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Mechanik (Theoretische Physik 1)

Sommersemester 2018

Abgabe bis Freitag, 27.04.18, 12:00 neben PH 3218.

Übungsblatt Nr. 2 Dieses Blatt wird in den Übungen vom 30.04. - 04.05.18 besprochen.

Aufgabe 1:
Conservative Fields **3 Punkte**

Given the vector field:

$$\mathbf{f}(x,y) = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, 0\right),$$

1. Show that it is irrotational in its maximal domain, i.e. at each point where it is well defined.

2. Up to a dimensional constant, $\mathbf{f}$ could represent a force field. Show that such a field is non-conservative on the maximal domain of $\mathbf{f}$ by computing the work from $A = (1,0,0)$ to $B = (-1,0,0)$ by taking two different paths on the unitary circle parametrised as

$$\gamma(t) = (\cos(t), \sin(t), 0).$$

3. Restrict the domain of $\mathbf{f}$ to positive y's only. Now compute the work from $A = (\sqrt{2}/2, \sqrt{2}/2, 0)$ to $B = (-\sqrt{2}/2, \sqrt{2}/2, 0)$ along the paths γ and ρ of figure 1: is the work still path-dependent? Is the field conservative in the restricted domain?

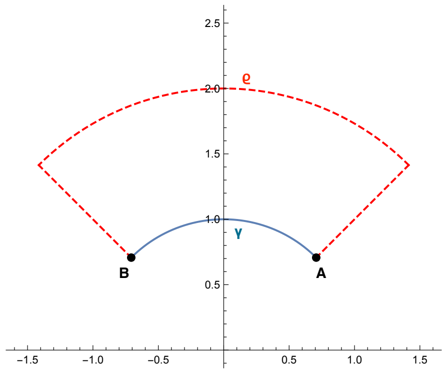

Abbildung 1: Two paths for working out some work.

Aufgabe 2:
Rocket science! **2 Punkte**

Consider a rocket with a time-dependent mass $m(t)$, with $m(t = 0) \equiv m_0$. The rocket accelerates by emitting exhaust gases, which escape with a velocity of magnitude v_e in the rocket's rest frame. Neglecting the effect of gravity, so that the total momentum of the rocket plus exhaust is conserved, derive the Tsiolkovsky rocket equation

$$v - v_0 = v_e \log \frac{m_0}{m}, \tag{1}$$

where v_0 and m_0 are the rocket's speed and mass at a reference time t_0. A Saturn V rocket, as used in the Apollo moon missions, had 3 stages –with each stage being detached from the rocket after spending the fuel– with the following characteristics:

Stage	Total mass (kg)	Dry mass (kg)	v_e (m/s)
I	2.3×10^6	1.3×10^5	2580
II	4.8×10^5	3.6×10^4	4130
III	1.2×10^5	1.0×10^4	4130

What is the maximum velocity that a Saturn V rocket could reach in the absence of gravity? Assume that each stage is detached with zero speed with respect to the rocket.

Aufgabe 3:
Spinning in the cold **2 Punkte**

An ice skater with an arm span of 160cm carries a dumbbell of $M = 4$kg in each of her hands, and finds herself spinning around her vertical axis with a frequency ω of half a revolution every second. The skater starts then bending her arms, so that each dumbbell approaches her vertical axis at a pace of 10 cm/s. Assuming that the contribution to the angular momentum of each of the skater's arms is given by

$$|\mathbf{L}|_{\text{arm}} = \frac{1}{3} m R^2 \omega, \tag{2}$$

where R is the horizontal distance between the skater's vertical axis and each of the hands, and $m = 3$ kg is the mass of each arm, what is the skater's rotational speed after 4 seconds? Is the kinetic energy of the dumbbells conserved, and why?

Aufgabe 4:
Area Law **3 Punkte**

Consider a point-like object of mass m tied to a string, and rotating horizontally around a point where the string enters a vertical hollow pole. Suppose that the string is pulled form inside the pole, so that the distance between the rotating mass and the tip of the pole varies with time as $l(t) = L \cdot e^{-\frac{t}{\tau}}$, with $\tau = 3$ s. Is $\mathbf{L}$ conserved and why? If the initial angular velocity is $\omega(0) = 1.5$ rad/s, what is the angle in degrees swept by the radius vector after $\Delta t = 2$s?