Exercises for Theoretical Particle Physics

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Tutorials Mondays at 12pm

SS 2017
Sheet 1, due on May 4th

Problem 1: Decay of a scalar particle **5 points**

Since the discovery of the Higgs boson, it is confirmed that nature contains at least one (apparently) fundamental scalar field. Theories beyond the Standard Model often contain many scalar particles. Their properties can be studied when they decay into other particles. Consider a scalar field ϕ of mass M that couples to Dirac fermions ψ_1 and ψ_2 of mass m_1 and m_2 via the a Yukawa interaction

$$y\phi\bar{\psi}_1\psi_2 \; + \; h.c., \tag{1}$$

where y is a dimensionless coupling constant. What is the lifetime of ϕ particles?

Problem 2: Perturbation theory with toy path integrals **5 points**

In quantum mechanics and quantum field theory, correlation functions and observables can be expressed in terms of infinite dimensional *path integrals*. In a free theory these are Gaussian and can be solved exactly. In the presence of interactions one usually has to resort to perturbation theory. To get familiar with the concept, let us study one- and two-dimensional integrals.

2a) The integral

$$\int_{-\infty}^{\infty} dx \exp\left(-ax^2 - bx^4\right) \tag{2}$$

can be solved exactly for $a, b > 0$. Compare the analytic solution to the solution obtained by expanding the integrand in b around $b = 0$ up to order b^2. Plot the solutions for $a = 1$ in the interval $0 < b < 1$.

2b) Let A be a symmetric positive definite matrix The integral

$$\int_{-\infty}^{\infty} dxdy \exp\left[-\frac{1}{2}(x\ y)A\begin{pmatrix} x \\ y \end{pmatrix} - bx^4\right] \tag{3}$$

can be solved analytically. Express the full solution in terms of $\det A$, A_{22} and b. Expand the integrand in b and express the solution to first order in b in terms of the (free) solution to zeroth order in b, A_{22} and b.