Exact Renormalization Group Equation, and Asymptotic Safety

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Contents

2 Asymptotic Safety (AS), and the critiques to this scenario in quantum gravity

In the second part of this talk, we will use the beta functions to discuss the proposal of Asymptotic Safety in gravity, as well as the criticisms to such scenario. This discussion will be based in [1, 2, 3, 4, 5, 6, 7, 8, 18].

2.1 Gravitational fixed point in the Einstein-Hilbert truncation

Let us consider the complete result in the Einstein-Hilbert truncation including the Cosmological constant (54), (55). For the RG flow, let us consider the dimensionless couplings,

$$\tilde{\Lambda} = k^{-2}\,\Lambda \qquad\qquad \tilde{G} = k^{d-2}\,G,.$$

and rewrite the beta functions (54), (55) as follows: let us start rewriting (55), as,

$$-\partial_t\left(\frac{1}{16\pi G}\right) = \frac{k^{d-2}}{16\pi}(B_1 + B_2\eta_N) \tag{56}$$

$$-k\partial_k\left(\frac{k^{d-2}}{\tilde{G}}\right) = k^{d-2}(B_1 + B_2\eta_N)$$

$$\partial_t\tilde{G} = (d-2)\tilde{G} + (B_1 + B_2\eta_N)\tilde{G}^2\,. \tag{57}$$

where B_1, B_2 are functions of $\tilde{\Lambda}$ and d, given in the appendix. On the other hand, rewriting (54),

$$\partial_t\left(\frac{2\Lambda}{16\pi G}\right) = \frac{k^d}{16\pi}(A_1 + A_2\eta_N) \tag{58}$$

$$k\partial_k\left(\frac{k^d\tilde{\Lambda}}{\tilde{G}}\right) = k^d(A_1 + A_2\eta_N)\partial_t\tilde{\Lambda} = -d\,\tilde{\Lambda}\frac{1}{2}\tilde{G}(A_1 + A_2\eta_N) + \frac{\tilde{\Lambda}}{\tilde{G}}\partial_t\tilde{G}$$

plugging-in (57) on the right hand side,

$$\partial_t\tilde{\Lambda} = -2\tilde{\Lambda} + \frac{1}{2}(A_1 + A_2\eta_N)\tilde{G} + (B_1 + B_2\eta_N)\tilde{G}\tilde{\Lambda} \tag{59}$$

where A_1, A_2 are functions of $\tilde{\Lambda}$ and d, given in the appendix.

2.1.1 Approximations: the "Simple Einstein-Hilbert flow equations"

Equations (57) and (59) are a system of ordinary, coupled differential equations for $\tilde{\Lambda}$, $\tilde{G}$. However, let us recall the anomalous dimension, which enters in the right hand side of these equations,

$$\eta_N = -\partial_t \log Z_N = -\partial_t \log\left(\frac{1}{16\pi G(k)}\right)\,,$$

which we have not written in terms of $\tilde{G}$. The reason is that for this discussion we will take another approximation,

- ignoring the running of the couplings on the right hand side of (57), (59). In such a case η_N is set to zero on the right hand side of (57), (59),

$$\partial_t\tilde{G} = (d-2)\tilde{G} + B_1\tilde{G}^2\,. \tag{60}$$

$$\partial_t\tilde{\Lambda} = -2\tilde{\Lambda} + \frac{1}{2}A_1\tilde{G} + B_1\tilde{G}\tilde{\Lambda} \tag{61}$$

  This can be justified only by numerical solutions of the complete flow equations, which show that these approximations still capture at least qualitatively the fixed point.

- We can take yet another approximation to solve this complicated system of coupled differential equations. Namely, let us consider the field equation for the background metric, which gives,

$$\bar{R} = \frac{2d}{d-2}\Lambda\,.$$

In other words, the last equation could be understood as stating that Λ is of the order $\mathcal{O}(\bar{R}^1)$ **on-shell** [1]. Since we have kept only $\mathcal{O}(\bar{R}^1)$ in the Einstein-Hilbert truncation (27), it is plausible to keep only the leading order terms in Λ. To do so, we set Λ to zero in the coefficients in the $\mathcal{O}(\bar{R}^1, \Lambda^1)$ expansion, in the left hand side of (53), which amounts to set Λ to zero in the terms A_i, B_i $(i = 1, 2)$ in (58), (56). Hence, from (60), (61) we obtain,

the "simple Einstein-Hilbert flow equations"

$$\partial_t\tilde{G} = (d - 2 + b_1\tilde{G})\,\tilde{G}\,, \tag{62}$$

$$\partial_t\tilde{\Lambda} = -2\tilde{\Lambda} + \frac{1}{2}a_1\tilde{G} + b_1\tilde{G}\tilde{\Lambda} \tag{63}$$

where, $a_1 = A_1(\Lambda = 0)$, $b_1 = B_1(\Lambda = 0)$ are pure constants depending only on d. Below, we will comment on some of the most evident failures in this approximation, which are, however, not essential to analyze the fixed point.

2.1.2 The fixed points

Let us first consider (62). The zeros of $\partial_t\tilde{G}$ are,

- $$\tilde{G} = 0\,,$$

  plugging it in (63), the zero of $\partial_t\tilde{\Lambda}$ is given by,

  $$\tilde{\Lambda} = 0$$

- $$\tilde{G} = -\frac{d-2}{b_1}\,,$$

  plugging it in (63), the zero of $\partial_t\tilde{\Lambda}$ is given by,

  $$\tilde{\Lambda} = -\frac{d-2}{2d}\frac{a_1}{b_1}\,.$$

All in all, within all the approximations we find a trivial, and a non-trivial fixed point, respectively,

$$\left(\tilde{G}_* = 0,\, \tilde{\Lambda}_* = 0\right) \qquad \left(\tilde{G}_* = -\frac{d-2}{b_1},\, \tilde{\Lambda}_* = -\frac{d-2}{2d}\frac{a_1}{b_1}\right) \tag{64}$$

From the expressions in the appendix, in $d = 4$, $a_1 = 1/\pi$, and $b_1 = -23/3\pi$,

$$\left(\tilde{G}_* = 0,\, \tilde{\Lambda}_* = 0\right) \qquad \left(\tilde{G}_* = \frac{6\pi}{23} \approx 0.82,\, \tilde{\Lambda}_* = \frac{3}{92} \approx 0.03\right)$$

such that the Newton's constant remains positive at the FP. The physical viability of the FP is further discussed in the Appendix 3.3 from a simpler approach.

2.1.3 The stability of the fixed points

We would like to examine the stability of these fixed points (FP), or in other words, their basins of attraction: let us define the UV-critical surface of a FP as the set of all points that flow to the FP as $t \to \infty$ (let us recall $t = \log(k/\Lambda_{UV})$). At least analitically we can extract some information by analyzing the linearized flow, namely the tangent space at the FP. For that, we will consider the stability matrix,

$$\left.\frac{\partial \beta_i}{\partial \tilde{g}_j}\right|_* , \tag{65}$$

Now, for the beta functions $\beta_{\tilde{G}}$ (62) and $\beta_{\tilde{\Lambda}}$ (63), we obtain the following matrix,

$$\left(\frac{\partial \beta}{\partial \tilde{g}}\right) = \begin{pmatrix} -2 + b_1 \tilde{G} & \frac{a_1}{2} + b_1 \tilde{\Lambda} \\ 0 & d - 2 + 2b_1 \tilde{G} \end{pmatrix}$$

Now, we must evaluate the components at the FPs (64) to obtain the corresponding eigenvalues and hence, the stability properties:

- Plugging-in the values of the nontrivial fixed point $\left(\tilde{G}_* = -\frac{d-2}{b_1},\, \tilde{\Lambda}_* = -\frac{d-2}{2d}\frac{a_1}{b_1}\right)$, the stability matrix is,

$$\left(\frac{\partial \beta}{\partial \tilde{g}}\right) = \begin{pmatrix} -d & \frac{a_1}{d} \\ 0 & -(d-2) \end{pmatrix}$$

  hence, for $d = 4$ the two eigenvalues $-d$, $-(d-2)$ are negative, which means that both directions are attracted by the FP as $t \to \infty$ (UV attractive) (Note the review on the stability matrix in the grey box, below). This is clearly seen in the figure 1.

- Plugging-in the values of the trivial fixed point $\left(\tilde{G}_* = 0,\, \tilde{\Lambda}_* = 0\right)$, the stability matrix is,

$$\left(\frac{\partial \beta}{\partial \tilde{g}}\right) = \begin{pmatrix} -2 & \frac{a_1}{2} \\ 0 & (d-2) \end{pmatrix}$$

  hence, for $d = 4$ the eigenvalues -2, $(d-2)$ are negative and positive respectively, which means that only one of the directions is attracted by the FP as $t \to \infty$, while the other is repelled. This is clearly seen in the figure 1.

Let us stress that the difference of eigenvalues for the stability matrix at the two different FPs arises as follows: we have computed the beta functions for the couplings $\tilde{G}$, $\tilde{\Lambda}$. However, we started with $Z_N \propto \tilde{G}^{-1}$ which is singular at the trivial FP $\tilde{G} = 0$, hence, the coordinate transformation is singular.

Let us also note that we can solve the simple system (62), (63) analitically. However, the explicit solutions $\tilde{\Lambda}(t)$, $\tilde{G}(t)$ are not much more illuminating than the previous analysis and their plot, which is given for $d = 4$ in figure 1.

Additional material: the stability matrix. Attractive and Repelling directions [1]

Consider coordinates centered about a fixed point $\tilde{g}_*$, in order to study deformations about it,

$$y_j = \tilde{g}_j - \tilde{g}_{j*} ,$$

and the flow equations $\partial_t y_i$. Let us expand these flow equations as,

$$\partial_t y_i = \frac{1}{n!} \sum_{n=0}^{\infty} \sum_j \left.\frac{\partial^n \partial_t y_i}{\partial \tilde{g}_j^n}\right|_* y_j^n \tag{66}$$

The linearized approximation amounts to assume y_j are small for every j, hence,

$$\partial_t y_i = \sum_j \partial_t y_i|_* + \sum_j \left.\frac{\partial \partial_t y_i}{\partial \tilde{g}_j}\right|_* y_j + \ldots$$

$$\partial_t y_i = \sum_j \left.\frac{\partial \partial_t \tilde{g}_i}{\partial \tilde{g}_j}\right|_* y_j - \sum_j \left.\frac{\partial}{\partial \tilde{g}_j} \partial_t \tilde{g}_{i*}\right|_* y_j + \ldots$$

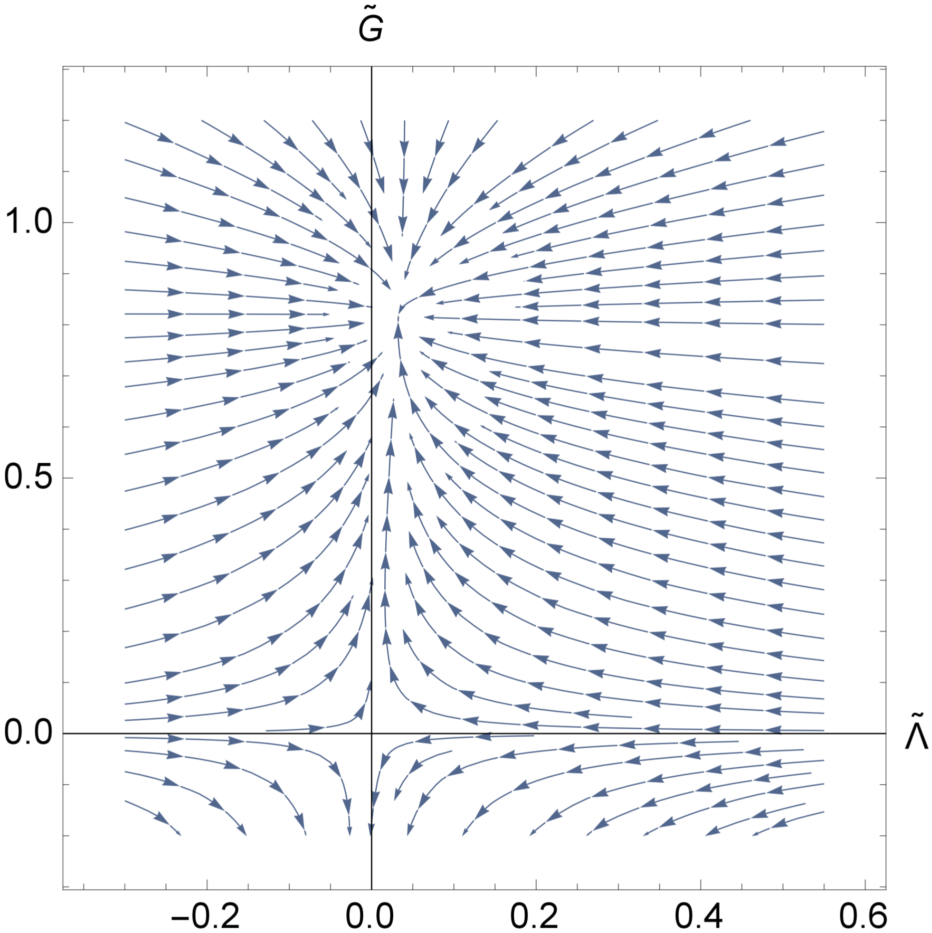

Figure 1: Plot of the Simple Einstein-Hilbert flow equations (62), (63) in $d = 4$. Figure taken from [1] chapter 7.

$$\partial_t y_i = \sum_j \left.\frac{\partial \beta_i}{\partial \tilde{g}_j}\right|_* y_j + \ldots \tag{67}$$

where we have used the definition of fixed point $\partial_t \tilde{g}_{i*} = 0$ multiple times, and introduced the beta functions $\beta_i = \partial_t \tilde{g}_i$. Diagonalizing the stability matrix,

$$\left(D^{-1} \left.\frac{\partial \beta_i}{\partial \tilde{g}_j}\right|_* D \right)_{mn} = \delta_{mn}\lambda_n \qquad\qquad s_i = \left(D^{-1} y\right)_i$$

with λ_i the eigenvalues of the stability matrix, we can write the linearized equation and the solution as,

$$\partial_t s_i = \lambda_i s_i \qquad\qquad s_i \propto e^{\lambda_i t}$$

It is then clear that the directions s_i that correspond to

- negative λ_i are attracted by the FP as $t \to \infty$ (UV attractive and IR repulsive).
- positive λ_i are repelled by the FP as $t \to \infty$ (UV repulsive and IR attractive).

2.2 A comment on the approximations, other truncations, and usefulness of the ERGE in gravity

We have assentially taken two approximations to obtain the "Simple E-H flow equations". Namely, we set η_N, and $\tilde{\Lambda}$ to zero on the right hand side of (54), (55). In particular, the latter approximation does not capture an important feature of the Einstein-Hilbert truncation. Namely, there is a singularity at

$$\tilde{\Lambda} = \frac{1}{2}\,,$$

which does not arise in the simple flow equations (62), (63). This feature, is, however, not necessarily a property of the flow but may be an artifact of all the other approximations. On the bright side, these approximations allowed us to study analytically the fixed points. The latter are only slightly modified in more complete studies. In particular, the fixed points continue to appear on more elaborate truncations including higher order terms, such as $f(R)$ (For instance in [19] the truncation already contained 35 couplings, also finding evidence for the fixed points), couplings to matter, as well as other types of approximations. An up to date review is given in [8] (See also [4, 6]).

Finally, let us note that the computation of the gravitational beta functions, and the evidence for the fixed points in $d = 4$ was done for the first time by Reuter, who applied the ERGE to this problem [3]. Before the application of ERGE to gravity, the evidence for nontrivial fixed points in gravity was restricted to $d = 2$.

2.3 The critiques to the AS scenario in quantum gravity

Below, we discuss some of the criticisms to the Asymptotic Safety (AS) scenario in quantum gravity. This section is amply based on the references [5, 8, 18].

2.3.1 The meaning of $G(k)$: is it really Newton's constant at the scale k?

Let us recall that k is the scale parameterizing the momentum-shell in the Wetterich flow equation for the averaged effective action Γ_k (1). In particular, we have understood the coupling dependent on the scale k, $G(k)$, as the coupling constant that includes the quantum fluctuations above the scale k integrated out. Hence, as we take the $k \to 0$ all quantum fluctuations are integrated out, and the corresponding G is the same that appears in the full effective action Γ.

Because of this k dependance, it is common practice in the AS literature to refer to these couplings as running couplings. In particular, one is led to call $G(k)$ as Newton's constant at the scale k. Nevertheless, in [5, 18], it is argued that $G(k)$ is not a running coupling in the usual, physically meaningful sense, and hence, it should not be understood as Newton's constant at the scale k. Instead, it is proposed that a more appropriate name for $G(k)$ would be "incomplete coupling constant" in the sense described above: namely, it does not include all quantum corrections integrated out.

The argument in [5, 18] can be summarized in the following way: "*To be useful, a running coupling must capture at least a significant common portion of the quantum corrections* " (Excerpt from [18]). The problem for couplings such as Newton's constant is that it is present in a non-renormalizable theory, where we find power-law divergences instead of logaritmic divergences. There is process dependance or *non-universality.* In other words, in such a non-renormalizable theory quantum corrections do not organize themselves into a running coupling. Hence, there is no reason to expect that $G(k)$ captures the effect of quantum corrections on Newton's constant [18].

As is pointed out in [5] the common interpretation in AS of $G(k)$ as Newton's constant at the scale k comes from extrapolating the many successes of FRG in condensed matter systems even with power-law scaling, and more fundamentally, from the physically clear picture of coarse-graining as a way to define coupling constants at different scales. However, it is also argued that the kinematic variety in Minkowski reactions is much more involved than in 3D condensed matter systems, hence, the extrapolation may be more complicated and find some obstructions.

On the other hand, also in [5] it is argued that a way to circumvent the problem of quantum effects not organizing themselves in running Λ and G may be just be including higher order operators in the truncation. However, this brings us to the next problem that AS may have.

2.3.2 The ghosts and tachyons in the higher derivative truncation

In this talk we have restricted to the Einstein-Hilbert truncation, however, the operator expansion for the averaged effective action Γ_k contains higher order operators, with higher order time derivatives.

It is well known that non-degenerate higher time derivative theories propagate ghosts (at least those with a finite number of derivatives). The problem can be tracked back to the fundamental Ostrogradskian instability which implies an unbounded energy from below even in the classical theory [20]. Besides, it can be seen in a propagator that scales as p^{-4} hence violating the unitarity constraints from the KL spectral representation [5].

Hence, even before trying to solve the above mentioned problems by extending the truncation, one should be able to learn how to deal with ghosts and tachyons arising in the truncation, or whether there is a mechanism controlling them (See also [8]).

2.3.3 The need for Lorentzian flow

Most of the AS literature computes the flow equation in the Euclidean setting [8]. This makes it straightforward to define the flow. Namely, first integrating out fluctuations with a large squared momentum $> k^2$, and proceeding towards lower momenta. However, the criticism is that for instance, a large Euclidean momentum can translate into a massless on-shell Lorentzian particle $k_0^2 - \vec{k}^2 = 0$.

On the other hand, let us cite, "*there are solid arguments to expect that the effective actions obtained from integrating out fluctuations in a Lorentzian and Euclidean signature setting will be different. The space of metrics of the two settings comes with different topological properties: while all Euclidean metrics can be connected by geodesics this property does not hold in the Lorentzian case.*" (Excerpt from [8].)

Furthermore, it is noted in [5] that this issue is made even more difficult with the higher derivative operators introducing ghosts.

2.3.4 The dependance on the regulator and the truncation

Let us recall that there is a free choice in the regulator, cutoff scheme. Thus, the precise details of for instance, the beta functions, depend on the latter. However, it has become clear that this dependance is only very mild [5, 6, 8].

We find the same situation with the truncations. Evidence has been gathered on the non-trivial fixed points from the gravitational beta functions despite the different truncations, and matter content. On this regard, please note the discussion in section 2.2.

3 Appendix

3.1 Some coefficients in the flow equations

The following terms arise when rewriting the flow equations (54), (55) in the convenient form (56), (58):

$$A_1 = \frac{16\pi(d - 3 + 8\tilde{\Lambda})}{(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)(1 - 2\tilde{\Lambda})}$$

$$A_2 = -\frac{16\pi(d + 1)}{(4\pi)^{\frac{d}{2}}(d + 2)\Gamma\left(\frac{d}{2}\right)(1 - 2\tilde{\Lambda})}$$

$$B_1 = -\frac{4\pi(5d^2 - 3d + 24 - 8(d + 6)\tilde{\Lambda})}{3(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)(1 - 2\tilde{\Lambda})}$$

$$B_2 = \frac{4\pi(5d - 7)}{3(4\pi)^{\frac{d}{2}}\Gamma\left(\frac{d}{2}\right)(1 - 2\tilde{\Lambda})}$$

3.2 Dimensionless coordinates in theory space

The ERGE incorporates the idea of Renormalization group transformations which include first, a rescaling of momenta and lengths, and second, a canonical normalization of the kinetic term after integration over a momentum shell [1]. The first is simply dealt with, by using dimensionless coordinates, as we show below. The second is dealt with by regarding the field renormalization constant as dependant on the IR cutoff,

$$Z_\phi(k)\,,$$

hence regarding it as another running coupling, keeping track of its beta function, or *anomalous dimension*,

$$\eta_\phi = -\frac{d}{dt}\log Z_\phi(k)\,, \tag{68}$$

while assuming the fields as $k-$independent.

If the coupling g_i has mass dimension d_i, we define the coupling measured in units of the IR cutoff, dimensionless parameter,

$$\tilde{g}_i = k^{-d_i} g_i\,.$$

We take $\{\tilde{g}_i\}$ as coordinates of theory space. Even though we do not need to keep track of rescaling of these coordinates, the Beta functions for $\{\tilde{g}_i\}$ are different from those of the dimensionful couplings $\{g_i\}$. Let us write the Beta functions for the dimensionless couplings in IR cutoff units,

$$\begin{aligned}
\tilde{\beta}_i(\tilde{g}_i) &= \tilde{\beta}_i(\tilde{g}_j, 1)\\
&= \frac{d\tilde{g}_i}{dt} = \frac{d}{dt}k^{-d_i}g_i(k)\\
&= \frac{dk}{dt}\frac{dk^{-d_i}}{dk}g_i(k) + k^{-d_i}\beta_i(g_j, k)\\
&= -d_i\,\Lambda e^t k^{-d_i-1}g_i + k^{-d_i}\beta_i(g_j,k)\\
&= -d_i\,k^{-d_i}g_i + k^{-d_i}\beta_i(g_j,k) = -d_i\,\tilde{g}_i + k^{-d_i}\beta_i(g_j,k)\\
\tilde{\beta}_i(\tilde{g}_i) &= -d_i\,\tilde{g}_i + \alpha_i(\tilde{g}_j)\,,
\end{aligned} \tag{69}$$

where we have defined,

$$\alpha_i(\tilde{g}_j) = k^{-d_i}\beta_i(g_j,k) = \beta_i(\tilde{g}_j, 1)\,,$$

which contains the contributions from quantum fluctuations, and depends only implicitly on k or t via its arguments, while the first term on the right hand side of (69) is only due to the classical scaling.

Additional material:

3.3 Is it reasonable to expect a fixed point for Newton's constant? (without Cosmological constant)

We will also follow the dimensional arguments given in [1] to show the case for a fixed point in the simplest truncation. In this discussion we will only use a part of the results in section 1.

Let us set the cosmological constant to zero in the Einstein-Hilbert truncation (27), $\Lambda \to 0$, such that the effective action takes the following form in the operator expansion,

$$-\frac{1}{16\pi G}\mathcal{O}_2\,,$$

where,

$$\mathcal{O}_2 = \int d^dx\sqrt{g}R\,.$$

The mass dimension of the operator $\mathcal{O}_2$ is $2-d$. Hence, evaluating the effective action in a background metric, with an UV [a] cutoff k, we expect a power divergence of the form,

$$k^{d-2}\mathcal{O}_2\,.$$

On the other hand, in perturbation theory the beta function is of the form [1],

$$k\partial_k\left(-\frac{1}{16\pi G(k)}\right)\propto k^{d-2}\,.$$

Choosing $B/16\pi$ as a proportionality constant, the beta function takes the form,

$$\partial_t G = BG^2k^{d-2}\,, \tag{70}$$

Now, let us recall that the RG flow has to be written for the dimensionless couplings measured in units of the cutoff k,

$$\tilde{G} = Gk^{d-2}\,,$$

such that we obtain a beta function of the form (69),

$$\partial_t\tilde{G} = (d-2)\tilde{G} + k^{d-2}\partial_t G = (d-2)\tilde{G} + B(Gk^{(d-2)})^2$$

$$\partial_t\tilde{G} = (d-2+B\tilde{G})\tilde{G}\,, \tag{71}$$

where, in the first line, we have used the beta function in the form (70).

Let us note that (71) indicates a trivial, and a non-trivial fixed point, respectively,

$$\tilde{G} = 0 \tag{72}$$

$$\tilde{G} = -\frac{d-2}{B}\,. \tag{73}$$

Physical feasibility of the non-trivial fixed point for Newton's constant (within the simplest truncation)

Let us note that the beta function (71) does not allow[b] $\tilde{G}$ to change signs. Since Newton's constant is positive at low energies, then

$$\tilde{G} > 0\,,$$

must hold at all energies. So, if $d = 4$ then, the non-trivial fixed point (73) is physically viable in this simplest truncation if,

$$B < 0\,.$$

Now, let us evaluate B in this argument using the computation of the previous sections with the ERGE. From (55) we recognize that the right hand side is not constant. Thus, let us ignore the running on the right hand side of (55), which automatically vanishes the terms containing the anomalous dimension η_N. Thus, we identify from (55),

$$B|_{\Lambda=0,\,\eta_N=0} = -\frac{16\pi}{(4\pi)^{\frac{d}{2}}12\Gamma\left(\frac{d}{2}\right)}\left(4(d+6)-d(7-5d)\right)\,,$$

and in $d = 4$,

$$B|_{\Lambda=0\,,\eta_N=0,\,d=4} = -\frac{23}{3\pi}\,.$$

Therefore, the fixed point discussed above seems physically viable.

Note, however, that the result (55) depends on the specific Litim regulator and the cutoff scheme. It is instructive to verify this critical sign in a regulator independent way, although still restricted to this particular cutoff scheme (28), (29). Namely, in (45) consider the second line, which is the only contribution in this truncation without the cosmological constant (namely, the only term proportional to $\bar{R}$),

$$+\frac{1}{(4\pi)^{\frac{d}{2}}}\left[\frac{d(7-5d)}{24}Q_{\frac{d}{2}-1}\left(W_2\right)-\frac{d+6}{6}\,Q_{\frac{d}{2}-1}\left(W_{(gh)}\right)\right]\,.$$

As before, setting Λ and η_N to zero, the Q-functionals are the same and factorize in the last line. In fact, using (52), they are,

$$Q_{\frac{d}{2}-1}\left(W_2\right)\Big|_{\Lambda=0,\,\eta_N=0} = Q_{\frac{d}{2}-1}\left(W_{(gh)}\right) = Q_{\frac{d}{2}-1}\left(\frac{\partial_t R_k(\Delta)}{\Delta - R_k(\Delta)}\right) = \frac{4k^{d-2}}{(d-2)\Gamma\left(\frac{d}{2}-1\right)}\,,$$

such that for $d > 2$ the relevant sign is determined by

$$\frac{d(7-5d)}{24} - \frac{d+6}{6} = (3-5d)\frac{d}{24} - 1\,,$$

which is negative for $d > 0$, hence $B < 0$ independent of the regulator, and **the fixed point discussed above seems physically viable.**

[a]Let us recall that above we considered k as an IR cutoff. See the section 2.3 regarding the critiques on ERGE, and the AS scenario for more comments on the meaning of the scale k.

[b]To recognize this, note the IR and UV stability of the two fixed points, and the fact that one of them is precisely $\tilde{G} = 0$.

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