**Technische Universität München** **Theoretische Physik T70**
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# Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 14.01.16, 12:00 next to PH 3218.

**Exercise Sheet No. 10** To be discussed from 18.01. - 22.01.16.

## Problem 1: Elastic scattering of fast electrons by atoms in the first Born approximation — 10 Points

(a) Transform the scattering amplitude

$$f^{(1)}(\theta) = -\frac{m}{2\pi\hbar^2}\int \mathrm{d}^3\mathbf{r}\, \mathrm{e}^{\mathrm{i}q|\mathbf{r}|\cos\theta} V(\mathbf{r})\,, \tag{1}$$

so that it explicitly depends on $\rho(\mathbf{r})$, by making use of:

$$\mathrm{e}^{\mathrm{i}q|\mathbf{r}|\cos\theta} = -\frac{1}{q^2}\nabla^2 \mathrm{e}^{\mathrm{i}q|\mathbf{r}|\cos\theta}\,, \tag{2}$$

and the Poisson's equation:

$$\nabla^2 V(\mathbf{r}) = e^2\left[Z\delta^3(\mathbf{r}) - \rho(\mathbf{r})\right]\,, \tag{3}$$

with the electron charge density $\rho(\mathbf{r})$.

(b) Express the scattering amplitude in terms of the electron charge form factor:

$$F(q) = \int \mathrm{e}^{\mathrm{i}q|\mathbf{r}|\cos(\theta)}\rho(\mathbf{r})\,\mathrm{d}^3\mathbf{r} \tag{4}$$

(c) Assuming that the radius $R$, below which $\rho(\mathbf{r})$ is significantly bigger than zero satisfies $Rq \ll 1$, expand the exponential in Eq. (4), to third order. For a spherically symmetric charge distribution $\rho(r)$, what is the physical interpretation of the first three terms? Write down the corresponding cross-section. What happened to the singularity that appeared in the Rutherford scattering for $\theta = 0$?

(d) Calculate the form factor $F(q)$ explicitly for the Hydrogen atom in its ground state,

$$\Psi_{1s}(r,\theta,\phi) = \frac{1}{\sqrt{\pi a_0^3}}\mathrm{e}^{-r/a_0}\,, \tag{5}$$

as well as the corresponding differential cross-section.