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Foreword

These notes are not original but mostly a combination of results found in books and lecture notes. My contribution is here foremost in the selection and the perspective provided. I have made liberal use of the following references

- Peskin, Schroeder - *An Introduction to Quantum Field Theory* ★★
- Schwartz - *Quantum Field Theory and the SM* ★★
- Martin Beneke - *Lecture notes on QFT* ★★★,
- Mandl, Shaw - *Quantum Field Theory* ★
- Zee - *Quantum Field Theory in a Nutshell* ★
- Srednicki - *Quantum Field Theory* ★★★
- Weinberg - *Quantum Field Theory I-III* ★★★
- Ramond - *Field Theory: a Modern Primer* ★★★
- Itzykson, Zuber - *Quantum Field Theory* ★★★
- Shifman, *Advanced topics in Quantum Field Theory* ★★★

and many more. I strongly suggest that you find a book (or books) you like from the ones above and study it as a complement to these lecture notes.

This course provides a hopefully gentle excursion into the beautiful world of advanced quantum field theory, building on the foundations we set during the last semester. You will learn to tackle the theoretical framework that underlies all of nature.

And remember: If you don't make mistakes, you're not working on hard enough problems. And that's a big mistake.

Let's start.

The ★ indicates the technical level of the book. It is roughly proportional to the time needed per page for a full understanding. Note, that the Zee book in particular is written in a conversational tone, but covers more concepts than any of the other books.