Technische Universität München **Theoretische Physik T70**
Physik Department **B. Garbrecht**
M. Drewes, D. Gueter, J. Klarić

Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 21.01.16, 12:00 next to PH 3218.

Exercise Sheet No. 11 To be discussed from 25.01. - 30.01.16.

Problem 1:
Current density **3 Points**

Calculate the current density of the wave function $\phi(\mathbf{r})$, which is the sum of the incoming plane wave $\phi_0(\mathbf{r})$, and the scattered wave $\phi_s(\mathbf{r})$:

$$\begin{aligned} \phi(\mathbf{r}) &= \phi_0(\mathbf{r}) + \phi_s(\mathbf{r}) \\ \phi_0(\mathbf{r}) &= \mathrm{e}^{\mathrm{i}kr\cos\vartheta} \\ \phi_s(\mathbf{r}) &= f(\vartheta)\frac{\mathrm{e}^{\mathrm{i}kr}}{r} \end{aligned}$$

Which interference terms appear?
Hint:
Neglect terms proportional to r^{-n} for $n \geq 3$.

Problem 2:

Asymptotic solution of the Scattering problem **3 Points**

Show that the asymptotic solution of the scattering problem,

$$\phi(\mathbf{r}) = \mathrm{e}^{\mathrm{i}kr\cos\vartheta} + f(\vartheta)\frac{\mathrm{e}^{\mathrm{i}kr}}{r}\,; \quad \left(k^2 = \frac{2mE}{\hbar^2}\right),$$

satisfies the Schrödinger equation, if the scattering potential falls off faster than $1/r$.

Problem 3:

Scattering on a central potential **4 Points**

Determine the scattering phases $\delta_l(k)$, as well as the scattering amplitude $f(\vartheta)$, for an elastic scattering on a central potnential:

$$V(\mathbf{r}) = V(r) = \frac{c}{r^2}\,; \quad c > 0\,,$$

with the simplification $c \ll \hbar^2/(2m)$.
Hints:
Use the sum:

$$\sum_{l=0}^{\infty} P_l(\vartheta) = \frac{1}{2\sin(\vartheta/2)}\,,$$

and the ansatz:

$$\phi(\mathbf{r}) = \sum_{l=0}^{\infty} R_l(r) P_l(\cos\vartheta)$$

The Laplace operator in spherical coordinates is given by:

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} + \frac{\mathbf{L}^2}{\hbar^2 r^2} \,.$$

The asymptotic behaviour of the radial part of the wave function is:

$$R_l(\rho) \to \frac{1}{\rho} \sin\left(\rho - \frac{l\pi}{2} + \delta_l\right) \mathrm{e}^{\mathrm{i}\delta_l} \,,$$

where $\rho = kr$.
Under what condition does the radial Schrödinger equation become Bessel's differential equation?
The asymptotic form of the Bessel functions is given by:

$$j_\lambda(\rho) \to \frac{1}{\rho} \sin\left(\rho - \frac{\lambda\pi}{2}\right)$$