Technische Universität München **Theoretische Physik T70**
Physik Department **B. Garbrecht**
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Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 29.10.15, 12:00 next to PH 3218.

Exercise Sheet No. 02 To be discussed from 02.11. - 06.11.15.

Problem 1: Sudden Approximation — 1 Point

If the Hamiltonian changes very quickly, the system "doesn't have time" to adjust to change, and remains in the same state it was in before. This is known as the sudden approximation.

(a) Write down the Schrödinger equation for the time evolution operator $U(t, t_0)$.

(b) For a Hamiltonian that changes between times t_0 and $t_0 + \Delta t$, approximate the time evolution operator $U(t, t_0)$ to the first non-zero term if $H\Delta t/\hbar \ll 1$.

Problem 2: Electrically Charged Linear Harmonic Oscillator — 4 Points

Consider an electrically charged linear harmonic oscillator in the ground state, which at a time t_0 is suddenly acted upon by a homogeneous electric $\mathbf{E}$ field, constant in time from then on:

$$\mathbf{E} = E_x \vartheta(t - t_0)\mathbf{e}_x \,. \tag{1}$$

Determine the the probability of exciting the particle into the n-th state by making use of the *sudden approximation.*

Hint: The potential corresponding to the electric field is given by $\phi(x) = -eE_x x$. Determine first the wave functions of the harmonic oscillator under the influence of this potential. The matrix elements occurring in the transition probability can be computed with the help of the generating functions for the Hermite polynomials.

Problem 3: Beta Decay — 5 Points

In β-decay, the nuclear charge number Z of a $(Z-1)$-times ionized atom changes suddenly to $(Z+1)$. The effect on the electron wave function can be described by with the help of the *sudden approximation.*

Using the wave functions for an electron in the Coulomb potential of the nucleus, calculate the probabilities for the transition of the electron into the $2s$- and $3s$- states, provided that the electron was in the ground state before the β-decay.