Advanced Quantum Field Theory SS 2019

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https://www.t75.ph.tum.de/teaching/ss19-quantum-field-theory-ii/

Sheet 2: Quantum Yang-Mills: FP, BRST, axial gauge (18.05.2019)

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1 Slavnov operator

BRST transformations of any field ϕ can be expressed as $\phi \to \phi + \theta \mathcal{Q}_{\text{BRS}}\, \phi$, where $\mathcal{Q}_{\text{BRS}}$ is known as the Slavnov operator and θ is a Grassmann number:

$$\begin{aligned} \mathcal{Q}_{\text{BRS}}\, \phi_i &= igc^a T^a_{ij} \phi_j \,, \\ \mathcal{Q}_{\text{BRS}}\, A^a_\mu &= D_\mu c^a \,, \\ \mathcal{Q}_{\text{BRS}}\, c^a &= -\frac{1}{2} g f^{abc} c^b c^c \,, \\ \mathcal{Q}_{\text{BRS}}\, \bar{c}^a &= -\frac{1}{\xi} \partial^\mu A^a_\mu \,, \end{aligned} \tag{1}$$

for matter, gauge, ghost, anti-ghost fields, respectively.

a) Show that the BRST transformation of any of the $\mathcal{Q}_{\text{BRS}}\phi$, $\mathcal{Q}_{\text{BRS}}A^a_\mu, \ldots$ vanishes, i.e. the Slavnov operator is nilpotent $\mathcal{Q}^2_{\text{BRS}} = 0$.

b) Show that a nilpotent operator that commutes with the Hamiltonian divides its eigenstates into three subspaces.

2 Axial gauges and QED ghosts

The axial gauge fixes $n^\mu A^a_\mu = 0$ with n^μ a 4-vector and gauge-fixing parameter ξ.

a) Derive the axial-gauge non-Abelian Lagrangian including gauge-fixing and ghost terms.

b) Derive the corresponding form of the gluon propagator (in momentum space) and show that the ghost fields decouple.

c) Consider the gauge choice $G(A) = \partial_\mu A^\mu + \lambda A_\mu A^\mu$ in QED. Derive the photon propagator (in momentum space) and the Feynman rule for the ghost-photon interaction.

3 Generalized Maxwell equation

a) Show that the gauge-fixed Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 + \partial^\mu A^a_\mu B^a + \frac{\xi}{2}(B^a)^2 - \bar{c}^a \partial_\mu D^\mu c^a + \mathcal{L}_M(\phi_i, D_\mu \phi_i) \tag{2}$$

is invariant under the *global* symmetry that corresponds to *space-time independent* gauge transformations. Note that under such a symmetry, A_μ, B, c and $\bar{c}$ all transform in the adjoint representation. The matter Lagrangian is taken to be invariant. Note also, that B is an auxiliary field, *i.e.* it does not have a kinetic term.

b) Derive the EOM for the gauge field and determine the Noether current J^a_μ associated to the aforementioned global symmetry.

c) Arrive at the generalized Maxwell equation

$$\partial^\nu F^a_{\nu\mu} + g J^a_\mu = i\{Q_{\mathrm{BRS}}, D_\mu \bar{c}^a\}\,, \tag{3}$$

where Q is the conserved charge associated with the BRST symmetry. Note, it is not actually necessary to calculate explicitly the charge operator, but only to know that it generates the BRST transformation on any field ϕ with

$$\theta \mathcal{Q}_{\mathrm{BRS}}\phi = i[\theta Q_{\mathrm{BRS}}, \phi]\,. \tag{4}$$

Note, that (3) reduces to the usual Maxwell equation if evaluated between physical states.