* (D^\mu_{ij}\phi_j) - M^2 \phi^*_i \phi_i \,,\end{aligned} \tag{1}$$

where

$$\begin{aligned} D^{ac}_\mu c^c &= \delta^{ac}\partial_\mu + g f^{abc} A^b_\mu c^c \,, \\ D^\mu_{ij}\{\psi_j, \phi_j\} &= (\delta_{ij}\partial^\mu - i g T^a_{ij} A^{a\,\mu})\{\psi_j, \phi_j\} \,. \end{aligned} \tag{2}$$

Derive the Feynman rules for the triple-gluon vertex, the ghost-gluon vertex and the scalar-scalar-gluon vertex.

2 Charge universality

The ghost field renormalization constant Z_{3c} and the ghost-gluon vertex renormalization constant Z_{1c} in the renormalized $SU(N)$ non-Abelian gauge theory are defined by

$$\mathcal{L} \supset -Z_{3c}\, \bar{c}^a \Box c_a + g Z_{1c}\, f^{abc} (\partial^\mu \bar{c}^a) A^b_\mu c^c \tag{3}$$

where g is the renormalized coupling constant.

Compute Z_{3c} and Z_{1c} in Feynman gauge at 1-loop and verify, at leading g^2 order, that $Z_{1c}/Z_{3c} = Z_1/Z_2$, where Z_2 and Z_1 are the renormalization constants of the fermion kinetic term and the fermion-gluon interaction respectively,

$$\begin{aligned} Z_1 &= 1 + \frac{1}{\varepsilon}\left(\frac{g^2}{16\pi^2}\right)\left[-2C_F - 2C_A\right] \,, \\ Z_2 &= 1 + \frac{1}{\varepsilon}\left(\frac{g^2}{16\pi^2}\right)\left[-2C_F\right] \,, \end{aligned} \tag{4}$$

in minimal subtraction with dimensional regularization. You will need to use the relation

$$f^{adf} f^{feb} f^{ced} = \frac{N}{2} f^{abc} \,. \tag{5}$$

3 Running coupling

Compute the contribution of a scalar multiplet in a generic irreducible representation of $SU(N)$, $\mathbf{R}$, to the corresponding non-Abelian β-function. You will need the relation

$$\mathrm{Tr}[T^a T^b] = T(\mathbf{R})\delta^{ab}\,, \tag{6}$$

where $T(\mathbf{R})$ is the index of the representation. Note that only the scalar-scalar-gluon vertex is needed in dimensional regularization. Argue why the seagull scalar graph can be ignored.