Technische Universität München **Theoretische Physik T70**
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Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 28.01.16, 12:00 next to PH 3218.

Exercise Sheet No. 12 To be discussed from 01.02. - 05.02.16.

Problem 1:
Hollow shell potential **10 Points**

Consider scattering of a particle of mass m represented by a plane wave off the potential

$$V(r) = \gamma^{-1}\delta(r-a)$$

with $r \equiv |\mathbf{x}|$. Scattring off spacially localized potentials is dominated by the low multipoles l. Let us focus on s-wave scattering.

(a) Make ansätze and identify the boundary conditions for inside and outside the shell for the radial equation.

(b) Determine the scattering amplitude and scattering phase.

If one considers the regions inside and outside the shell as two different physical systems, then large γ^{-1} (or small γ) corresponds to weak coupling between these systems.[1] In the limit $\gamma^{-1} \to \infty$ one would expect that the inner part behaves like a potential well, with a tower of stable bound states inside. Based on physical intuition, there should be quasistable states (lifetime much larger than inverse energy) for large finite γ^{-1}. Based on what you learned in the lecture, such states should apear as resonances in the scattering cross section.

(c) Calculate the scattering cross section in this limit.

[1] This may initially be surprising, as usually weak coupling means that the coefficient of the perturbation is small. That is the case if we have one soluble system that is exposed to a small perturbation. In the example here, the limit $\gamma^{-1} \to \infty$ splits the whole system into two soluble subsystems *inside* and *outside* that do not interact with each other. For finite γ^{-1} the wave function can penetrate the shell and the inside and outside parts do interact. Hence, large γ^{-1} corresponds to weak coupling between the two subsystems.