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Mechanik (Theoretische Physik 1)

Sommersemester 2018

Abgabe bis Freitag, 18.05.18, 12:00 neben PH 3218.

Übungsblatt Nr. 6 Dieses Blatt wird in den Übungen vom 21.05. - 25.05.18 besprochen.

Aufgabe 1:
Lenz vector **3 Punkte**

Prove that, aside from angular momentum $\mathbf{L}$ and energy E, the Kepler potential

$$V(r) = -\frac{\alpha}{r}$$

conserves an additional quantity, the so called "Lenz vector"

$$\mathbf{\Lambda} = \frac{1}{m\alpha}\,\mathbf{p} \times \mathbf{L} - \frac{1}{r}\,\mathbf{r}.$$

[Hint: Express $\dot{\mathbf{L}}$ in terms of the vectors $\dot{\mathbf{p}}, \mathbf{p}, \mathbf{r}$ and their scalar products, and use the equations of motion for the above potential. Consider the scalar product $\mathbf{\Lambda} \cdot \mathbf{r}$ and show that $\mathbf{\Lambda}$ points in the direction of the perihelion of the trajectory, as well as the fact that $|\mathbf{\Lambda}|$ coincides with the eccentricity ϵ.]

Aufgabe 2:
Guided gyroscope **3 Punkte**

A thin ring of radius r and mass m spins with constant angular velocity ω_s around its axis of symmetry (see Fig. 1, in which the ring's massless spokes have not been drawn). The ring's symmetry axis is attached to a frame, which makes the axis spin with constant angular velocity ω_p around the vertical z axis.

Consider a reference frame (x, y, z) that moves with the rotating frame, and whose origin lies at the center of the ring. An infinitesimal mass element of the ring, with $dm = m d\phi/(2\pi)$, is then subjected to a Coriolis force $d\mathbf{F} = 2dm\mathbf{v} \times \boldsymbol{\omega}_p$, as illustrated in Fig. 1. It can be then seen that the Coriolis force tends to align the angular velocities $\boldsymbol{\omega}_s$ and $\boldsymbol{\omega}_p$. Such movement is prevented by the bearing forces $\mathbf{F_1}$ and $\mathbf{F_2}$, which exert a compensating torque with respect to the ring's center,

$$\mathbf{N} = I_s\,\boldsymbol{\omega}_p \times \boldsymbol{\omega}_s, \tag{1}$$

which points in the negative x direction. In the formula above, I_s is the momentum of inertia of the ring with respect to its axis of symmetry.

Prove Eq. (1) by integrating over the ring the infinitesimal torques due to the Coriolis forces $d\mathbf{F}_c$ acting on the mass elements dm.

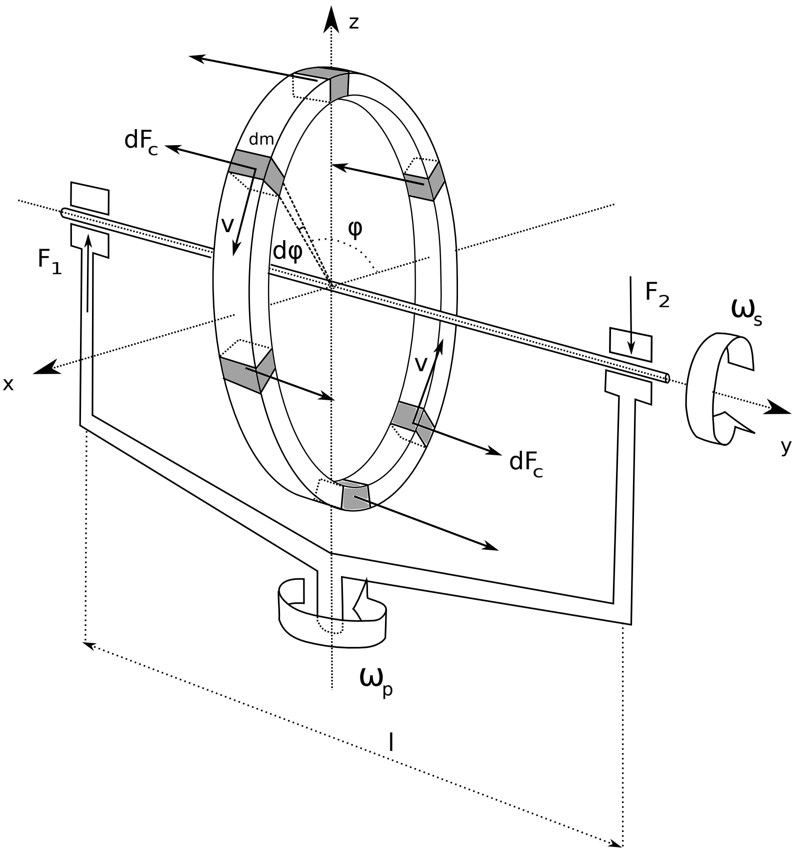

Abbildung 1: Guided gyroscope

Aufgabe 3:
The Precession of the Perihelion of Mercury **4 Punkte**

In Newtonian mechanics, when ignoring the effects from other planets, the trajectory of Mercury appears to be a perfect ellipse. Many-body effects can lead to a precession of the perihelium, but the measurements show that, the celestial body's perihelion shifts in 100 years by $43''$ more than it can be accounted for with Newtonian mechanics. In the context of Einstein's General Relativity, the effective gravitational potential to order one in relativistic corrections is given by:

$$U_{eff}(r) = -\frac{GmM}{r} + \frac{l^2}{2mr^2} - \frac{1}{c^2}\frac{GMl^2}{2mr^3}\,.$$

Using this approximation, show that for each revolution there is a nonzero angular displacement of the perihelium of Mercury regardless of the presence of other celestial bodies. Given that

$$\begin{aligned}\frac{GM_\odot}{c^2} =& 1.48\times 10^5\mathrm{cm}\,,\\ a =& 5.78\times 10^{12}\mathrm{cm}\,,\\ \varepsilon =& 0.2056\,,\end{aligned}$$

can this account for the shift of $43''$ per 100 years?
Hint: The exercise should be solved in the same spirit of the computation shown during lectures

(Newtonsche Mechanik file, page 43) except for the solution for the differential equation for $s = 1/r$, which should be performed with perturbation theory defining $s = s_0 + s_1$, where s_0 is Kepler's solution.