Exercises for Theoretical Particle Physics I

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Sheet 8

Problem 1: The seesaw mechanism **5 points**

The neutrinos are nearly massless neutral particles. Their interactions violate parity: only the left-handed neutrinos interact with W and Z bosons.

a) The Z boson has an associated $U(1)$ gauge invariance which is broken in nature. If only left-handed neutrinos ν_L exist, the only possible dimension four mass term is a Majorana mass of the form $\mathcal{L} \subset \overline{\nu_L^c} M \nu_L$. Show that this mass term is forbidden by the $U(1)$ symmetry.

This motivates the introduction of "right-hadnded" neutrinos ν_R. The kinetic Lagrangian involving ν_L and ν_R is:

$$\mathcal{L}_{\text{kin.}} = \overline{\nu_L} \partial\!\!\!/ \nu_L + \overline{\nu_R} \partial\!\!\!/ \nu_R + -\overline{\nu_{L,a}} m_{ai}\, \nu_{R,i} + \frac{1}{2} \nu_{R,i}^T C^{-1} M_i \nu_{R,i}\,, + \text{h.c.}$$

where m is a $3 \times n_s$ matrix, and M is a $n_s \times n_s$ matrix with $n_s \geq 2$ being the nubmer of new right-handed states.

b) Show that each ν_R^c transforms as a left-handed spinor under the Lorentz group, so that it can mix with ν_L.

c) Rewrite the Lagrangian in terms of a doublet $\Psi = (\nu_L, \nu_R^c)$. This is not a Dirac spinor, but a doublet of left-handed Weyl spinors. Using $\mathcal{L}_{\text{kin}}$, show that this doublet satisfies the Klein-Gordon equation. Find the mass eigenstates for the neutrinos by block-diagonalizing the mass matrix and neglecting terms which are higher order in m/M.

d) Assume $M = 10^{10}\,\text{GeV}$ and $m = vY \approx 100\,\text{GeV}$. What are the masses of the physical particles? Considering that the lower bound on the observed neutrino masses is at the $\sim 10\,\text{meV}$ scale, what value of M can you expect if m is similar to the electron mass $m_e = 0.511\text{MeV}$?

e) The left-handed neutrino couples tot he Z boson and also to the electron through the W boson. The relevant part of the interaction lagrangian is:

$$\mathcal{L}_{\text{int.}} = g_w \left(\overline{\nu_L} W\!\!\!\!/\, \ell_L + \overline{\ell_L} W\!\!\!\!/\, \nu_L \right) + g_Z \overline{\nu_L} Z\!\!\!/\, \nu_L\,.$$

Using these interactions, draw a Feynman diagram for neutrinoless double β-decay, in which two neutrons decay to two protons and two electrons.

f) Which of the terms in $\mathcal{L}_{\text{kin.}}$ and $\mathcal{L}_{\text{int.}}$ respect a global symmetry (lepton number) under which $\nu_L \to e^{i\theta}\nu_L$, $\nu_R \to e^{i\theta}\nu_R$ and $\ell_L \to e^{i\theta}\ell_L$? Define arrows on e and ν lines in the neutrinoless double β-decay diagram to respect lepton number flow. Show that you cannot connect the arrows on the diagram without violating lepton number. Does this imply that neutrinoless double β-decay can tell if the neutrino has a Majorana mass?

Problem 2: Neutrinos and leptonic CP violation **5 points**

Br repeating a procedure similar to the one for quarks done in the lecture one can find that a necessary and sufficient condition for CP-conservation in the leptonic sector is the existence of unitary matrices W,U and V such that equations:

$$U^\dagger m W = m^* ,$$
$$U^\dagger m_\ell V = m_\ell^* ,$$
$$W^T M W = M^* ,$$

are all satisfied simultaneously. With m_ℓ being the mass matrix of the charged leptons.

a) show that the requirement for the existance of V is trivially satisfied if $m_\ell^\dagger m_\ell$ has real eigenvalues.

b) replace the requirement on m_ℓ by a requirement on $h_\ell = m_\ell m_\ell^\dagger$.

c) in the last problem you have shown that the light neutrino mass matrix is given by $m_\nu = m M^{-1} m^T$. What are CP-conserving requirements for this mass matrix?

d) show that you can separate the requirements on U and W by looking at the low-energy m_ν, h_ℓ and high-energy M, $h = m^\dagger m$ parts of the theory separately.

Traces of various combinations of these matrices can be used to state the requirements for CP-conservation without having to find the particular matrices U and W.

e) show that $\operatorname{Im}\operatorname{Tr}(h_\ell m_\nu m_\nu^* m_\nu h_\ell^* m_\nu^*) = 0$ is a necessary condition for CP invariance

f) Consider the invariants

$$I_1 = \operatorname{Im}\operatorname{Tr}[h(M^\dagger M) M^* h^* M] , \tag{1}$$
$$I_2 = \operatorname{Im}\operatorname{Tr}[h(M^\dagger M)^2 M^* h^* M] , \tag{2}$$
$$I_3 = \operatorname{Im}\operatorname{Tr}[h(M^\dagger M)^2 M^* h^* M H] , \tag{3}$$

in the basis where M is diagonal. Under what conditions can you reconstruct $\operatorname{Im}(h_{12}^2)$, $\operatorname{Im}(h_{23}^2)$ and $\operatorname{Im}(h_{13}^2)$ from the values of I_1, I_2 and I_3?