



Sheet 1: Special Relativity (2.11.2020)

1 Addition of velocities

Derive the law of addition of velocities in general, when \vec{u} and \vec{v} point in different directions. Note, the law must obey rotational invariance.

2 Ordering of space-time events

Three events A, B, C are seen by observer \mathcal{O} to occur in the order ABC. Another observer, \mathcal{O}' , sees the events to occur in the order CBA. Can there always be a third observer who sees the events in the order ACB? Restrict your considerations to one space dimension and support your conclusion by drawing a spacetime diagram.

3 Atmospheric muon lifetime

Muons are unstable subatomic particles with an approximate mass of $m_\mu = 100 \text{ MeV}/c^2$ and a lifetime of $\tau = 1.5 \mu\text{s}$. They can be created when cosmic rays enter the Earth's atmosphere. Suppose that a muon is created at 4 km above sea level with an energy of 1000 MeV with respect to the Earth, taking it $1.5 \mu\text{s}$ to disintegrate in the muon's rest frame. Will the muon reach the sea level for

- an observer in the Earth's rest frame?
- an observer in the muon's rest frame?

4 Length contraction and time dilation

A train of proper length L enters a tunnel of proper length D , travelling at a velocity v .

- Find the condition between L , D and v , such that the train is totally inside the tunnel at some time for an observer A standing outside the tunnel. What is the corresponding condition for an observer B inside the train?
- When the back end of the train enters the tunnel, observer B realizes that the exit of the tunnel is closed. Show that the condition on the time observer B will have, in his/her rest frame, to react and send a laser signal from the back end of the train to the exit of the tunnel, in order to make a hole for the train to pass through, is given by

$$\tau_B < \frac{D\sqrt{1-\beta^2} - L(1+\beta)}{v}, \quad (1)$$

where $\beta = v/c$.