Exercises for Theoretical Particle Physics I

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Sheet 3

Problem 1: Ward Identity **5 points**

In the lecture you learned the formula

$$i\mathcal{M} = -ie^2\varepsilon^\star_\mu(k')\varepsilon_\nu(k)\bar{u}(p',s')\left[\frac{\gamma^\mu \not{k}\gamma^\nu + 2\gamma^\mu p^\nu}{2pk} + \frac{\gamma^\nu \not{k}'\gamma^\mu - 2\gamma^\nu p^\mu}{2pk'}\right]u(p,s) =$$
$$= \varepsilon^\star_\mu(k')\varepsilon_\nu(k)M^{\mu\nu} \;.$$

for the total leading-order amplitude for Compton Scattering. Check explicitly that $M^{\mu\nu}$ fulfils the Ward identity, i.e.

$$k_\nu M^{\mu\nu} = 0 \;.$$

Keep in mind that you can use the Dirac equation, e.g.

$$(\not{p} - m)u(p,s) = 0$$

and energy momentum conservation, i.e. sum of ingoing momenta is equal to the sum of outgoing momenta, to simplify the expression.

Problem 2: γ_5 and a decaying scalar **5 points**

The product of Dirac matrices $i\gamma^0\gamma^1\gamma^2\gamma^3$ plays an important role in particle physics. In fact has been given its own name:

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3.$$

The matrix has several interesting properties, e.g:

- it is hermitian $\gamma_5^\dagger = \gamma_5$
- it anti-commutes with all other Dirac matrices $\{\gamma^\mu, \gamma_5\} = 0$
- all traces containing only one γ_5 and less than four standard Dirac matrices vanish
- and $(\gamma_5)^2 = 1$.

Now let us consider the following scenario:
There is a scalar particle S with mass $M = 4.00$ GeV that decays into the following fermion—anti-fermion pairs: e^+e^- (electron mass $m_e \approx 0.511 \cdot 10^{-3}$ GeV), $\mu^+\mu^-$ (muon mass $m_\mu \approx 0.1057$ GeV) and $\tau^+\tau^-$ (tau mass $m_\tau \approx 1.778$ GeV).
The fermion—anti-fermion—scalar vertex has the Feynman rule:

$$i(g \cdot i\gamma_5 + g' \cdot 1).$$

The precise values of g and g' are unknown, but it is known that they are rather small (i.e. perturbation theory is applicable). The couplings do **not** depend on the fermion species, i.e. e, μ or τ.

A cunning experimentalist managed to measure (with unrealistic accuracy) the ratio of taus produced in decays of the scalar S to the number of events where electrons are produced. That is, he determined the ratios of total decay widths

$$\frac{\Gamma(S \to \tau^+\tau^-)}{\Gamma(S \to e^+e^-)} = 0.4579$$

Use this result to obtain some information on the pseudo-scalar coupling g and the scalar coupling g'.

A possible strategy would be:

- Recall from the last exercise sheet that the differential decay rate in the S rest frame given by
$$d\Gamma = \frac{1}{32\pi^2} \sum_{\text{Spins}} |\mathcal{M}(P_S \to \vec{p}\vec{p}')|^2 \frac{|\vec{p}|}{M^2} d\Omega \ .$$
- First determine $\sum_{\text{Spins}} |\mathcal{M}(P_S \to \vec{p}\vec{p}')|^2$ to lowest order in g and g' analogously to the calculation of $e^+e^- \to \mu^+\mu^-$ in the lecture.
- Make use of the identities for γ_5 given above
- Determine the expression for the decay rate ratio in terms of masses and couplings.