Technische Universität München **Theoretische Physik T70**
Physik Department **B. Garbrecht**
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Quantum Mechanics II

Winter Term 2015/16

Hand in until Thursday, 22.10.15, 12:00 next to PH 3218.

Exercise Sheet No. 01 To be discussed from 26.10. - 30.10.15.

Problem 1: Equation of Motion in the Heisenberg Picture — 7 Points

Consider the one-dimensional Hamiltonian in the Schrödinger picture for a particle with mass m:

$$H = \frac{p^2}{2m} + V(x)\,. \tag{1}$$

(a) Verify the commutation relations in the Heisenberg picture

$$[x(t), p(t)] = \mathrm{i}\hbar \qquad [x(t), x(t)] = [p(t), p(t)] = 0\,. \tag{2}$$

(b) Show that the Hamiltonian in the Heisenberg picture is given by

$$H_H(t) = \frac{1}{2m} p_H^2(t) + V(x_H(t))\,. \tag{3}$$

(c) Using the equation of motion in the Heisenberg picture, derive a general expression for the time evolution of expectation values of observables. Write down the corresponding equations for the expectation values of x and p.

(d) Consider the harmonic oscillator with $V(x) = \frac{m}{2}\omega^2 x^2$. Write down the differential equation for the operators $x(t)$ and $p(t)$ and find the solutions. Calculate the commutators $[x(t_1), x(t_2)]$, $[p(t_1), p(t_2)]$ and $[x(t_1), p(t_2)]$. What happens for $t_1 = t_2$?

Problem 2: Creation and Annihilation Operators in the Heisenberg Picture — 3 Points

In the Heisenberg picture, an arbitrary operator Ω_H can be expressed by the corresponding time independent operators in the Schrödinger picture Ω_S

$$\Omega_H(t) = \mathrm{e}^{\frac{\mathrm{i}}{\hbar}Ht}\Omega_S \mathrm{e}^{-\frac{\mathrm{i}}{\hbar}Ht}\,, \tag{4}$$

where H is the Hamiltonian of the system.
Consider a N-dimensional harmonic oscillator:

$$H = \sum_{\nu=1}^{N} \hbar\omega_\nu \left(a_{S\nu}^\dagger a_{S\nu} + \frac{1}{2} \right)\,. \tag{5}$$

Using Hadamard's Lemma:

$$\mathrm{e}^X Y \mathrm{e}^{-X} = Y + [X, Y] + \frac{1}{2!}[X, [X, Y]] + \frac{1}{3!}[X, [X, [X, Y]]] + \ldots\,, \tag{6}$$

express $a_{H\nu}^\dagger$, $a_{H\nu}$ in terms of $a_{S\nu}^\dagger$ and $a_{S\nu}$.