M. Drewes, D. Gueter, J. Klarić

Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 12.11.15, 12:00 next to PH 3218.

Exercise Sheet No. 04 To be discussed from 16.11. - 20.11.15.

Problem 1: Fermi's Golden Rule — 10 Points

Recall that the transition probability per unit time (rate) from a energy eigenstate $|m\rangle$ into other energy eigenstates $|f\rangle$ in a continuum under the influence of a interaction $V_{\rm I}$ is given by *Fermi's golden rule*

$$\Gamma_{m\to\{f\}} = \frac{2\pi}{\hbar}\rho(f)|\langle f|V_{\rm I}|m\rangle|^2 \,, \tag{1}$$

with $\rho(f)$ is the number of final states.

One can use *Fermi's golden rule* to calculate the decay rate of a β-decaying neutron: $n \to p^+ + e^- + \bar{\nu}_e$. A simple form of the perturbation is given by *Fermi's ansatz*:

$$\langle p^+ e^- \bar{\nu}_e|V_{\rm I}|n\rangle = \frac{G_F\mathcal{M}}{V} \,, \tag{2}$$

with the *Fermi constant* G_F, the nuclear matrix element $\mathcal{M}$ and the normalization volume $V = \int d^3x$. For now it can be assumed that $|\mathcal{M}|^2 \approx 1$. In the following calculation use relativistic expressions for the energies.

(a) Express the number of final states $\rho(E)$ for which the electron energy E_e is smaller than E as an integral.

Hint: What is the corresponding volume of the phase space element $d^3p_e d^3p_{\nu_e} d^3x_e d^3x_{\nu_e}$ for electrons and neutrinos? Use a delta function $\delta(E_i - E_f)$ to account for energy conservation between the initial and final states with energy E_i and E_f, respectively. Moreover, the neutrino mass can be neglected.

(b) Show that the kinetic energy of the proton T_p can be neglected compared to the energies of the neutrino E_{ν_e} and the electron E_e.

(c) Express the rate $\Gamma(E)$ for the neutron to decay into electrons with energy smaller than E.

(d) What is the differential rate $d\Gamma(E)/dE$?

(e) Plot $d\Gamma(E)/dE$ over E.

(f) Compute the total rate Γ. (You may use *Mathematica* for solving the integral.)