Technische Universität München **Theoretische Physik T70**
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Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 17.12.15, 12:00 next to PH 3218.

Exercise Sheet No. 09 To be discussed from 11.01. - 15.01.16.

Problem 1: Hard Sphere Scattering — 4 Points

Imagine a classical point-like particle elastically scattering off a stationary hard sphere with finite radius R and scattering angle θ. The distance between the trajectory of the incoming particle and the center of the stationary hard sphere is b, the so-called impact parameter. Assume the hard sphere to be that massive that its position does not change during the scattering process.

(a) Express the impact parameter b in terms of the radius R and the scattering angle θ.

(b) Give $\mathrm{d}\sigma/\mathrm{d}\Omega$ as a function of the impact parameter b and the angle θ.

(c) Find the total scattering cross section for a hard sphere:

$$\sigma = \int d\Omega \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \,. \tag{1}$$

Problem 2: Rutherford Scattering — 6 Points

Rutherford scattering describes elastic scattering of charged particles by the Coulomb interaction. One example is the classical Rutherford scattering of α-particles against gold nuclei. In the following you are supposed to determine the **classical** differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ for Rutherford scattering under the assumption of point-like particles with Ω being the solid angle.
Assume that the incoming particles enter the interaction region within a ring of section $(b, b + \Delta b)$ before they scatter into the angle interval $(\theta, \theta + \Delta\theta)$.

(a) For the Coulomb force being a central force, express $\mathrm{d}\Omega$ in terms of $\mathrm{d}\theta$.

The whole scattering can be described by the energy E of the incoming particle and its angular momentum $l = |\mathbf{r} \times \mathbf{p}|$ with momentum $\mathbf{p}$.

(b) Write down the angular momentum l as a function of the impact parameter b and the energy E.

Assume that different values for b cannot yield the same scattering angle θ.

(c) Express $\mathrm{d}\sigma/\mathrm{d}\Omega$ as a function of the impact parameter b and the angle θ.

From classical mechanics we know that the solution to the equation of motion is hyperbolic

$$\frac{1}{r} = \frac{mZZ'e^2}{l^2}\left(\epsilon \sin\frac{\theta}{2} + 1\right) , \tag{2}$$

with the eccentricity

$$\epsilon = \sqrt{1 + \left(\frac{2Eb}{ZZ'e^2}\right)^2}\,. \tag{3}$$

The direction of the incoming asymptote is determined by the limit $r \to \infty$.

(d) Give the functional relationship between the impact parameter b and $\cot\theta$.

(e) Use the solution from (c) to finally determine the classical differential cross section $\mathrm{d}\sigma/\mathrm{d}\Omega$ for Rutherford scattering in terms of the angle θ and the energy E.