4. Electroweak Interactions

4.1 Abelian Higgs Model

We have seen that adding a mass term for a gauge boson breaks the gauge invariance of a Lagrangian. In turn, gauge symmetry protects the vanishing of the gauge boson masses from radiative corrections. Nonetheless, there are massive spin-1 bosons in Nature. Some of these have been identified as bound states of a quark and an anti-quark with angular momentum one. (It was for these particles, that the Higgs mechanism was proposed originally, even though it turns out that it does not apply there.) The important specimen in this Chapter are the force carriers of the weak interactions, $W^\pm$ and Z^0. As force carriers, these should be categorised as gauge particles, however, they are rather massive (80,39 GeV and 91,188 GeV).

The <u>Higgs mechanism</u> is a way to reconcile masses with gauge invariance which is based on <u>spontaneous symmetry breaking</u>. We explain the salient features on the Abelian $U(1)$ model and then adapt these to the relevant case of $SU(2) \times U(1)$.

Consider therefore the $U(1)$ symmetric Lagrangian for a complex scalar field Φ:

$$\mathcal{L} = (\partial_\mu \Phi)(\partial^\mu \Phi)^* + \mu^2 |\Phi|^2 - \lambda |\Phi|^4$$

The potential is of the "Mexican Hat" type and has minima for

$$2\mu^2 - 4\lambda |\Phi|^2 = 0 \Rightarrow \langle |\Phi| \rangle = \frac{v}{\sqrt{2}} = \sqrt{\frac{\mu^2}{2\lambda}}$$

that is, Φ acquires a vacuum expectation value (VEV).

Now decompose $\Phi = \frac{1}{\sqrt{2}}(\Phi_R + i\Phi_i)$ in two real degrees of freedom. Then,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \Phi_R)(\partial^\mu \Phi_R) + \frac{1}{2}(\partial_\mu \Phi_I)(\partial^\mu \Phi_I) + \frac{1}{2}\mu^2(\Phi_R^2 + \Phi_I^2) - \frac{\lambda}{4}(\Phi_R^2 + \Phi_I^2)^2$$

Next, we make use of the $U(1)$ symmetry to have Φ pointing into the real direction:

$$\sqrt{2}\langle \Phi \rangle = \langle \Phi_R \rangle = v$$

$$0 = \langle \Phi_I \rangle$$

Since v is a large classical vacuum expectation value, we should expand around it in order to identify the quantum particle excitations:

$$\Phi = \frac{1}{\sqrt{2}}(v + \phi_R + i\phi_I)$$

The Lagrangian then becomes:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_R)(\partial^\mu \phi_R) + \frac{1}{2}(\partial_\mu \phi_I)(\partial^\mu \phi_I) - \mu^2 \phi_R^2 - \sqrt{\lambda}\,\mu\,\phi_R(\phi_R^2 + \phi_I^2) - \frac{\lambda}{4}(\phi_R^2 + \phi_I^2)^2 + \frac{1}{4\lambda}\mu^4$$

When a Lagrangian has a symmetry that is broken by a particular field configuration, one speaks of spontaneous symmetry breaking. In the present case, a global $U(1)$ symmetry is spontaneously broken. The real field ϕ_R with mass $\sqrt{2}\,\mu$ is called a

Higgs boson, whereas the massless field ϕ_I is a Goldstone boson. One can prove that the number of Goldstone bosons equals the number of broken symmetries and that furthermore, the vanishing of the Goldstone boson mass is protected against radiative corrections.

Now, we promote the global $U(1)$ to a local one by gauging it, i.e. we add the gauge kinetic term $-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ and replace $\partial_\mu \longmapsto D_\mu = \partial_\mu + ieA_\mu$.

Expanding around the VEV, the kinetic term becomes

$$\frac{1}{2}\left[(\partial_\mu + ieA_\mu)(v+\phi_R+i\phi_I)\right]\left[(\partial^\mu - ieA^\mu)(v+\phi_R-i\phi_I)\right]$$

$$= \frac{1}{2}(\partial_\mu\phi_R)(\partial^\mu\phi_R) + \frac{1}{2}(\partial_\mu\phi_I)(\partial^\mu\phi_I) + evA_\mu\partial^\mu\phi_I + \frac{1}{2}e^2v^2A_\mu A^\mu$$
$$+ \text{ interaction terms}$$

Hence, we obtain a gauge boson mass ev. Now we go to unitary gauge, i.e. we perform a gauge transfor= mation such that $\Phi \mapsto \Phi\, e^{\frac{i\phi_I(x)}{v}}$ points into the real direction:

$$A_\mu \longmapsto A_\mu - \frac{1}{ev}\partial_\mu\phi_I(x)$$

$$evA_\mu\partial^\mu\phi_I \longmapsto evA_\mu\partial^\mu\phi_I - (\partial^\mu\phi_I)(\partial_\mu\phi_I)$$

$$\frac{1}{2}e^2v^2A_\mu A^\mu \longmapsto \frac{1}{2}e^2v^2A_\mu A^\mu - \frac{1}{ev}A_\mu\partial^\mu\phi_I + \frac{1}{2}(\partial_\mu\phi_I)(\partial^\mu\phi_I)$$

$$(D_\mu\Phi)(D^\mu\Phi)^* \longmapsto \frac{1}{2}(\partial_\mu\phi_R)(\partial^\mu\phi_R) + \frac{1}{2}e^2v^2A_\mu A^\mu + \text{interaction terms}$$

We have thus removed the Goldstone-boson entirely from the theory! However, a remnant is still present, because the number of field degrees of freedom is conserved and there is a new longitudinal polarisation state of the gauge boson. One colloquially speaks of the gauge boson haven "eaten" the Goldstone boson.

The disappearance of the Goldstone boson relies on unitary gauge. We it comes to loop calculations, this is no longer convenient as it obscures the renormalisability of the theory. For the Electroweak gauge interactions, a systematic gauge fixing may again be achieved by the means of the Faddeev Popov gauge fixing procedure, but for the tree-level effects that we will discuss, the unitary gauge will often prove applicable.

4.2 Electroweak Gauge Theory

Electroweak theory is based on the gauge group $SU(2)_L \times U(1)_Y$, where L indicates that only left-handed quarks and leptons form $SU(2)_L$-doublets and Y denotes the weak hypercharge of matter particles. The gauge boson Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4} W^{i\mu\nu} W^i_{\mu\nu} - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} \qquad (i = 1,2,3)$$

where

$$W^i_{\mu\nu} = \partial_\mu W^i_\mu - \partial_\nu W^i_\mu - g_W \, \varepsilon^{ijk} W^j_\mu W^k_\nu$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

and where g_W is the $SU(2)_L$ gauge coupling.

Matter couples via the covariant derivative

$$D^\mu_{ij} = \delta_{ij} \, \partial^\mu + i \, g_W \, (T \cdot W^\mu)_{ij} + i \, Y \delta_{ij} \, g'_W \, B^\mu$$

The representation matrices of the representation R of $SU(2)_L$ are given by T, and $i,j = 1, \ldots, d(R)$. Note that for $SU(2)_L$, the structure constants are given by the ε-tensor:

$$[T^i, T^j] = i\,\varepsilon^{ijk}\,T^k$$

Now, define

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2) \quad \text{and} \quad T^\pm = T^1 \pm i T^2$$

$$\Longrightarrow W_\mu \cdot T = W_\mu^3 T^3 + \frac{1}{\sqrt{2}} W_\mu^+ T^+ + \frac{1}{\sqrt{2}} W_\mu^- T^-$$

$$T^3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}, \quad T^+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

$$[T^+, T^-] = 2T^3, \quad [T^3, T^\pm] = \pm T^\pm$$

The matrix T^3 is called the weak isospin operator.

Next, we aim to spontaneously break the $SU(2)_L \times U(1)_Y$ symmetry such that there are three massive gauge bosons ($W^\pm$, Z^0) and one massless (the photon A).

Now we introduce the Higgs field, which is an $SU(2)_L$ doublet (i.e. it is in the fundamental representation) with weak hypercharge $Y = +\frac{1}{2}$:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

The superscripts of the components denote the electric charge which we will identify below.

Explicitly, the Lagrangian for this field with a potential that leads to spontaneous symmetry breaking is given by

$$\mathcal{L} = (\partial^\mu \phi^+ - i g_W W^\mu \cdot T \phi^+ - \frac{1}{2} i g_W' B^\mu \phi^+)$$
$$* (\partial^\mu \phi + i g_W W^\mu \cdot T \phi + \frac{1}{2} i g_W' B^\mu \phi) + \mu^2 \phi^+ \phi - \lambda (\phi^+ \phi)^2$$

We use the freedom of $SU(2)_L \times U(1)_Y$ rotations to express the VEV as

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

We note that transformations generated by T^3+Y leave this VEV invariant:

$$(T^3+Y)\langle\phi\rangle = \left[\begin{pmatrix}\frac{1}{2} & 0\\ 0 & -\frac{1}{2}\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\right]\begin{pmatrix}0\\ v\end{pmatrix} = 0$$

This combination of the symmetry remains therefore unbroken. There should be a massless photon and we identify the electric charge

$$Q = T^3 + Y$$

Now we expand ϕ around the minimum of the potential as

$$\phi = U^{-1}(\xi)\begin{pmatrix}0\\ \frac{H+v}{\sqrt{2}}\end{pmatrix} \quad \text{where} \quad U(\xi) = e^{-i\frac{T\cdot\xi}{v}}$$

There are still four real degrees of freedom, the three Goldstone modes ξ and the Higgs field H. The gauge transformation

$$\phi \longmapsto U(\xi)\phi$$
$$T\cdot W^\mu \longmapsto U\, T\cdot W^\mu U^{-1} + \frac{i}{g_W}(\partial^\mu U)U^{-1}$$

brings us to unitary gauge and removes the ξ from the game. We define $\chi = \begin{pmatrix}0\\1\end{pmatrix}$ and are left with

$$\mathcal{L} = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}\mu^2(H+v)^2 - \frac{\lambda}{4}(H+v)^4$$
$$+ \frac{(v+H)^2}{8}\chi^\dagger(2g_W T\cdot W_\mu + g_W' B_\mu)(2g_W T\cdot W^\mu + g_W' B^\mu)\chi$$

The last term is the mass term for the gauge bosons. We expect three massive modes and find them in

$$\frac{v^2}{8}\left[(g_W W^3_\mu - g_W' B_\mu)(g_W W^{3\mu} - g_W' B^\mu) + 2g_W^2 W^-_\mu W^{+\mu}\right]$$

We denote the massive neutral field by

$$Z^{0\mu} = \frac{g_W W^{3\mu} - g_W' B_\mu}{\sqrt{g_W^2 + g_W'^2}}$$ and identify the massless photon with $$A^\mu = \frac{g_W' W^{3\mu} + g_W B_\mu}{\sqrt{g_W^2 + g_W'^2}}$$

Conversely, $$W^{3\mu} = \frac{g_W Z^{0\mu} + g_W' A^\mu}{\sqrt{g_W^2 + g_W'^2}}, \quad B^\mu = -\frac{g_W' Z^{0\mu} - g_W A^\mu}{\sqrt{g_W^2 + g_W'^2}}$$

With these new fields, the covariant derivative is

$$D_\mu = \partial_\mu - i\frac{g_W}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{1}{\sqrt{g_W^2 + g_W'^2}} Z_\mu (g_W^2 T^3 - g'^2 Y) - i\frac{g g'}{\sqrt{g_W^2 + g_W'^2}} A_\mu (T^3 + Y)$$

This suggests the definition of the <u>Weinberg angle</u> or <u>electroweak mixing angle</u>

$$\sin^2 \vartheta_W = \frac{g_W'^2}{g_W^2 + g_W'^2} \approx 0{,}23 \quad \text{and} \quad |e| = \frac{g_W g_W'}{\sqrt{g_W^2 + g_W'^2}} = g_W \sin \vartheta_W$$

$$\Rightarrow \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix} = \begin{pmatrix} \cos\vartheta_W & \sin\vartheta_W \\ -\sin\vartheta_W & \cos\vartheta_W \end{pmatrix} \begin{pmatrix} Z^{0\mu} \\ A^\mu \end{pmatrix}$$

The mass terms then read

$$\frac{g_W^2 v^2}{4} W_\mu^+ W^{-\mu} + \frac{(g_W^2 + g_W'^2) v^2}{8} Z_\mu Z^\mu$$

such that we may identify

$$M_W = \frac{1}{2} v g_W \qquad M_Z = \frac{1}{2} v \sqrt{g_W^2 + g_W'^2} = \frac{M_W}{\cos\vartheta_W}$$

This constraint is usually expressed as the <u>ρ parameter</u>

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\vartheta_W}$$

At tree-level, $\rho = 1$. With the observed

$M_Z = 91{,}884 \pm 0{,}0022$ GeV $\qquad M_W = 80{,}22 \pm 0{,}26$ GeV

$\sin^2 \vartheta_W = 0{,}2325 \pm 0{,}0013$

it follows

$$\rho = 1{,}008 \pm 0{,}007$$

We therefore see that the coupling strengths or the mixing angle determine the ratios of the gauge boson masses, which is a distinctive feature of the Higgs mechanism. Moreover, the observed vector boson masses allow to infer the Higgs VEV $v = 246$ GeV.

The self-couplings of the Higgs boson are

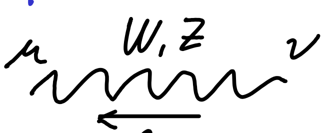
$$\mathcal{L}_H = \frac{1}{2}(\partial_\mu H)(\partial^\mu H) + \frac{1}{2}\mu^2 (H+v)^2 - \frac{\lambda}{4}(H+v)^4$$

$$\underset{v=\sqrt{\frac{\mu^2}{\lambda}}}{\overset{\uparrow}{=}} -\mu^2 H^2 - \lambda v H^3 - \frac{\lambda}{4} H^4 + \frac{1}{4}\frac{\mu^4}{\lambda}$$

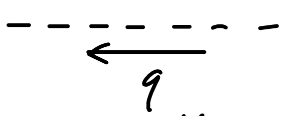
such that the Higgs mass is $M_H = \sqrt{2}\,\mu = \sqrt{2\lambda}\, v$

A measurement of these self-couplings would be an important check of the theory.

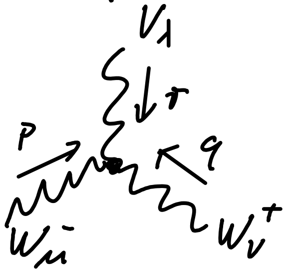
The unitary gauge is good enough to calculate tree-level processes. The Feynman rules are



$$= \left[-g^{\mu\nu} + \frac{q^\mu q^\nu}{M^2} \right] \frac{i}{q^2 - M^2 + i\varepsilon}$$

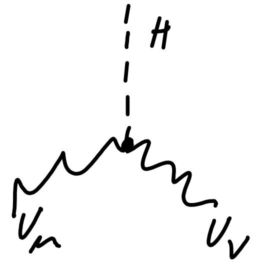


$$= \frac{i}{q^2 - M^2 + i\varepsilon}$$



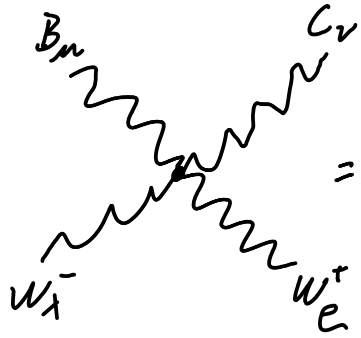
$$= i g_V \left[(p-q)_\lambda g_{\mu\nu} + (q-r)_\mu g_{\nu\lambda} + (r-p)_\nu g_{\lambda\mu} \right]$$

$$g_A = -e, \quad g_Z = g_W \cos\theta_W$$



$$= i g_{VH} M_W g_{\mu\nu}$$

$$g_{WH} = g_W \qquad g_{ZH} = \frac{g_W}{\cos^2\vartheta_W}$$



$$= i\left[2 g_{\mu\nu} g_{\lambda\rho} - g_{\mu\lambda} g_{\nu\rho} - g_{\mu\rho} g_{\nu\lambda}\right] \begin{cases} g_W^2 & \text{for } (B_\mu, C_\nu) = (W_\mu^+, W_\nu^-) \\ -g_W^2 \cos^2\vartheta_W & \text{for } (B_\mu, C_\nu) = (Z_\mu, Z_\nu) \\ -e^2 & \text{for } (B_\mu, C_\nu) = (A_\mu, A_\nu) \\ -e g_W & \text{for } (B_\mu, C_\nu) = (A_\mu, Z_\nu) \end{cases}$$

In nature, we observe that $SU(2)_L$ couples only to the left handed components of fermions and that these fermions can be grouped in doublets (fundamental representation). Furthermore, left- and right handed fermions have different weak hypercharges. Interactions with that property can be built with the help of the chiral projection operators

$$P_L = \frac{1-\gamma^5}{2}, \qquad P_R = \frac{1+\gamma^5}{2}$$

$$Q_L^{1,2,3} = P_L \begin{pmatrix} u \\ d \end{pmatrix}, P_L \begin{pmatrix} c \\ s \end{pmatrix}, P_L \begin{pmatrix} t \\ b \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \begin{pmatrix} c_L \\ s_L \end{pmatrix}, \begin{pmatrix} t_L \\ b_L \end{pmatrix}$$

$$u_R^{1,2,3} = P_R u, P_R c, P_R t = u_R, c_R, t_R$$

$$d_R^{1,2,3} = P_R d, P_R s, P_R b = d_R, s_R, b_R$$

$$L_L^{1,2,3} = P_L \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}, P_L \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}, P_L \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} = \begin{pmatrix} \nu_{eL} \\ e^-_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu^-_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau^-_L \end{pmatrix}$$

$$e_R^{1,2,3} = P_R e, P_R \mu, P_R \tau = e_R, \mu_R, \tau_R$$

$$\psi_L = L_L, Q_L$$

$$\psi_R = e_R, u_R, d_R$$

We observe that accordin to thisy scheme, the matter fermions fall into three generations.

The Lagrangian for the coupling of the Electroweak interactions to matter is given by

$$\mathcal{L} = \overline{\psi_R^i}\, i(\partial\!\!\!/ + i g_W' Y_i B\!\!\!/)\psi_R^i + \overline{\psi_L^j}\, i(\partial\!\!\!/ + i g_W T\cdot W\!\!\!\!/ + i g_W' Y_j B\!\!\!/)\psi_L^j$$

The quantum numbers of matter and Higgs fields are given by

	$SU(3)\times SU(2)_L\times U(1)_Y$
Q_L	$(3, 2, \frac{1}{6})$
u_R	$(3, 1, \frac{2}{3})$
d_R	$(3, 1, -\frac{1}{3})$
L_L	$(1, 2, -\frac{1}{2})$
e_R	$(1, 1, -1)$
H	$(1, 2, \frac{1}{2})$

For the groups SU, we are giving here the dimension of the representation, whereas for $U(1)$, the weak hypercharge Y.

Recall that the electric charge operator is $Q^i = T^3 + Y^i$ and verify that these assignment are in consistency with the electric charges.

Now for the fermion masses. Mass terms violate chirality, i.e. they mix left- and right-handed compo= nents. Writing down terms of the form

$$m_u \bar{u} u + m_d \bar{d} d$$

does not fly, because these are neither $SU(2)_L$ invariants (singlets) nor $U(1)_Y$ invariant (weak hypercharge does not add up to zero.

What works however are Lagrangian terms of the form

$$\mathcal{L} = -y_d^{ij}\,\overline{Q}_{La}^i\,\phi_a\,d_R - y_u^{ij}\,\varepsilon^{ab}\,\overline{Q}_{La}^i\,\phi_b^\dagger\,u_R^j - y_e^{ij}\,\overline{L}_{La}^i\,\phi_a\,e_R^j + \text{h.c.}$$

The y are matrices of Yukawa couplings. Notice that δ^{ab} and ε^{ab} are $SU(2)$ invariant tensors and that furthermore, in each term, the weak hypercharge adds up to zero. The Yukawa matrices are in general non-diagonal. They can however be diagonalised through the biunitary transformation

$$y_{u,d,e} = U_{u,d,e}\, y^D_{u,d,e}\, W^\dagger_{u,d,e}$$ where y^D is diagonal and where U, W diagonalise $y y^\dagger = U y^{D2} U^\dagger$, $y^\dagger y = W y^{D2} W^\dagger$.

The Lagrangian terms read (suppressing ε^{ab})

$$-\phi\,\overline{\psi_L}\,y\,\psi_R = -\phi\,\overline{\psi_L}\,U\,y^D\,W^\dagger\,\psi_R\,,$$

such that we can eliminate W by redefining

$$\psi_R \longmapsto W\psi_R$$

In addition, we get rid of U by redefining

$$u_L \longmapsto U_u u_L \qquad d_L \longmapsto U_d d_L$$
$$e_L \longmapsto U_e e_L$$

The interactions with the gauge bosons B and W^3 (and therefore A and Z) are invariant under these transformations.

However

$$\frac{1}{\sqrt{2}}\,\overline{u}_L\,\gamma^\mu d_L \longmapsto \frac{1}{\sqrt{2}}\,\overline{u}_L\,\gamma^\mu\, U_u^\dagger U_d\, d_L$$

We define the <u>Cabibbo-Kobayashi-Maskawa (CKM) matrix</u> $V = U_u^\dagger U_d$.

The u and d quarks are now defined such that the Yukawa couplings are diagonal, such that the quarks are mass eigenstates, but they are no longer eigenstates of the exchange of charged W bosons. This has the important consequence that W bosons can mediate between the different generations. Now back to the question of fermion masses. Since $\langle \varphi^0 \rangle = \frac{v}{\sqrt{2}}$, we find that these are given by $m_{u,d,e} = y^D_{u,d,e} \frac{v}{\sqrt{2}}$. Spontaneous symmetry breaking therefore gives rise to mass terms mixing fermions in different representations of $SU(2)_L \times U(1)_Y$ by gauge invariant Lagrangian terms.

The model gives rise to the weak interactions at low energies. Calculating an "effective action" (in Quantum Field Theory, there is a well-defined method for this), and "integrating" out the W bosons from the path integral, we obtain the effective interaction:

$$\sum_{\pm} i \frac{1}{\sqrt{2}} \overline{\psi_L^j} g_W T^{\pm} \psi_L^j \frac{-i}{M_W^2} \frac{1}{\sqrt{2}} \overline{\psi_L}^i g_W T^{\mp} \psi_L^i$$

$$= \sum_{\pm} \frac{4}{\sqrt{2}} G_F \overline{\psi_L^j} T^{\pm} \psi_L^j \overline{\psi_L}^i T^{\mp} \psi_L^i$$

Where $G_F = \frac{\sqrt{2} g_W^2}{8 M_W^2}$ is the Fermi constant, that can be observed from radioactive decay.

This is valid, provided the fermion momenta are much below M_W. Notice that it is then of the form that we had for the interactions between leptons and quarks in DIS.

Above interaction explains weak radioactive decays:

$d \longrightarrow u + e + \bar{\nu}_e$

Besides, due to the replacement $d_L \longmapsto V d_L$, it mediates between the different generations of quarks, as we will discuss later in these lectures.

Neutral currents mediated by the Z boson were first observed in $\nu N \rightarrow \nu +$ hadron scatterings (N stands for nucleon) in 1973 by the Gargamelle bubble chamber at CERN. This way, the weak mixing angle could be determined and the strength of the weak interactions through $|e| = g_W \sin \vartheta_W$. Given these observations, the theory of Electroweak unification not only predicts the existence of $W^\pm$ and Z but also their masses. Eventually, $W^\pm$ and Z were first directly produced in 1983 at SPS (Super Proton Synchrotron) at CERN and their masses were found to be in agreement with low-energy observations.

Hence, Electroweak symmetry breaking (EWSB) is a satisfactory model to explain weak interactions at low energy in terms of a gauge theory. In addition, it <u>predicts</u> the existence of the $W^\pm$, Z and the Higgs boson. Besides the existence of these particles, there are also consistency checks, such as the ρ-parameter.

4.3 The Vector Bosons $W^\pm$, Z

Due to the gauge quantum numbers of the fermions, there are rather distinctive predictions for the couplings of vector bosons to matter fields. To study these, we express the couplings between the physical fields of the Electroweak sector to the fermions as

$$\mathcal{L}_f = \sum_f \overline{\psi}_f \left(i\not{\partial} - m_f - g_W \frac{m_f H}{2 M_W} \right) \psi_f$$

$$- \frac{g_W}{\sqrt{2}} \sum_f \overline{\psi}_f \left(\gamma^\mu T^+ W_\mu^+ P_L + \gamma^\mu T^- W_\mu^- P_L \right) \psi_f$$

$$- |e| \sum_f Q_f \overline{\psi}_f \gamma^\mu A_\mu \psi_f - \frac{g_W}{2 \cos\theta_W} \sum_f \overline{\psi}_f \gamma^\mu Z_\mu (V_f - A_f \gamma^5) \psi_f$$

The fermion masses and their interactions with the Higgs boson have been expressed as

$$y_f^D \, \overline{\psi}_f \frac{v + H}{\sqrt{2}} \psi_f = \overline{\psi}_f m_f \psi + \overline{\psi}_f \, g_W \frac{m_f H}{2 M_W} \psi_f$$

$$\uparrow$$

$$v = \frac{2 M_W}{g_W}$$

For the couplings to the Z boson, we have defined vectorial and axial vectorial couplings

$$V_f = T_f^3 - 2 Q_f \sin^2\theta_W$$
$$A_f = T_f^3$$

Now, consider the process $Z(q) \longrightarrow f(p) \bar{f}(p)$. The matrix element is

$$\frac{-i g_W}{2 \cos\theta_W} \bar{u}(p) \gamma^\mu (V_f - A_f \gamma^5) v(p') \varepsilon_\mu(q)$$

The sum over the three polarisation of the massive Z boson is $\displaystyle\sum_{\text{Polarisations}} \varepsilon_\mu^{(i)}(q) \varepsilon_\nu^{(i)*}(q) = -g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2}$

such that we need to evaluate the trace

$$\mathrm{tr}\left[\not{p}\gamma^\mu(V_f - A_f\gamma^5)\not{p}'(V_f + A_f\gamma^5)\gamma^\nu\right]\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2}\right)$$

$$= \mathrm{tr}\left[\not{p}\gamma^\mu(V_f^2 + A_f^2)\not{p}'\gamma^\nu\right]\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_Z^2}\right)$$

$\not{p}\gamma^\mu = -\gamma^\mu\not{p} + 2p^\mu$

$$= \mathrm{tr}\Big[\underbrace{\gamma^\mu\gamma_\mu \not{p}\not{p}' - 2\not{p}\not{p}'}_{\substack{\gamma^\mu\gamma_\mu = 4 \\ 4 * M_Z^2}} - \frac{1}{M_Z^2}\underbrace{\not{p}\not{q}\not{p}'\not{q}}_{4(2p\cdot q\, p'\cdot q - p\cdot p' q^2) = 0}\Big](V_f^2 + A_f^2)$$

$(p+p')^2 = 2p\cdot p' = M_Z^2$

$(q-p)^2 = 0 = M_Z^2 - 2q\cdot p$

$$= 4M_Z^2(V_f^2 + A_f^2)$$

Now, we average over initial and sum over final polarisations:

$$\frac{1}{3}\sum_{\text{polarisations}} |i\mathcal{M}|^2 = \frac{1}{3}\frac{g_W^2}{4\cos^2\vartheta_W}4M_Z^2(V_f^2 + A_f^2) = \frac{1}{3}\frac{8}{\sqrt{2}}G_F M_Z^4(V_f^2 + A_f)^2$$

Where we made use of the definition of the Fermi-constant

$$G_F = \frac{\sqrt{2}\,g_W^2}{8M_W^2} = \frac{\sqrt{2}\,g_W^2}{8M_Z^2\cos^2\vartheta_W} \Longleftrightarrow \frac{M_Z^2}{\cos^2\vartheta_W} = \frac{8G_F}{\sqrt{2}\,g_W^2}M_Z^4$$

$$\rho = \frac{M_W^2}{M_Z^2\cos^2\vartheta_W} \approx 1$$

In order to obtain the decay rate, we must divide by $2M_Z$ and multiply by the two-body phase-space factor

$$\int \frac{d^4k}{(2\pi)^4}\frac{d^4(k')}{(2\pi)^4}(2\pi)^4\delta^4(q-k-k')\,2\pi\,\delta(k^2)\,2\pi\,\delta(k'^2) = \frac{1}{8\pi}$$

such that

$$\Gamma(Z\to f\bar{f}) = C\,\frac{G_F M_Z^3}{6\sqrt{2}\,\pi}(V_f^2 + A_f^2)$$

Here, we have introduced a colour factor, $C=3$ for quarks & $C=1$ for leptons.

Similarly, for the decays of W bosons, one obtains

$$\Gamma(W^+ \longrightarrow f\bar{f}') = C\,\frac{G_F M_W^3}{6\sqrt{2}\,\pi}$$

where we refer to the sum of the decays to a given quark of charge $\frac{2}{3}$ and all anti-quarks of charge $\frac{1}{3}$, e.g. $W^+ \rightarrow u\bar{d} + u\bar{s} + u\bar{b}$. For an individual mode $W \longrightarrow u_i \bar{d}_j$, there is an additional factor $|V_{ij}|^2$ from the CKM matrix.

The following branching fractions therefore test the gauge interactions of matter in the Standard Model of EWSB:

	Relative Coupling	Branching Ratio
$W^+ \rightarrow e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$	1	3 * 11,1%
$W^+ \longrightarrow u\bar{d} + u\bar{s} + u\bar{b}$	3	33,3%
$W^+ \longrightarrow c\bar{d} + c\bar{s} + c\bar{b}$	3	33,3%

There is no decay into top quarks because of their large mass $m_t \simeq 172$ GeV.

	V_f	A_f	Rel. Coupling	Br. Ratio
$Z \rightarrow \nu_e\bar{\nu}_e, \nu_\mu\bar{\nu}_\mu, \nu_\tau\bar{\nu}_\tau$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3 * 6,8%
$Z \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$	$-\frac{1}{2} + 2\sin^2\vartheta_W$	$-\frac{1}{2}$	$\frac{1}{4} + (\frac{1}{2} - 2\sin^2\vartheta_W)^2$	3 * 3,4%
$Z \rightarrow u\bar{u}, c\bar{c}$	$\frac{1}{2} - \frac{4}{3}\sin^2\vartheta_W$	$\frac{1}{2}$	$3[\frac{1}{4} + (\frac{1}{2} - \frac{4}{3}\sin^2\vartheta_W)^2]$	2 * 11,8%
$Z \longrightarrow d\bar{d}, s\bar{s}, b\bar{b}$	$-\frac{1}{2} + \frac{2}{3}\sin^2\vartheta_W$	$-\frac{1}{2}$	$3[\frac{1}{4} + (\frac{1}{2} - \frac{2}{3}\sin^2\vartheta_W)^2]$	3 * 15,2%

These tree-level branching ratios compare well with the experimentally observed values listed in the Particle Physics Booklet pp. 9, 10 $\ddot{\triangledown}$

For the Z boson, you also find in that table "invisible" decays. These go into neutrinos.
The theoretical tree-level ratio for the decay into one species of neutrinos over decays into one species of leptons is

$$\frac{\Gamma(Z \to \nu\bar{\nu})}{\Gamma(Z \to l^+ l^-)} = \frac{2}{1+(1-4\sin^2\vartheta_W)^2} \approx 1{,}99$$

Experimentally, one finds that

$$\frac{\Gamma(Z \to \text{invisible})}{\Gamma(Z \to l^+ l^-)} = 5{,}956 \pm 0{,}031,$$

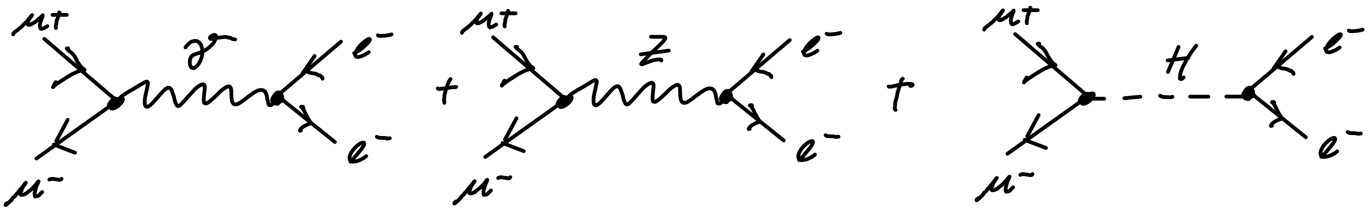
such that the number of light neutrinos coupling to Z is $2{,}991 \pm 0{,}016$.

We can also use above matrix element in order to obtain the production cross sections for the vector bosons:

$$\sigma_{f\bar{f}' \to W} = \frac{\pi}{3}\sqrt{2}\, G_F M_W^2 \, |V_{f\bar{f}'}|^2 \, \delta(\hat{s} - M_W^2)$$

$$\sigma_{f\bar{f} \to Z} = \frac{\pi}{3}\sqrt{2}\, G_F M_Z^2 \, (V_f^2 + A_f^2) \, \delta(\hat{s} - M_Z^2)$$

When protons (e.g. LHC) [or protons & anti-protons (e.g. SPS, Tevatron) are collided, the full cross section can be obtained from the partonic one using the methods applied to the Drell-Yan process.
Another classic observation is the process $e^+e^- \to \mu^+\mu^-$. We discussed the production via photons in the chapter on QED. At high energies close to the Electroweak scale, one has to account for the new particles by these diagrams:



Since the Yukawa couplings of the Higgs to μ and e are very small, we can neglect these for the present purposes. Calculating the cross-section is straightforward using the methods introduced in these lectures, but get somewhat tedious. The details can be found e.g. in the book by Greiner & Müller. The result is

$$\sigma_{e^+e^- \to \mu^+\mu^-} \approx \frac{4\pi\alpha^2}{3s}\left[1 + \frac{s^2}{16\,\sin^4 2\vartheta_W \left(s - \left|M_Z - \frac{i}{2}\Gamma_Z\right|^2\right)^2}\right]$$

Here, Γ_Z is the total decay rate of the Z-boson. It is related to the Z-boson self-energy as

$$\Gamma_Z = \frac{1}{M_Z}\,\mathrm{Im}\,\Pi(p^0 = M_Z, \vec{p} = 0)$$

and therefore occurs in the resummed propagator (cf. Chapter 2). There it regulates the divergence that would otherwise occur for $s = M_Z^2$. The Z-boson occurs as a <u>resonance</u> peak on top of the $\frac{1}{s}$-behaviour of the cross section. The maximum is reached for $s = M_Z^2 - \frac{\Gamma_Z^2}{4}$ where $\sigma_{max} \approx \frac{4\pi\alpha^2}{3\Gamma_Z^2}$

For larger (smaller) values of Γ_Z, the peak becomes wider (more narrow), such that instead of a decay rate, one often speaks of a decay width.

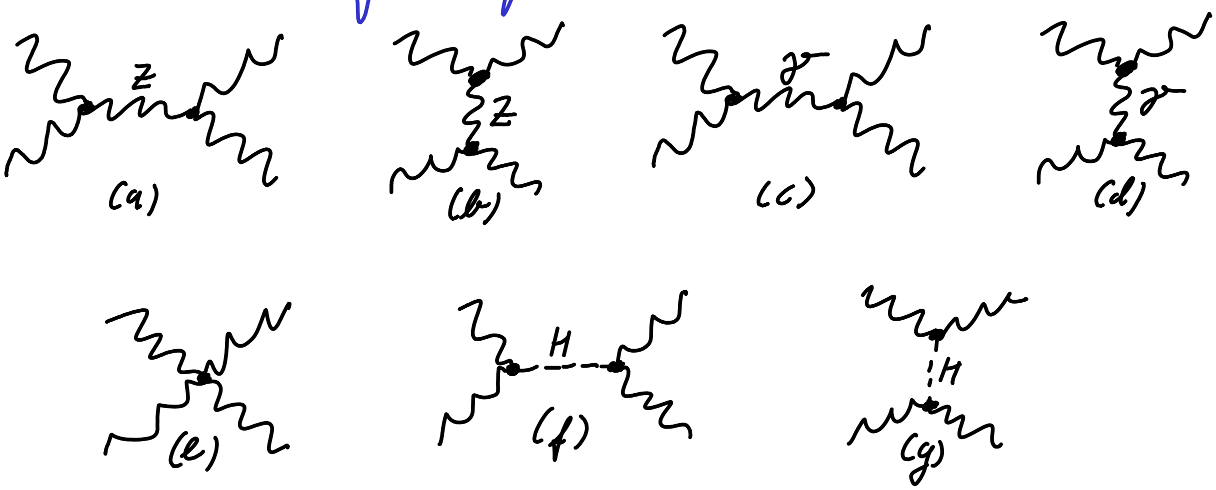
All these results provide a strong evidence that the Electric and Weak interactions unify to a single, spontaneously broken gauge theory. Yet, the charge assignments and the value of the ρ-parameter may rely on coincidences, such that there could be no additional degrees of freedom in the Electroweak sector beyond $W^\pm$ and Z. Or the Higgs boson could be too heavy to be observed at the LHC.

However, consideration of the high-energy behaviour of the scattering amplitudes of vector boson indicates that $W^\pm$ and Z are not the full story.

We therefore consider the process

$$W^+(p_+) + W^-(p_-) \longrightarrow W^+(q_+) + W^-(q_-)$$

The contributing diagrams are



In the centre of mass frame, we parametrise the momenta as

$$p_\pm = (E, 0, 0, \pm p) \qquad E^2 - p^2 = M_W^2$$

$$q_\pm = (E, 0, \pm p \sin\vartheta, \pm p \cos\vartheta)$$

The interesting behaviour at high energies occurs for the longitudinal polarisations (cf. the polarisation sum above)

$$\varepsilon_L(p_\pm) = \left(\frac{p}{M_W}, 0, 0, \pm\frac{E}{M_W}\right)$$

$$\varepsilon_L(q_\pm) = \left(\frac{p}{M_W}, 0, \pm\frac{E}{M_W}\sin\vartheta, \pm\frac{E}{M_W}\cos\vartheta\right)$$

These satisfy the Lorentz gauge condition $k \cdot \varepsilon_L(k) = 0$ and are normalised such that $\varepsilon_L^2 = -1$.

The pure gauge diagrams exhibit the disastrous high-energy behaviour.

$$|i\mathcal{M}^{(a-d)}|^2 = g_W^2\left\{\frac{p^4}{M_W^4}\left[3 - 6\cos\vartheta - \cos^2\vartheta\right] + \frac{p^2}{M_W^2}\left[\frac{9}{2} - \frac{11}{2}\cos\vartheta - 2\cos^2\vartheta\right]\right\}$$

$$|i\mathcal{M}^{(e)}|^2 = g_W^2\left\{\frac{p^4}{M_W^4}\left[-3 + 6\cos\vartheta + \cos^2\vartheta\right] + \frac{p^2}{M_W^2}\left[-4 + 6\cos\vartheta + 2\cos^2\vartheta\right]\right\}$$

While the most offensive terms $\propto p^4$ cancel, still the remaining terms indicate a breakdown of perturbation theory for $p \gg M_W$.

This growth of $|i\mathcal{M}|^2$ leads to a growth of the S-matrix as well, which is why it is sometimes referred to unitarity violation in W^+W^- scattering, but one should bear in mind that this is due to the application of perturbation theory beyond its domain of validity.

But where poison grows, there also does the cure. For the Higgs exchanges, one finds:

$$|i\mathcal{M}^{(f-g)}|^2 = g_W^2\left\{\frac{p^2}{M_W^2}\left[-\frac{1}{2} - \frac{1}{2}\cos^2\vartheta\right] - \frac{M_H^2}{4M_W^2}\left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right]\right\}$$

Field Theory holds another spectacular cancellation in stock

$$|i\mathcal{M}^{(a-g)}|^2 = -g_W^2\,\frac{M_H^2}{4M_W^2}\left[\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right]$$

Which has a benign behaviour up to high energies.

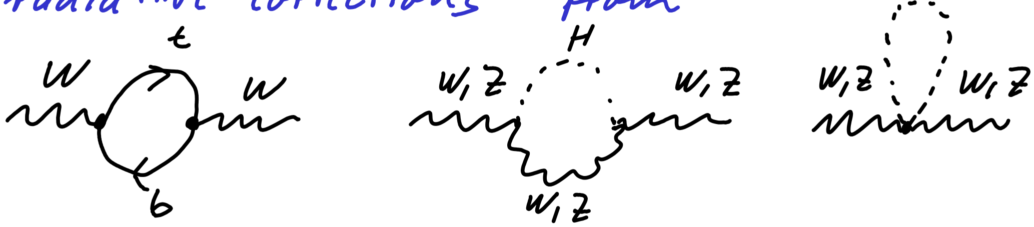
The meaning of these results is that at high energy colliders that probe the Electroweak scale, we will either

- discover the Higgs boson H
- or discover other new degrees of freedom that ensure the perturbativity of W^+W^- scattering at high energies
- or observe a breakdown of perturbation theory in W^+W^- scatterings, the details of which will hint to another mechanism different from the Standard Model of EWSB realised in Nature.

It is therefore believed (and explained to the funding bodies) that a high energy collider such as the LHC must necessarily discover the Higgs boson or uncover new laws of Nature.

4.4 The Higgs Boson

Now, where is it? A detailed analysis of W^+W^- scattering leads to the conclusion that unitarity is fixed if $M_H < 1$ TeV. In addition, there are other more powerful indications. The gauge boson masses receive radiative corrections from



Regarding the first diagram, recall that the superficial degree of divergence of the vacuum polarisation is quadratic. This divergence is cancelled by the anti-fermion in the loop, so we may expect this correction to be $\propto (m_t^2 - m_b^2) \approx m_t^2$. For the Z, where a $t\bar{t}$ pair runs in the loop, there is no quadratic dependence on the top mass. As a result, the ρ-parameter receives the corrections

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = \left[1 + \frac{3}{16\pi^2}\left(\frac{m_t}{v}\right)^2 - \frac{11 \tan\theta_W}{96\pi^2} g^2 \log\frac{M_H}{M_W} + \dots \right]$$

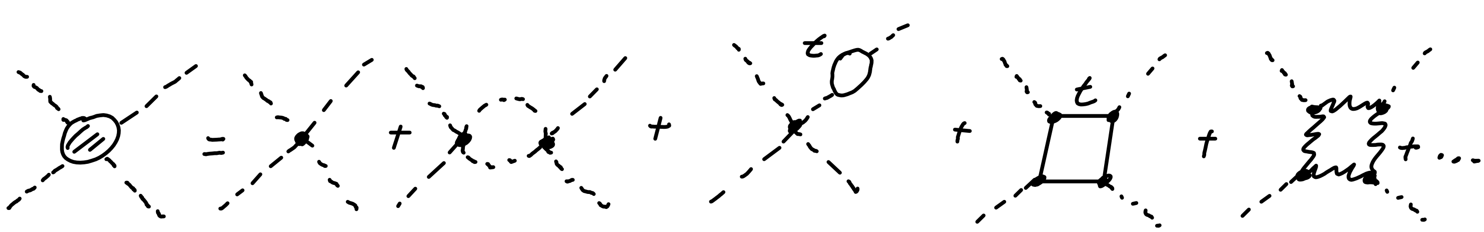
There is a number of other electroweak precision observables that are sensitive to the Higgs mass. From these, one can determine (cf. Figure)

$$M_H = 95 \begin{smallmatrix} +30 & +74 \\ -24 & -43 \\ 1\sigma & 2\sigma \end{smallmatrix} \text{ GeV} \quad \text{or} \quad M_H < 162 \text{ GeV} \ @ \ 95\% \text{ c.l.}$$

Another more philosophical indication is vacuum stability and triviality. It derives from the running quartic coupling

$$\frac{\partial\lambda}{\partial\log\mu} = \frac{3}{2\pi^2}\left[\lambda^2 + \frac{1}{2} y_t^2 \lambda - \frac{1}{4} y_t^4 + B(g,g') + \dots \right]$$

$$B(g,g') = -\frac{1}{8}\lambda(3g_W^2 + g_W'^2) + \frac{1}{64}(3g_W^4 + 2g_W^2 g_W'^2 + g_W'^4)$$



As the reference scale, where we feed the couplings into the evolution equations, we pick the VEV v.

The triviality bound applies when $\lambda \gg g_W, g'_W, y_t$, and it yields an upper bound on M_H:

$$\frac{\partial \lambda}{\partial \log \mu} = \frac{3}{2\pi^2} \lambda^2 \Rightarrow \lambda(\mu) = \frac{\lambda(v)}{1 - \frac{3\lambda(v)}{4\pi^2} \log\left(\frac{\mu^2}{v^2}\right)}$$

For increasing μ, $\lambda(\mu)$ is growing and hits a Landau pole when $\mu = \mu_L$

$$\log \frac{\mu_L^2}{v^2} = \frac{4\pi^2}{3\lambda(v)}$$

Now μ corresponds to a cutoff scale which could be the Planck scale $2{,}4 * 10^{18}$ GeV, the Grand Unified scale 10^{16} GeV or any other scale up to which the Standard Model should be well defined perturbatively. Hence, we require $\mu < \mu_L$. Therefore, we must impose

$$\lambda(v) < \frac{4\pi^2}{3 \log \frac{\mu^2}{v^2}} \underset{M_H = \sqrt{2\lambda}\, v}{\Longleftrightarrow} M_H < \sqrt{\frac{8\pi^2}{3 \log \frac{\mu^2}{v^2}}}\, v$$

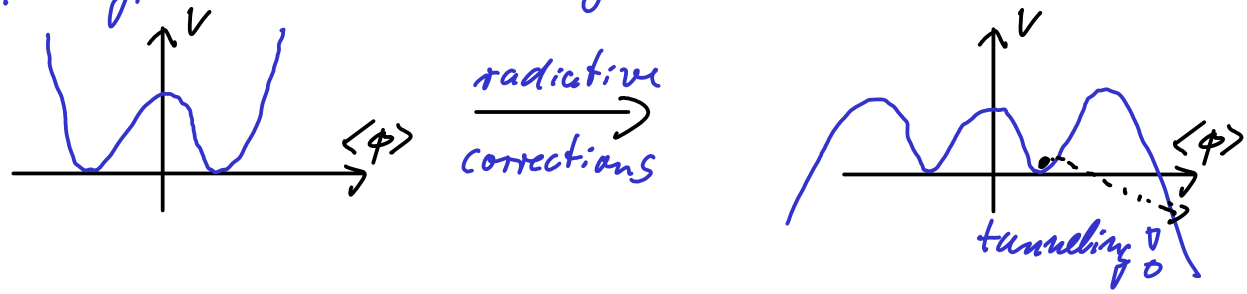
For $\mu = 10^{16}$ GeV, this translates into $M_H < 160$ GeV, a bound which becomes logarithmically less restrictive for smaller values of μ, cf. Figure.

When $\lambda \ll g_W, g'_W, y_t$, the vacuum instability bound is of importance. One finds now that

$$\frac{\partial \lambda}{\partial \log \mu} \approx \frac{3}{8\pi^2}\left[-3y_t^4 + \frac{3}{16}\left(2g_W^4 + (g_W^2 + g_W'^2)^2\right)\right]$$

$$= \frac{3}{8\pi^2 v^4}\left[2M_W^4 + M_Z^4 - m_t^4\right] < 0 .$$

The heavy quark mass destabilises the potential, such that the minimum at v is no longer the global minimum, which leads to the possibility of apocalyptic vacuum decay



To avoid this unpleasant consequence, we require the coupling to remain positive up to the cutoff scale μ:

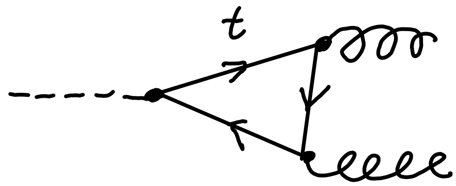
$$\lambda(\mu) - \lambda(v) = \frac{\partial \lambda}{\partial \log \mu} \log \frac{\mu}{v}$$

$$\lambda(\mu) \overset{?}{>} 0 \Rightarrow \lambda(v) > -\frac{\partial \lambda}{\partial \log \mu} \log \frac{\mu}{v} \Longleftrightarrow$$

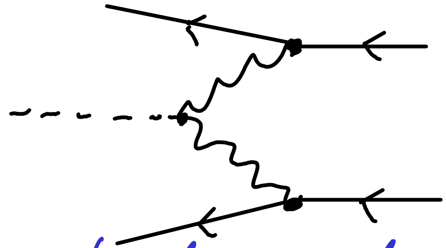
$$M_H > \sqrt{\frac{3}{8\pi^2 v^4}\left[2M_W^4 + M_Z^4 - m_t^4\right] \log \frac{\mu^2}{v^2}}\; v$$

A detailed 2-loop analysis sets $M_H > 130$ GeV as the lower bound for the Standard Model Higgs mass from vacuum stability. (cf. Figure).
There is however a loophole. If the Higgs were discovered, say, at 125 GeV, our vacuum would be unstable, but its life time would exceed the age of the Universe.

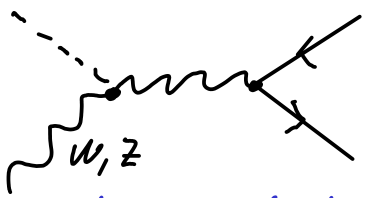
Now, we turn to the production of Higgs bosons and their detection at the Large Hadron Collider (LHC). The dominant production mechanisms are represented by the following diagrams:



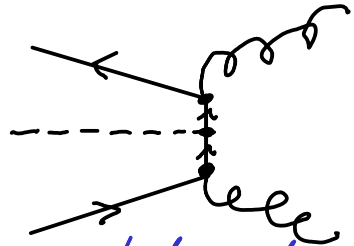
gluon fusion



vector boson fusion



associated production with W, Z ("Higgs strahlung")



associated production with $t\bar{t}$

Details about these processes along with references to the original calculations can be found, e.g., in Ellis, Stirling & Webber.

At all energies, gluon-gluon fusion is dominating.

The summed and averaged matrix element is given by

$$|i\mathcal{M}_{gg\to H}|^2 = \frac{\alpha_s^2(M_H^2)\, G_F\, M_H^4}{288\sqrt{2}\,\pi^2} \left| I\left(\frac{m_t^2}{M_H^2}\right)\right|^2$$

where

$$I(x) = 3x\left[2 + (4x-1)F(x)\right]$$

$$F(x) = \vartheta(1-4x)\,\frac{1}{2}\left[\log\left(\frac{1+\sqrt{1-4x}}{1-\sqrt{1-4x}}\right) - i\pi\right]^2$$

$$-\vartheta(4x-1)\, 2\left[\arcsin\left(\frac{1}{2\sqrt{x}}\right)\right]^2$$

For $x>1$, one may approximate $I(x) \approx 1 + \frac{1}{4x}$.

The parton-level cross section is

$$\sigma_{gg\to H} = \frac{\pi}{\hat{s}}\,\delta(\hat{s}-M_H^2)\,|i\mathcal{M}_{gg\to H}|^2$$

Vector boson fusion gives rise to a subdominant contribution (cf. Figure) with the squared matrix element for $q(p_1)+q'(p_2)\to q(p_3)+q'(p_4)$

$$|i\mathcal{M}_{qq\to Hqq}|^2 = 128\sqrt{2}\,G_F^3 M_V^8 \frac{C_1^V\, p_1\cdot p_2\;\, p_3\cdot p_4 + C_2^V\, p_1\cdot p_4\;\, p_2\cdot p_3}{(2p_3\cdot p_1+M_V^2)^2\,(2p_4\cdot p_2+M_V^2)^2}$$

$$C_1^W=0$$
$$C_2^W=1$$
$$C_1^Z=\frac{1}{4}\left[(V_q-A_q)^2(V_{q'}-A_{q'})^2+(V_q+A_q)^2(V_{q'}-A_{q'})^2\right]$$
$$C_2^Z=\frac{1}{4}\left[(V_q-A_q)^2(V_{q'}+A_{q'})^2+(V_q+A_q)^2(V_{q'}-A_{q'})^2\right]$$

The initial states must be convoluted with the parton distribution functions and for the final states, a three body integration is necessary.
Even more suppressed are the associated productions. However, the Higgs may be more efficiently identified when detecting a vector boson or a top-quark pair in addition. Moreover, the ratio of the different production mechanisms is an important prediction that may be used in order to put the model to a test.
The parton-level cross sections with associated W, Z production are

$$\sigma(q\bar{q}'\to WH)=\frac{(G_F M_W^2)^2}{9\pi}\,|V_{qq'}|^2\,\frac{p_W}{\sqrt{\hat{s}}}\,\frac{3M_W^2+p_W^2}{(\hat{s}-M_W^2)^2}$$

$$\sigma(q\bar{q}\to ZH)=\frac{(G_F M_Z^2)^2}{9\pi}\,(V_q^2+A_q^2)\,\frac{p_Z}{\sqrt{\hat{s}}}\,\frac{3M_Z^2+p_Z^2}{(\hat{s}-M_Z^2)^2}$$

$$p_V^2=\frac{1}{4\hat{s}}\left(\hat{s}^2+M_V^4+M_H^4-2\hat{s}M_H^2-2M_V^2M_H^2\right)$$

Besides looking at the individual Higgs production cross sections varying over M_H, it is also interesting to look at the dependence of the cross section on the collision energy $\sqrt{s}$. The impact of increasing $\sqrt{s}$ from 1,8 TeV (Tevatron) over 7 TeV and 8 TeV (LHC before 2013) up to 14 TeV (maximal energy end of 2014) is sizeable.

To detect Higgs bosons, one needs to know and quantify its various decay channels.

The tree level partial decay widths are

$$\Gamma(H \to f\bar{f}) = \frac{c\, G_F\, m_f^2\, M_H}{4\pi\sqrt{2}} \left(1 - \frac{4 m_f^2}{M_H^2}\right)^{\frac{3}{2}}$$

$$\Gamma(H \to W^+W^-) = \frac{G_F M_H^3}{8\pi\sqrt{2}} \left(1 - \frac{4 M_W^2}{M_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{4 M_W^2}{M_H^2} + \frac{12 M_W^4}{M_H^4}\right)$$

$$\Gamma(H \to ZZ) = \frac{G_F M_H^3 M_W^2}{16\pi\sqrt{2} M_Z^2} \left(1 - \frac{4 M_Z^2}{M_H^2}\right)^{\frac{1}{2}} \left(1 - \frac{4 M_Z^2}{M_H^2} + \frac{12 M_Z^4}{M_H^4}\right)$$

The cross sections for the decay into vector bosons are only valid above the threshold for two-body decays.

For the decay into off-shell gauge bosons one obtains

$$\Gamma(H \to VV^*) = \frac{3 M_V^4}{32\pi^2 v^4} M_H\, \delta_V'\, R(x) \qquad x = 4\frac{M_V^2}{M_H^2}$$

$$\delta_W' = 1$$

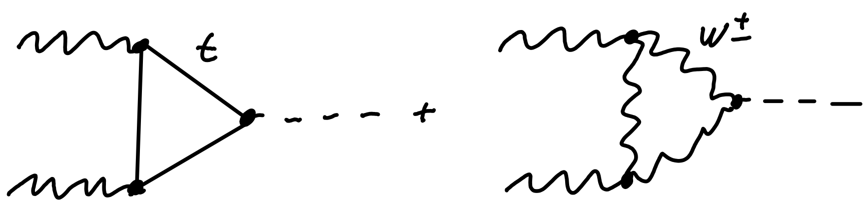
$$\delta_Z' = \frac{7}{12} - \frac{10}{9}\sin^2\vartheta_W + \frac{40}{27}\sin^4\vartheta_W$$

$$R(x) = \frac{3(1-8x+20x^2)}{\sqrt{4x-1}} \arccos\frac{3x-1}{2x^{\frac{3}{2}}} - \frac{1-x}{2x}(2-13x+47x^2)$$

$$- \frac{3}{2}(1-6x+4x^2)\log x$$

Whether there are on-shell or off-shell, the gauge bosons decay into charged leptons or quarks and are registered by the detector, or they escape when they are neutrinos.

Another rather subdominant but important decay channel is $H \to \gamma\gamma$ (two photons), which proceeds via the diagrams



The partial decay rate is given by

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2 G_F M_H^3}{128\sqrt{2}\,\pi^3}\left|\sum_q 3e_q^2 I_q\left(\frac{m_q^2}{M_H^2}\right) + I_W\left(\frac{M_W^2}{M_H^2}\right)\right|^2$$

$$I_q(x) = 4x\left[2 + (4x-1)\,F(x)\right]$$

$$I_W(x) = -2\left[6x + 1 + 6x(2x-1)\,F(x)\right]$$

and $F(x)$ as we had for $gg \to H$.

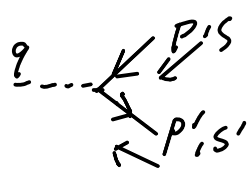
The typical particle produced at LHC is however not a Higgs boson, but hadrons from proton-proton collisions or leptons and vector bosons from Drell-Yan processes. These particles constitute backgrounds, and if the back=ground is not distinguishable from the signal, it is called irreducible. At a hadron collider, clearly hadronic backgrounds are dominating and therefore, for early detection of the Higgs, it is favourable to look for non-hadronic signals such as $H \to ZZ^* \to 4\ell$ or $H \to \gamma\gamma$. While the first process has a much higher signal-to-background ratio, the latter brings more events and better statistics, which is presently crucial. The recent discovery of a Higgs-like particle is therefore due to the decays $H \to \gamma\gamma$, but eventually all production and decay channels will be used in order to test the consistency of the Standard Model.

4.5 Unstable Particles

We have encountered production cross sections for the Higgs boson as well as its decay rate. Among the decay products are unstable on-shell vector bosons. It is therefore in order to discuss in what sense these particles can be understood as final states. Standard Model interactions encompass interferences as well as some tedious numerator algebra. So instead, consider the toy model

$$\mathcal{L} = \overline{\psi_i}\, i\not\partial\, \psi_i + \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 - \sum_i y_i \overline{\psi_i}\, \phi\, \psi_i$$

The field ϕ represents a vector boson or a Higgs boson, i.e. it serves as a model for an unstable particle. Now consider the cross section for $\overline{\psi_i}\psi_i \longrightarrow \phi$



$$\left.\begin{aligned} p &= (p^0, \vec{p}) \\ p' &= (p^0, -\vec{p}) \end{aligned}\right\} \text{CMS} \qquad p^2 = p'^2 = 0$$

$$i\mathcal{M} = -i y_i\, u(p,s)\, \bar{v}(p',s')$$

$p^0 = \frac{m}{2}$

$$\frac{1}{4}\sum_{s,s'} |i\mathcal{M}|^2 = \frac{1}{4} y_i^2\, \overbrace{\mathrm{tr}[\not p \not p']}^{4\,\frac{m^2}{2}} = 2 y_i^2\, p^{0\,2}$$

$$\sigma = \frac{1}{8 p^{0\,2}} \int \frac{d^4 q}{(2\pi)^4}\, 2\pi\, \delta(q^2 - m^2)\, (2\pi)^4 \delta^4(p + p' - q)\; 2 y_i^2\, p^{0\,2}$$

That is one δ-function too many in order to give rise to a well-defined & finite cross section. On the other hand, we know that since ϕ has a finite life-time only, there should be no sharp on-shell δ-function due to the uncertainty relation. Hence, we expect that within the correct expression, we should replace the on-shell δ-function by some finite-width representation, where the width is given by the life time of ϕ. This is what we derive in the following.

The decay rate of ϕ is given by

$$\Gamma_\phi(\vec{q}) = \frac{m^2}{\omega_\phi} \sum_i g_i^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{4|\vec{p}||\vec{p}'|} (2\pi)^4 \delta^4(q-p-p')$$

Note that this expression is valid as well for $\vec{q} \neq 0$, and we have $q^0 = \omega_\phi = \sqrt{\vec{q}^{\,2} + m^2}$, $p^0 = |\vec{p}|$, $p'^0 = |\vec{p}'|$.

Now, we can evaluate

$$\Gamma_\phi(\vec{q}) = \frac{m^2}{\omega_\phi} \sum_i g_i^2 \int \frac{d^3p}{(2\pi)^3} \frac{1}{4p^0p'^0} 2\pi\,\delta(q^0 - p^0 - p'^0)$$

$$\vec{p}' = \vec{q} - \vec{p} \iff p'^0 = \sqrt{\vec{q}^{\,2} + \vec{p}^{\,2} - 2|\vec{q}||\vec{p}|\cos\vartheta}$$

$$q^0 - |\vec{p}| - \sqrt{\vec{q}^{\,2} + \vec{p}^{\,2} - 2|\vec{q}||\vec{p}|\cos\vartheta} = 0$$

$$q^{02} - 2q^0|\vec{p}| + \vec{p}^{\,2} = \vec{q}^{\,2} + \vec{p}^{\,2} - 2|\vec{q}||\vec{p}|\cos\vartheta$$

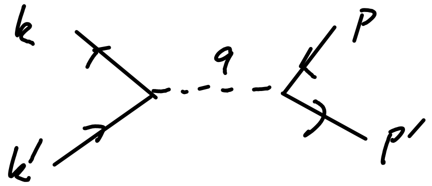
$$\cos\vartheta = -\frac{q^2 - 2q^0|\vec{p}|}{2|\vec{q}||\vec{p}|}$$

$$\cos\vartheta < 1 \Rightarrow -q^2 < -2|\vec{p}|(q^0 - |\vec{q}|) \iff |\vec{p}| < \tfrac{1}{2}(q^0 + |\vec{q}|)$$

$$\cos\vartheta > -1 \Rightarrow -q^2 > -2|\vec{p}|(q^0 + |\vec{q}|) \iff |\vec{p}| > \tfrac{1}{2}(q^0 - |\vec{q}|)$$

$$\Gamma_\phi(\vec{q}) = \frac{m^2}{q^0} \sum_i g_i^2 \int_{\frac{1}{2}(q^0-|\vec{q}|)}^{\frac{1}{2}(q^0+|\vec{q}|)} \cancel{\vec{p}^{\,2}}\, d|\vec{p}|\, \frac{1}{2\pi} \frac{1}{4\cancel{p^0}\cancel{p'^0}} \frac{\cancel{p'^0}}{|\vec{q}|\cancel{|\vec{p}|}} = \frac{m^2 \sum_i g_i^2}{8\pi q^0}$$

Consider next the squared matrix element for $\bar{\psi}_i \psi_i \longrightarrow \bar{\psi}_j \psi_j$, i.e. the diagram



From the scalar propagator, $|i\mathcal{M}|^2$ would contain a factor

$$\frac{1}{q^2-m^2+i\varepsilon}\,\frac{1}{q^2-m^2-i\varepsilon} = \frac{1}{[q^2-m^2]^2+\varepsilon^2}$$

(Note: $\dfrac{\varepsilon}{[q^2-m^2]^2+\varepsilon^2} = \pi\,\delta(q^2-m^2)$)

This is ill defined for $q^2=m^2$, but the problem is fixed because being unstable, ϕ has a self-energy with an imaginary part. We assume that the the real part of the self-energy is cancelled by appropriate counterterms for $q^2=m^2$.
The imaginary part can be obtained by evaluating the self-energy introducing Feynman parameters and performing a Wick rotation. In the Feynman parameter integrals, there will be logarithms with negative argument and therefore, there will result an imaginary part. In the present context, it is however more instructive to go directly for the imaginary part without performing a Wick rotation.

$$i\Pi(q) = \sum_i g^2\, \mathrm{tr} \int \frac{d^4p}{(2\pi)^4}\, \frac{i\not{p}}{p^2+i\varepsilon}\, \frac{i(\not{q}-\not{p})}{(q-p)^2+i\varepsilon}$$

$$\rightsquigarrow \operatorname{Im}\Pi(q) = 2\sum_i g^2 \int \frac{d^4p}{(2\pi)^4}\, p\cdot(q-p)\left[\frac{1}{p^2+i\varepsilon}\,\frac{1}{(q-p)^2+i\varepsilon} + \frac{1}{p^2-i\varepsilon}\,\frac{1}{(q-p)^2-i\varepsilon}\right]$$

$$\frac{2}{x\pm i\varepsilon} = \frac{1}{x\pm i\varepsilon}+\frac{1}{x\mp i\varepsilon}+\frac{1}{x\pm i\varepsilon}-\frac{1}{x\mp i\varepsilon} = 2\,PV\frac{1}{x}\mp 2\pi i\,\delta(x)$$

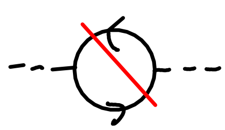
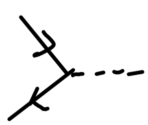
$$\mathrm{Im}\,\Pi(q) = 2\sum_i g_i^2\int\frac{d^4p}{(2\pi)^4}\,p\cdot(q-p)\quad 2\pi\delta(p^2)\,2\pi\delta((q-p)^2)$$

$$= q^2\sum_i g_i^2\int\frac{d^3p}{(2\pi)^3}\frac{d^3p'}{(2\pi)^3}\frac{1}{4p^0p'^0}(2\pi)^4\delta^4(q-p-p') = \omega_\phi\Gamma_\phi$$

The replacement of the propagators by the δ-functions can be justified when considering two subsequent integrations using the residue theorem. A simpler and more powerful method derives from the so called Closed-Time-Path formalism, which we however not discuss in these lectures.

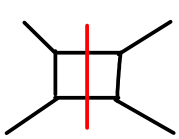
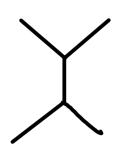
Now recall that the decay rate was $\frac{1}{2\omega_\phi}$ times the the phase space integral ($d\Pi$) over the $1\to 2$ diagram. The present calculation can be generalised to the <u>optical theorem.</u>

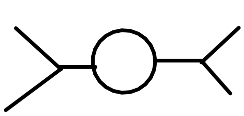
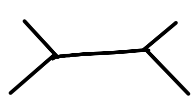
For our present example, it can be stated dia= grammatically that

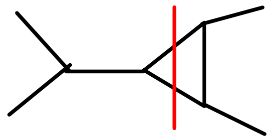
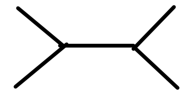
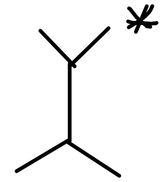
$2\,\mathrm{Im}$  $= \int d\Pi\,\Big|$  $\Big|^2$

Where the red line indicates an on-shell <u>cut.</u>

To give another example, it can also be applied to

$2\ \mathrm{Im}$  $= \int d\Pi\,\Big|$  $\Big|^2$ tt interference

$2\ \mathrm{Im}$  $= \int d\Pi\,\Big|$  $\Big|$ ss interference

2 Im  $= \int d\Pi$  $*$ * st interference

as well as to other possibilities.

Now back to the process $\psi_i \overline{\psi_i} \longrightarrow \psi_j \overline{\psi_j}$

$$i\mathcal{M}_{\psi_i\overline{\psi}_i \to \psi_j\overline{\psi}_j} = -y_i\, u(p,s)\,\overline{v}(p',s') \frac{i}{(p+p')^2 - m^2 + i\,\mathrm{Im}[\Pi(p+p')]}\, y_j\, \overline{u}(k,r)\, v(k',r')$$

$$\frac{1}{4}\sum_{\substack{s,s',\\ r,r'\\ j}} \left| i\mathcal{M}_{\psi_i\overline{\psi}_i \to \psi_j\overline{\psi}_j} \right|^2 = y_i^2 \sum_j y_j^2 \frac{(p+p')^4}{[(p+p')^2 - m^2]^2 + [\mathrm{Im}\,\Pi(p+p')]^2}$$

$$\mathrm{Im}\,\Pi(q) = q^2 \sum_i y_i^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3p'}{(2\pi)^3} \frac{1}{4p^0p'^0} (2\pi)^4 \delta^4(q-p-p')$$

$$\sigma = \frac{y_i^2 \sum_j y_j^2}{2p^0\, 2p'^0\, 2} \int \frac{d^3k}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{(p+p')^4}{2k^0\, 2k'^0} \frac{(2\pi)^4 \delta^4(p+p'-k-k')}{[(p+p')^2-m^2]^2 + [\mathrm{Im}\,\Pi(p+p')]^2}$$

$$= \frac{y_i^2}{8p^0p'^0} (p+p')^2 \frac{\mathrm{Im}\,\Pi(p+p')}{[(p+p')^2-m^2]^2 + [\mathrm{Im}\,\Pi(p+p')]^2}$$

When we recall the representation $\delta(x) = \frac{1}{\pi}\frac{\varepsilon}{x^2+\varepsilon^2}$ we observe that indeed this formula generalises the standard cross section to an expression suitable for unstable particles by a replacement of the on-shell δ-function by a finite-width distribution associated with the decay rate of the particle. The behaviour of the cross section near "resonances" is thus quantitatively explained.

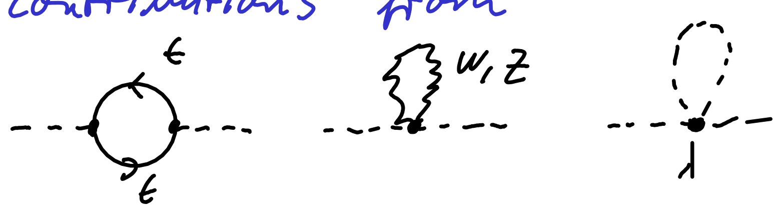
4.6 An Example for Beyond the Standard Model Theory: The Two-Higgs-Doublet Model

While the scope of these lectures does not encompass a comprehensive review of Beyond the Standard Model (BSM) Theory, we discuss here a simple extension of the Standard Model SM, that illustrates that the LHC may uncover phenomena beyond the SM. Besides, we introduce the intrinsic transformation properties of particles under discrete symmetries, which we will use at a later point of these lectures.

To give a brief overview of the motivations for considering BSM theory, we state a number of phenomena that are not explained within the SM and as well as some aesthetic deficiencies:

Hierarchy Problem / Naturalness Problem

The Higgs boson mass receives quadratically divergent contributions from



When the cutoff scale is of order the Planck scale (10^{18} GeV) or Grand Unified Theory (GUT) scale (10^{15} GeV) we need to choose the counterterm very carefully in order to achieve a Higgs mass & VEV of order 10^2 GeV.

Many BSM theories that occupy the theorist's imagination are motivated by this problem, such as

Supersymmetry (SUSY), Compositeness/Technicolor and Large Extra Dimensions. One of the distinctive features of SUSY is the presence of two Higgs doublets.

Grand Unified Theories (GUTs)

Electroweak unification may be taken as a blueprint for attempts to embed $U(1)_Y \times SU(2)_L \times SU(3)_C$ in larger groups such as $SU(5)$ or $SO(10)$. In the latter case, also two Higgs doublets are predicted. In SUSY frameworks, GUTs can be unified with gravitation within string theory.

Neutrino Masses

In a narrow definition of the Standard Model (no non-renormalisable operators, no right-handed neutrinos), there are no neutrino masses that are indirectly observed from neutrino oscillations.

Strong CP problem

The parameter ϑ in the interaction $\vartheta\, n_f \frac{g^2}{32\pi} F^a_{\mu\nu} \tilde{F}^{a\mu\nu}$, where F is the gluon field strength tensor is (when strong CP violation is removed from the Yukawa couplings) smaller than 10^{-11}, as inferred from current bounds on the neutron electric dipole moment. The problem is why this parameter is not of order one.

Cosmology

The CP violation and the deviation from thermodynamic equilibrium during the expansion of the Universe is not enough to account for the baryon asymmetry of the Universe (BAU)

$\frac{n_B}{n_\gamma} \simeq 6 * 10^{-10}$ [Big Bang Nucleosynthesis (BBN), Cosmic Microwave Background (CMB)]

This problem may be addressed by adding extra sterile neutrinos or by extending the Electroweak sector.

Besides, observations of the CMB, Large Scale Structure (LSS) and galaxy rotation curves imply that there is Dark Matter contributing five times more mass to the Universe than baryons. If the Dark Matter is associated with the Electroweak Symmetry breaking, it should be directly produced at the LHC, be directly detectable in underground experiments as well as yield indirect annihilation signals [γ-rays ($\rightarrow$ FERMI), neutrinos ($\rightarrow$ IceCube) and anti-matter ($\rightarrow$ Pamela, AMS)].

Finally, the Dark Energy problem is why the Universe is currently accelerating. It can be phenomenologically resolved by setting the vacuum energy $\Lambda \simeq (2 * 10^{-3}\, eV)^4$, much smaller than "natural" values such as the Electroweak or the Planck scale.

We conclude that there is a plethora of more or less well-motivated / plausible BSM scenarios. To pick a specific one, we consider two Higgs doublet model (2HDM).

We take two doublets $\Phi_{1,2}$, each with weak hypercharge $+\frac{1}{2}$. The most general potential is

$$V = m_{11}^2 \, \Phi_1^\dagger \Phi_1 + m_{22}^2 \, \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right)$$
$$+ \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2$$
$$+ \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 + \frac{\lambda_5}{2} \left[(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2 \right]$$

Minimisation of this potential is straightforward, but does not lead us to new insights. So we just assume that in unitary gauge, the VEVs are

$$\langle \Phi_1 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_1}{\sqrt{2}} \end{pmatrix}, \quad \langle \Phi_2 \rangle_0 = \begin{pmatrix} 0 \\ \frac{v_2}{\sqrt{2}} \end{pmatrix} \quad \text{and define} \quad \tan\beta = \frac{v_2}{v_1}$$

The observed couplings g and g' as well as $M_{W^\pm}$ and M_Z imply that $v = \sqrt{v_1^2 + v_2^2} = 246$ GeV.

Expanding around the VEV, we introduce the eight fields:

$$\Phi_a = \begin{pmatrix} \phi_a^+ \\ \frac{v_a + h_a + i\eta_a}{\sqrt{2}} \end{pmatrix}, \quad a = 1,2$$

The mass matrix for the charged fields can be expressed as

$$\mathcal{L}_{\phi^\pm \text{mass}} = \left[m_{12}^2 - (\lambda_4 + \lambda_5) v_1 v_2 \right] (\phi_1^- \ \phi_2^-) \begin{pmatrix} \frac{v_2}{v_1} & -1 \\ -1 & \frac{v_1}{v_2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}$$

This matrix has one zero eigenvalue, giving rise to the Goldstone bosons

$$G^{\pm} = \frac{1}{\sqrt{v_1^2 + v_2^2}} \left(v_1 \phi_1^{\pm} + v_2 \phi_2^{\pm} \right) = \cos\beta\, \phi_1^{\pm} + \sin\beta\, \phi_2^{\pm}$$

that are eaten by $W^{\pm}$. The non-zero eigenvalue gives rise to the mass-square

$$m_{\pm}^2 = \left[\frac{m_{12}^2}{v_1 v_2} - \lambda_4 - \lambda_5 \right] (v_1^2 + v_2^2)$$

and the charged scalars

$$H^{\pm} = \frac{1}{\sqrt{v_1^2 + v_2^2}} \left(v_2 \phi_1^{\pm} - v_1 \phi_2^{\pm} \right) = \sin\beta\, \phi_1^{\pm} - \cos\beta\, \phi_2^{\pm}$$

Now before we look into the neutral fields, we consider in what sense we may characterise these by their charge conjugation properties. For that purpose, consider the Noether current of a complex field χ,

$$\mathcal{L} = (\partial_\mu \chi)(\partial^\mu \chi^*) - m^2 |\chi|^2$$

under the transformation $\chi \longmapsto e^{i\alpha} \chi \approx \chi + i\alpha\chi = \chi + \Delta\chi$

$$j^\mu = \frac{1}{\alpha} \frac{\partial \mathcal{L}}{\partial(\partial_\mu \chi)} \Delta\chi = i \chi \partial^\mu \chi^* - i (\partial^\mu \chi) \chi^*$$

We can thus infer that the charge conjugation C of a scalar field is implemented by

$$\chi(x) \overset{C}{\longmapsto} \chi^*(x)$$

Besides, we define the intrinsic parity of the Higgs fields to be

$$\Phi_a(x) \overset{P}{\longmapsto} + \Phi_a(-x)$$

In combination, we can state that the fields η are CP-odd.

Notice that this is a convention, because if we had chosen a different unitary gauge with an imaginary VEV, we would have obtained a different CP-transformation property. The present choice is however "natural" as it preserves C and P in the electromagnetic and strong interactions, whereas in the weak interactions these symmetries are maximally violated.

The mass matrix for the $\eta_{1,2}$ is

$$\mathcal{L}_{\eta\,mass} = \left(\frac{m_{12}^2}{v_1 v_2} - 2\lambda_5\right)(\eta_1 \;\; \eta_2)\begin{pmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{pmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}$$

There is again one massless mode

$$G^0 = \frac{1}{\sqrt{v_1^2 + v_2^2}}\left(v_1 \eta_1 + v_2 \eta_2\right) = \cos\beta\, \eta_1 + \sin\beta\, \eta_2$$

that is eaten by the Z boson and one massive, CP-odd scalar

$$A = \frac{1}{\sqrt{v_1^2 + v_2^2}}\left(v_2 \eta_1 - v_1 \eta_2\right) = \sin\beta\, \eta_1 - \cos\beta\, \eta_2 .$$

with mass square

$$m_A^2 = \left(\frac{m_{12}^2}{v_1 v_2} - 2\lambda_5\right)(v_1^2 + v_2^2)$$

Finally, the neutral, CP-even excitations e obtain the masses:

$$\mathcal{L}_{e\,mass} = -(e_1 \;\; e_2)\begin{pmatrix} m_{12}^2 \dfrac{v_2}{v_1} + \lambda_1 v_1^2 & -m_{12}^2 + \lambda_{345} v_1 v_2 \\ -m_{12}^2 + \lambda_{345} v_1 v_2 & m_{12}^2 \dfrac{v_1}{v_2} + \lambda_2^2 v_2^2 \end{pmatrix}\begin{pmatrix} e_1 \\ e_2 \end{pmatrix}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

This matrix can be diagonalised. Let the mixing angle be α and the mass eigenstates

$$h = \varrho_1 \sin\alpha - \varrho_2 \cos\alpha$$

$$H = -\varrho_1 \cos\alpha - \varrho_2 \sin\alpha$$

Without loss of generality, we assume that h is lighter than H.

In summary, the two Higgs-doublet model does not only feature one God particle, but rather, a pantheon:

$$\left.\begin{array}{l} h \\ H \end{array}\right\} \text{neutral, CP even}$$

$H^\pm$ charged

A neutral, CP odd

Notice that these are five degrees of freedom, as the two-doublet model originally features eight, three of which are eaten by the massive gauge bosons.

The couplings of these bosons to $W^\pm$ & Z are given e.g. in hep-ph/0503173. It is interesting to quote

$$Z_\mu Z_\nu h: \; i \frac{g_2}{\cos\theta_W} M_Z \sin(\beta-\alpha) g_{\mu\nu}$$

$$Z_\mu Z_\nu H: \; i \frac{g_2}{\cos\theta_W} M_Z \cos(\beta-\alpha) g_{\mu\nu}$$

$$W_\mu^+ W_\nu^- h: \; i g_2 M_W \sin(\beta-\alpha) g_{\mu\nu}$$

$$W_\mu^+ W_\nu^- H: \; i g_2 M_W \cos(\beta-\alpha) g_{\mu\nu}$$

Note that the CP odd boson A does not couple to two vector bosons at tree-level.

It is now interesting to consider how these particles are different from the SM Higgs boson.
For that purpose, notice that we could simply redefine (along with a rotation of the original couplings & masses)

$$H_1 = \cos\beta\, \Phi_1 + \sin\beta\, \Phi_2$$
$$H_2 = -\sin\beta\, \Phi_1 + \cos\beta\, \Phi_2$$

such that

$$\langle H_1 \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \text{ and } \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The SM excitation (which is not a mass eigenstate in general) is therefore

$$H^{SM} = \cos\beta\, \rho_1 + \sin\beta\, \rho_2 = h \sin(\alpha-\beta) - H\cos(\alpha-\beta)$$

So when $\alpha - \beta \simeq \frac{\pi}{2}$, h behaves very much like the SM Higgs boson, cf. also the couplings quoted above. Such a situation would indeed not be too contrived, because we can achieve large $\tan\beta$ ($\sin\beta \simeq 1$) by $m_{11} \gg m_{22}$ s.th. $v_2 \gg v_1$.
It is also possible to suppress α, such that h corresponds to the excitation of Φ_2 around its VEV, and $H^\pm, A, H$ are very heavy. This is called the "decoupling scenario." In other words, we may continuously "tune" the model in order to appear more or less like the SM.
The remaining important feature of the 2HDM are the couplings to fermions. Recall that in order to

diagonalise the Yukawa couplings, we transform the weak charged currents as

$$\frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L \longmapsto \frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu \underbrace{U_u^\dagger U_d}_{=V} d_L$$

Clearly, for the neutral currents, the CKM-matrix V does not appear. Moreover, since the Yukawa couplings define the quark-mass basis, a Higgs boson always decays at tree-level into fermions & anti-fermions of the same flavour. One may state this observation as the absence of flavour-changing neutral currents (FCNCs) in the SM at tree level.

This aspect is in general no longer true in the 2HDM, because the masses of the fermions and the various Yukawa couplings cannot be simultaneously diagonalised. The bounds on FCNCs are so strong that unless we impose a particular symmetry on the FCNCs, the 2HDM is constrained to be in the decoupling scenario with the new scalar particles to to have masses at the multi-TeV scale (cf. hep-ph/1106.0034).

One particular choice (that is realised in supersymmetric models) in order to forbid tree-level FCNCs, is to have $\Phi_2 = H_u$ only couple to right-handed u-type quarks and charged leptons, and $\Phi_1 = H_d$ only to d-type right-handed quarks. The Yukawa couplings are again proportional to the fermion masses, such that we may express these as

$$\mathcal{L}_{Yuk} = -\sum_{f=u,d,l} \frac{m_f}{v}\left(\xi_h^f \bar{f} f h + \xi_H^f \bar{f} f H - i \xi_A^f \bar{f} \gamma^5 f A \right)$$

$$- \left\{ \frac{\sqrt{2} V_{ud}}{v} \bar{u} \left(m_u \xi_A^u P_L + m_d \xi_A^d P_R \right) d H^+ + \frac{\sqrt{2} m_l \xi_A^l}{v} \bar{\nu}_L l_R H^+ + h.c. \right\}$$

where

$$\xi_h^u = \xi_h^d = \xi_h^l = \frac{\cos\alpha}{\sin\beta}, \qquad \xi_H^u = \xi_H^d = \xi_H^l = \frac{\sin\alpha}{\sin\beta}$$

$$\xi_A^u = \cot\beta, \quad \xi_A^d = -\cot\beta, \quad \xi_A^l = -\cot\beta$$

Again, we observe that in the decoupling limit, h behaves like the SM Higgs boson.

In summary, the 2HDM may illustrate that

- there may yet be more new particles discovered at the LHC,
- it is important to measure the production and decay channels of the 125 GeV particle, in order to distinguish this particle from a possible impostor like A and in order to derive bounds on possible deviations from the SM. Note that due to the clear detection of $h \to ZZ^{(*)} \to 4l$, we may readily exclude A of the 2HDM as an impostor.

4.7 Remarks on Current Status & Future Prospects

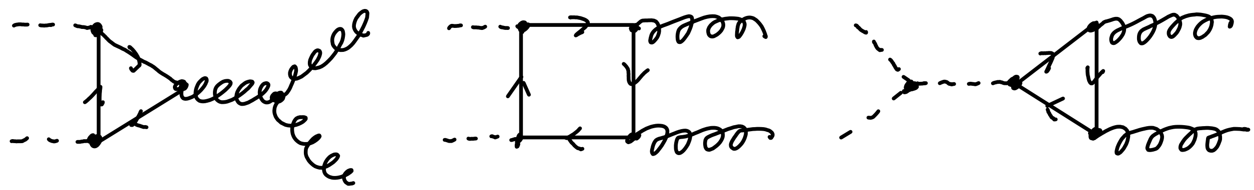
The LHC has by now gathered a multiple of the data compared to the amount that was the basis for the announcement of the Higgs discovery on 04/07/2012. The data until the end of 2012 has been

analysed and early in 2014, collisions will resume with 7 TeV per beam (up to now: 4 TeV).
Analyses include the test of impostor hypotheses. Different spin & parity properties may be excluded by studying the distributions of four leptons and diphotons. In particular, a spin 2 boson is excluded at 99% CL (ATLAS-CONF-2013-029), a parity-odd scalar at 97,8% CL (ATLAS-CONF-2013-013).
In turn, the properties of the new particle are so far in agreement with the SM, compare the ATLAS signals from 07/2012 to 03/2013. More aggressive analyses (e.g. 1303.3879) that combine ATLAS & CMS & Tevatron confirm the SM as well.
The discovery of BSM phenomena by the LHC (or other experiments) would also indicate directions of future experimental & theoretical efforts. If there are no BSM discoveries a future task may be to probe the cause of spontaneous symmetry breaking. The use of a simple scalar potential may be one of the less plausible aspects of the SM. The relation $M_H = \sqrt{2\lambda}\, v$ offers a direct probe of this mechanism. The Lagrangian contains the interactions

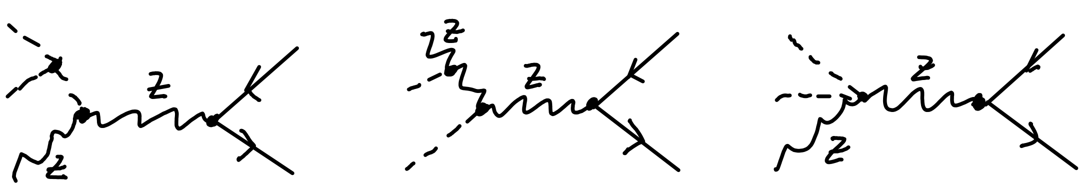
$$\mathcal{L} \supset -\lambda v H^3 - \frac{1}{4} \lambda H^4$$

Since v is known from the vector boson masses, M_H, the trilinear & the quartic coupling allow for independent ways of measuring λ (and thereby of testing the SM).

At the LHC, the relevant contributions to Dihiggs production are



Note that these diagrams lead to interferences.
It is challenging to distinguish between a signal from the trilinear interaction and the background from top-quark Yukawa couplings. For a 125 GeV SM Higgs boson, it is usually concluded that the trilinear interaction will not be detected at the LHC.
There are brighter prospects at a future linear e^+e^- collider where it would be possible to measure the trilinear coupling (cf. hep-ph/9903229). (The quartic coupling would remain beyond reach.) There, the relevant contributions to Dihiggs production arise from:
double Higgs-Strahlung:



WW Dihiggs-fusion: