7 Mathematical framework for QED

Main reference for this section is [17, Chapter 7].

7.1 Classical field theory

Consider a Lagrangian $\mathcal{L}(\phi, \phi^*, \partial\phi, \partial\phi^*; A, \partial A)$ where A is a vector field, ϕ a complex scalar field. Suppose first that $\mathcal{L}$ is invariant under the transformations

$$\phi_\varepsilon(x) = e^{i\varepsilon(x)}\phi(x), \quad \phi^*_\varepsilon(x) = e^{-i\varepsilon(x)}\phi^*(x), \quad A_{\mu,\varepsilon}(x) = A_\mu(x) + \partial_\mu\varepsilon(x), \tag{108}$$

for some function ε. Thus the corresponding variation of the Lagrangian must vanish. Exploiting the Euler-Lagrange equations we get

$$\delta\mathcal{L} = (\partial_\mu j^\mu)\varepsilon + (j^\nu - \partial_\mu F^{\mu\nu})\partial_\nu\varepsilon - (F^{\mu\nu})\partial_\mu\partial_\nu\varepsilon, \tag{109}$$

where

$$j^\mu(x) := \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi)}(x)i\phi(x) - \frac{\partial\mathcal{L}}{\partial(\partial_\mu\phi^*)}(x)i\phi^*(x), \quad F^{\mu\nu} := -\frac{\partial\mathcal{L}}{\partial(\partial_\mu A_\nu)} \tag{110}$$

- From the $\partial_\mu\partial_\nu\varepsilon$-term of the (109) we obtain that the symmetric part of $F^{\mu\nu}$ vanishes, i.e.

$$F^{\mu\nu} = -F^{\nu\mu} \tag{111}$$

- From the $\partial_\nu\varepsilon$-term of the (109) we get the *local Gauss Law*

$$\partial_\mu F^{\mu\nu} = j^\nu \tag{112}$$

- From the ε-term of (109) we get $\partial_\mu j^\mu = 0$ and $\partial_t Q = 0$, where

$$Q = \int d^3y\, j^0(0, \vec{y}). \tag{113}$$

  (Noether's theorem)

- Furthermore, Q is the infinitesimal generator of the global $U(1)$ symmetry, i.e.

$$\{Q, \phi(0,\vec{x})\} = -\frac{d}{d\varepsilon}\phi_\varepsilon(0,\vec{x})|_{\varepsilon=0} = -i\phi(0,\vec{x}), \tag{114}$$

$$\{Q, \phi^*(0,\vec{x})\} = -\frac{d}{d\varepsilon}\phi^*_\varepsilon(0,\vec{x})|_{\varepsilon=0} = i\phi^*(0,\vec{x}). \tag{115}$$

  Here the Poisson bracket is defined by

$$\{F, G\} = \int d^3z \left(\frac{\delta F}{\delta\phi(0,\vec{z})}\frac{\delta G}{\delta\pi(0,\vec{z})} - \frac{\delta F}{\delta\pi(0,\vec{z})}\frac{\delta G}{\delta\phi(0,\vec{z})}\right) + \cdots \tag{116}$$

  where $\pi(z) = \frac{\partial\mathcal{L}}{\partial(\partial_0\phi)(z)}$ is the canonical momentum and the omitted terms correspond to ϕ^* and A. (Note, however, that terms corresponding to A are not relevant for (115)).

- On the other hand using $\partial_\mu F^{\mu\nu} = j^\nu$ we can compute

$$\begin{aligned} -i\phi(0,\vec{x}) = \{Q, \phi(0,\vec{x})\} &= \int d^3y\, \{j^0(0,\vec{y}), \phi(0,\vec{x})\} \\ &= \int d^3y\, \{\partial_i F^{i,0}(0,\vec{y}), \phi(0,\vec{x})\} = \lim_{R\to\infty} \int_{B_R} d^3y\, \{\vec{\nabla}\cdot\vec{E}(0,\vec{y}), \phi(0,\vec{x})\} \\ &= \lim_{R\to\infty} \int_{\partial B_R} d\vec{\sigma}(\vec{y}) \cdot \{\vec{E}(0,\vec{y}), \phi(0,\vec{x})\}, \end{aligned} \tag{117}$$

  where $\vec{E} := (F^{1,0}, F^{2,0}, F^{3,0})$, B_R is a ball of radius R centered at zero, ∂B_R its boundary (a sphere) and we used the Stokes theorem. Note that in quantum theory, where $\{\,\cdot\,,\,\cdot\,\} \to -i[\,\cdot\,,\,\cdot\,]$ above[6], the last expression would be zero by locality, giving a contradiction!

- One possible way out (which we will not follow) is to abandon locality of charged fields but keep $\partial_\mu F^{\mu\nu} = j^\nu$ (Quantisation in the Coulomb gauge).

- We will follow instead the Gupta-Bleuler approach, where all fields are local, but $\langle\Psi_1|(\partial_\mu F^{\mu\nu} - j^\nu)\Psi_2\rangle = 0$ only for Ψ_1, Ψ_2 in some 'physical subspace' $\mathcal{H}' \subset \mathcal{H}$. This will enforce $\langle\Psi|\Psi\rangle < 0$ for some $\Psi \in \mathcal{H}$ thus we have to use 'indefinite metric Hilbert spaces' (Krein spaces)

- Incidentally, local, Poincaré covariant massless vector fields A^μ do exist on Krein spaces (which is not the case on Hilbert spaces). Thus we will have candidates for the electromagnetic potential.

7.2 Strocchi-Wightman framework [18, 19]

Definition 7.1 *An 'indefinite metric Hilbert space' (Krein space) $\mathcal{H}$ is a vector space equipped with a sesquilinear form $\langle\,\cdot\,|\,\cdot\,\rangle$ s.t.*

- *$\langle\,\cdot\,|\,\cdot\,\rangle$ is non-degenerate, i.e. for any $\Psi \neq 0$ there is some $\Phi \in \mathcal{H}$ s.t. $\langle\Psi|\Phi\rangle \neq 0$.*

- *$\mathcal{H}$ carries an auxiliary positive-definite scalar product $(\,\cdot\,|\,\cdot\,)$ w.r.t. which it is a Hilbert space.*

- *There is a bounded, invertible operator η on $\mathcal{H}$, self-adjoint w.r.t. $(\,\cdot\,|\,\cdot\,)$, s.t. $\langle\Psi_1|\Psi_2\rangle = (\Psi_1|\eta\Psi_2)$.*

Only the first property above is physically important. The role of the last two properties is to provide a topology on $\mathcal{H}$ which is needed for technical reasons (e.g. density of various domains).

Definition 7.2 *A Strocchi-Wightman relativistic quantum mechanics is given by:*

[6] It should be mentioned that for gauge theories the standard quantisation prescription $\{\,\cdot\,,\,\cdot\,\} \to -i[\,\cdot\,,\,\cdot\,]$ may fail in general and a detour via 'Dirac brackets' is required. However, in the above situation the simple analogy can be maintained.

1. *A Krein space* $\mathcal{H}$.

2. *A physical subspace* $\mathcal{H}' \subset \mathcal{H}$ *s.t.* $\langle\Psi|\Psi\rangle \geq 0$ *for* $\Psi \in \mathcal{H}'$.

3. *The physical Hilbert space* $\mathcal{H}_{\mathrm{ph}} := (\mathcal{H}'/\mathcal{H}'')^{\mathrm{cpl}}$, *where* $\mathcal{H}'' := \{\Psi \in \mathcal{H}' \,|\, \langle\Psi|\Psi\rangle = 0\}$. *Its elements are equivalence classes* $[\Psi] = \{\Psi + \Psi_0 \,|\, \Psi_0 \in \mathcal{H}''\}$, *where* $\Psi \in \mathcal{H}'$.

4. *A* $\langle\cdot|\cdot\rangle$*-unitary representation* $\widetilde{\mathcal{P}}_+^\uparrow \ni (\widetilde{\Lambda}, a) \mapsto U(\widetilde{\Lambda}, a)$ *in* $\mathcal{H}$ *s.t.* $\mathcal{H}'$ *is invariant under* U. *Then* U *induces a unitary representation on* $\mathcal{H}_{\mathrm{ph}}$ *by* $U_{\mathrm{ph}}(\widetilde{\Lambda}, a)[\Psi] = [U(\widetilde{\Lambda}, a)\Psi]$. *We assume that* U_{ph} *is continuous and satisfies the spectrum condition.*

5. *A unique (up to phase) vacuum vector* $\Omega \in \mathcal{H}'$ *s.t.* $\langle\Omega|\Omega\rangle = 1$ *and* $U(\widetilde{\Lambda}, a)\Omega = \Omega$.

Definition 7.3 *A Strocchi-Wightman QFT is given by:*

1. *A Strocchi-Wightman relativistic QM* $(\mathcal{H}, \mathcal{H}', U, \Omega)$.

2. *A family of operator-valued distributions* $(\phi_\ell^{(\kappa)}, D)$, $\kappa \in \mathbb{I}$, $\ell = 1, 2, \ldots r_\kappa$ *s.t.*

   - $\mathbb{I}$ *is some finite or infinite collection of indices numbering the types of the fields corresponding to finite-dimensional representations* $D^{(\kappa)}$ *of* $\widetilde{\mathcal{L}}_+^\uparrow = SL(2, \mathbb{C})$.
   - *For a fixed* $\kappa \in \mathbb{I}$ *the field* $\phi^{(\kappa)} = (\phi_\ell^{(\kappa)})_{\ell=1,\ldots r_\kappa}$ *transforms under* $D^{(\kappa)}$.
   - *For any* κ, ℓ *there exists some* $\overline{\kappa}, \overline{\ell}$ *s.t.* $\phi_\ell^{(\kappa)}(f)^\dagger = \phi_{\bar{\ell}}^{(\bar{\kappa})}(\bar{f})$.
   - $\Omega \in D$ *and* $U(\widetilde{\Lambda}, a)D \subset D$ *for all* $(\widetilde{\Lambda}, a) \in \widetilde{\mathcal{P}}_+^\uparrow$.

   satisfying:

   (a) *(Locality) If supp* f_1 *and supp* f_2 *are spacelike separated, then*

   $$[\phi_\ell^{(\kappa)}(f_1), \phi_{\ell'}^{(\kappa')}(f_2)]_- = 0 \text{ or } [\phi_\ell^{(\kappa)}(f_1), \phi_{\ell'}^{(\kappa')}(f_2)]_+ = 0 \tag{118}$$

   in the sense of weak commutativity on D. *(Here* $-/+$ *refers to commutator/anti-commutator).*

   (b) *(Covariance) For all* $(\widetilde{\Lambda}, a) \in \widetilde{\mathcal{P}}_+^\uparrow$ *and* $f \in S$

   $$U(\widetilde{\Lambda}, a)\phi_\ell^{(\kappa)}(f)U(\widetilde{\Lambda}, a)^\dagger = \sum_{\ell'} D_{\ell\ell'}^{(\kappa)}(\widetilde{\Lambda}^{-1})\phi_{\ell'}^{\kappa}(f_{(\Lambda,a)}). \tag{119}$$

   Here $f_{(\Lambda,a)}(x) = f(\Lambda^{-1}(x-a))$.

   (c) *(Cyclicity of the vacuum)* $\mathcal{D} = \mathrm{Span}\{\, \phi_{\ell_1}^{(\kappa_1)}(f_1) \ldots \phi_{\ell_m}^{(\kappa_m)}(f_m)\Omega \,|\, f_1, \ldots f_m \in S, m \in \mathbb{N}_0 \,\}$ *is a dense subspace of* $\mathcal{H}$ *in the topology given by* $(\cdot|\cdot)$.

The distributions $(\phi_\ell^{(\kappa)}, D)$ *are called the Strocchi-Wightman quantum fields.*

7.3 Free field examples

7.3.1 Free Wightman fields

1. Suppose first that $\mathcal{H}$ is a Hilbert space w.r.t $\langle \cdot \,|\, \cdot \rangle$ (i.e., $\langle \cdot \,|\, \cdot \rangle$ is a positive-definite scalar product and we can choose $\mathcal{H}' = \mathcal{H}$). Then the above setting is called the Wightman framework for fields with arbitrary (finite) spin.

2. Let us stay for a moment in this Hilbert space framework. Recall that finite-dimensional irreducible representations $D^{(\kappa)}$ of $\widetilde{\mathcal{L}}_+^\uparrow$ are labelled by two numbers (A, B). From the physics lecture you know the following free field examples:

   - $(0,0)$: scalar field ϕ
   - $\left(\frac{1}{2}, \frac{1}{2}\right)$: massive vector field j^μ
   - $\left(\frac{1}{2}, 0\right) \oplus \left(0, \frac{1}{2}\right)$: Dirac field ψ
   - $(1,0) \oplus (0,1)$: Faraday tensor $F^{\mu\nu}$.

3. It is however not possible to construct a massless free vector field A^μ on a Hilbert space which is local and Poincaré covariant [25]. Such fields turn out to exist on Krein spaces, which is usually given as the main reason to introduce them.

7.3.2 Free massless vector field A^μ on Krein space

The Gupta-Bleuler electromagnetic potential has the form

$$A_\mu(x) = \int \frac{d^3k}{2k_0(2\pi)^3} \sum_{\lambda=0}^{3} [a^{(\lambda)}(k)\varepsilon_\mu^{(\lambda)}(k)e^{-ikx} + a^{(\lambda)\dagger}(k)\varepsilon_\mu^{(\lambda)*}(k)e^{ikx}], \tag{120}$$

where $k_0 = |k|$ and $\varepsilon_\mu^{(\lambda)}$ are polarisation vectors which satisfy the orthogonality and completeness relations

$$\varepsilon^{(\lambda),\mu}(k) \cdot \varepsilon_\mu^{(\lambda')*}(k) = g^{\lambda\lambda'}, \quad \sum_\lambda (g^{\lambda\lambda})^{-1} \varepsilon_\mu^{(\lambda)}(k) \cdot \varepsilon_\nu^{(\lambda)*}(k) = g_{\mu\nu}. \tag{121}$$

For the $a^\lambda, a^{(\lambda)\dagger}$ we have

$$[a^{(\lambda)}(k), a^{(\lambda')\dagger}(k')] = -g^{\lambda\lambda'} 2k^0 (2\pi)^3 \delta(\vec{k} - \vec{k}'). \tag{122}$$

Due to $-g^{00} = -1$ we have $\langle a^{(0)\dagger}(f)\Omega | a^{(0)\dagger}(f)\Omega \rangle < 0$ thus our 'Fock space' $\mathcal{H}$ turns out to be a Krein space. Furthermore, the 'photons' above have four polarisations and not two. These unphysical degrees of freedom will be eliminated by the Gupta-Bleuler subsidiary condition (127) below.

Let us point out another peculiarity of this potential: We can form $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ so that $\varepsilon^{\alpha\beta\mu\nu}\partial_\beta F_{\mu\nu}(x) = 0$ is automatic. But the remaining free Maxwell equations fail:

$$\partial_\mu F^{\mu\nu}(x) = -\partial^\nu(\partial_\rho A^\rho)(x) \neq 0. \tag{123}$$

This can be expected on general grounds:

Theorem 7.4 *(Strocchi [20, 21]) Any Strocchi-Wightman vector field A_μ with $\partial_\mu F^{\mu\nu}(x) = 0$ is trivial, i.e. $\langle\Omega|F^{\mu\nu}(x)F^{\alpha\beta}(y)\Omega\rangle = 0$.*

This highlights the necessity of a gauge-fixing term in the Lagrangian, which is another point to which we will come below in the context of interacting QED.

7.4 Quantum Electrodynamics

Definition 7.5 *QED is a Strocchi-Wightman QFT whose fields include $F^{\mu\nu}, j^\mu$ and some 'charged fields' $\phi^{(\kappa)}$ s.t. the physical subspace $\mathcal{H}'$ satisfies:*

(i) There is a dense domain $D' \subset \mathcal{H}'$ s.t. $F^{\mu\nu}(f)D' \subset D'$, $j^\mu(f)D' \subset D'$ and $U(a, \widetilde{\Lambda})D' \subset D'$.

(ii) For $\Psi_1 \in \mathcal{H}'$ and $\Psi_2 \in D'$

$$\langle\Psi_1|(\partial_\mu F^{\mu\nu} - j^\nu)(f)\Psi_2\rangle = 0, \quad \langle\Psi_1|(\varepsilon_{\mu\nu\rho\sigma}\partial^\nu F^{\rho\sigma})(f)\Psi_2\rangle = 0. \qquad (124)$$

For QED defined as above, one can define the electric charge operator by suitably regularizing $Q = \int d^3y\, j^0(0, \vec{y})$.

Theorem 7.6 *(Strocchi-Picasso-Ferrari [22]) Suppose that Q is an infinitesimal generator of the global U(1) symmetry, i.e. for some field ϕ*

$$\phi(x) = [Q, \phi(x)] \text{ on } \mathcal{H} \qquad (125)$$

and that $\langle\Psi_1|\phi(x)\Psi_2\rangle \neq 0$ for some $\Psi_1, \Psi_2 \in \mathcal{H}'$. Then

1. *There is $\Psi \in D'$ s.t. $(\partial_\mu F^{\mu\nu} - j^\nu)(x)\Psi \neq 0$.*
2. *There is $0 \neq \Psi \in \mathcal{H}'$ s.t. $\langle\Psi|\Psi\rangle = 0$, i.e. $\mathcal{H}'' \neq \{0\}$.*
3. *There is $\Psi \in \mathcal{H}$ s.t. $\langle\Psi|\Psi\rangle < 0$.*

The proof is simple and relies on a Stokes theorem computation analogous to (117). This theorem shows that the Maxwell equations can hold on $\mathcal{H}'$ at best in matrix elements and that the Krein space framework is needed in (local) QED also in the presence of interactions.

Definition 7.7 *We say that QED is in the Gupta-Bleuler gauge if it contains (in addition to other fields) a vector field A_μ s.t. $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and*

$$\partial_\mu F^{\mu\nu} - j^\nu = -\partial^\nu(\partial_\rho A^\rho) \qquad (126)$$

holds as an operator identity on $\mathcal{H}$. Furthermore, the physical subspace is chosen as

$$\mathcal{H}' := \{\Psi \in \mathcal{H} \,|\, (\partial_\rho A^\rho)^{(+)}(f)\Psi = 0 \text{ for all } f \in S\}, \qquad (127)$$

where $(\partial_\rho A^\rho)^{(+)}$ is the positive frequency part of $(\partial_\rho A^\rho)$.

We add several remarks on this definition:

1. Note that by applying ∂_ν to (126) and using current conservation we obtain $\Box(\partial_\rho A^\rho) = 0$, thus the decomposition into positive and negative frequency parts is meaningful. For this reason, (124) formally hold. But positivity of the scalar product on $\mathcal{H}'$ needs to be assumed. (Known only for the free electromagnetic field).

2. The equation (126) comes from a classical Lagrangian with the gauge-fixing term, e.g.

$$\mathcal{L}_{\mathrm{gf}} = (D^\mu\phi)^*(D_\mu\phi) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2. \tag{128}$$

   $\mathcal{L}_{\mathrm{gf}}$ is still invariant under 'residual' local gauge transformations s.t. $\Box\varepsilon(x) = 0$. Denote infinitesimal transformations of the fields as $\delta_\varepsilon\phi(x) = i\varepsilon(x)\phi(x)$, $\delta_\varepsilon\phi(x)^* = -i\varepsilon(x)\phi(x)^*$ and $\delta_\varepsilon A_\mu(x) = \partial_\mu\varepsilon(x)$.

3. Def. Let $\mathcal{A}$ denote the algebra spanned by polynomials of quantum fields A^μ, j^μ, ϕ, ϕ^* smeared with smooth, compactly supported functions. We extend δ_ε to $\mathcal{A}$ via the Leibniz rule and denote the subalgebra of (residual) gauge-invariant elements by $\mathcal{A}_{\mathrm{gi}}$.

4. It turns out, that a 'local vector' (i.e. $\Psi = A\Omega$, where $A \in \mathcal{A}$) belongs to $\mathcal{H}'$ iff $A \in \mathcal{A}_{\mathrm{gi}}$ [17].

The material below was not covered in the lecture, but we leave it here for interested readers.

7.5 Electrically charged states of QED

Important problem in QED is a construction of physical electrically charged states. Vectors of the form $\phi(f)\Omega$, where ϕ is a charged field, are not in $\mathcal{H}'$, because ϕ is not invariant under residual gauge transformations. Moreover:

Proposition 7.8 *[17] For any local vectors $\Psi, \Phi \in \mathcal{H}'$ in QED we have $\langle\Psi|Q\Phi\rangle = 0$.*

We face the problem of constructing a field ϕ_C which is invariant under (residual) local gauge transformations (so that $\phi_C(f)\Omega$ is 'close' to $\mathcal{H}'$) and non-invariant under global gauge transformations (so that $\phi_C(f)\Omega$ is charged). Here is a candidate:

$$\phi_C(x) := e^{i[(\Delta)^{-1}\partial_i A^i](x)}\phi(x) = e^{i[(\Delta)^{-1}\partial_i(A^i+\partial^i\varepsilon)](x)}e^{i\varepsilon(x)}\phi(x), \tag{129}$$

where the last equality holds for local gauge transformations but fails for global (i.e. $\varepsilon(x)$=const). ϕ_C is simply the (non-local) charged field in the Coulomb gauge expressed in terms of the Gupta-Bleuler fields. Indeed, we can see (129) as a gauge transformation. Then the corresponding transformation applied to the potential gives

$$A_{\mu,C}(x) = A_{\mu}(x) + \partial_{\mu}[(\Delta)^{-1}\partial_i A^i](x). \tag{130}$$

which satisfies $\vec{\nabla} \cdot \vec{A}_C = 0$. Then $\phi_C(f)\Omega$ is a candidate for an electrically charged states. Since $\phi_C(f)$ is a very singular objects, this vector 'escapes' from $\mathcal{H}$ and its control (in the perturbative or axiomatic setting) requires subtle mathematical methods (cf. [17, 18]). These are outside of the scope of these lectures.

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