Technische Universität München **Theoretische Physik T70**
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Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 10.12.15, 12:00 next to PH 3218.

Exercise Sheet No. 07 To be discussed from 14.12. - 18.12.15.

Problem 1:
Two-Photon Decay of the $2s$ State of the Hydrogen Atom — 10 Points

Determine the lifetime of the $2s$ state of the hydrogen atom assuming a decay with the emission of *two* photons.

(a) Write down the interaction Hamiltonian H_I

(b) Insert the electric field operator $\mathbf{A}(\mathbf{x})$ into the Hamiltonian, separate it into two terms, $H_{1\gamma}$,containing a single photon creation operator $a^\dagger(\mathbf{p})$, and $H_{2\gamma}$ containing two creation operators.

From this you can see that at first order perturbation theory, $H_{1\gamma}$ creates a single photon. To calculate two photon production, we need to use Fermi's golden rule at second order, which gives us the emission rate:

$$\begin{aligned} R_{i\to f} &= \frac{\mathcal{N}^4}{\hbar^2}\left|\langle f|H_I|i\rangle + \sum_n \frac{\langle f|H_I|n\rangle\langle n|H_I|i\rangle}{E_i - E_n}\right|^2 \delta\left(\frac{E_f - E_i}{\hbar}\right), \\ &= \frac{\mathcal{N}^4}{\hbar^2}|\mathcal{M}_{1\gamma} + \mathcal{M}_{2\gamma}|^2\delta\left(\frac{E_f - E_i}{\hbar}\right), \end{aligned}$$

where $\mathcal{N}$ is the normalization constant for the electromagnetic field, and E_f is the energy of the final state including the energies of the two photons.

(c) Write down the matrix element $\mathcal{M}_{2\gamma}$, which corresponds to $H_{2\gamma}$, to first order in perturbation theory.

(d) The matrix element for creating two photons $\mathcal{M}_{1\gamma}$ vanishes at first order. Write down the second order result.

(e) Express the total matrix element $\mathcal{M} = \mathcal{M}_{1\gamma}^{(2)} + \mathcal{M}_{2\gamma}^{(1)}$, and extract the dimensionless part as $\mathcal{M}'$.

(f) Assuming that $\sum_{\alpha_1,\alpha_2} |\mathcal{M}'|^2 \approx 1$, estimate the lifetime of the $2s$ state. Compare with the decay rate through emission of a single photon.