Technische Universität München
Physik-Department **B. Garbrecht, C. Tamarit, G. Zattera**

Mechanik (Theoretische Physik 1)

Sommersemester 2018

Abgabe bis Freitag, 15.06.18, 12:00 neben PH 3218.

Übungsblatt Nr. 10 Dieses Blatt wird in den Übungen vom 18.06. - 22.06.18 besprochen.

Aufgabe 1:
Geodesics on a cylinder and on a sphere **2.5 Punkte**

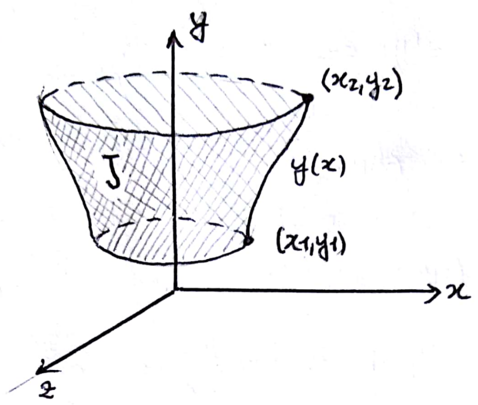
1. What is the curve of minimal distance that conects two points $(z_{1,2}, \varphi_{1,2})$ on the surface of a cylinder of radius R? What is its length?

2. What is the curve of minimal distance between two points A and B on a sphere? Hint: You may use the symmetry to place A without loss of generality at the pole, i.e. the polar angle $\vartheta = 0$. Show that the Euler-Lagrange equation implies that the derivative of the azimuthal angle φ with respect to θ vanishes.

Aufgabe 2:
Catenary **2.5 Punkte**

Under the influence of gravity, high voltage power lines adopt a shape which minimizes the height of their center of mass. Assuming a constant linear mass density, calculate such a shape for a power line segment of length l, hanging between two posts of equal height separated by the distance Δx. [Hint: One can use a first integral of the Lagrange equations].

Aufgabe 3:
Minimization of a rotational surface **2.5 Punkte**



Consider two points on the xy-plane (x_1, y_1) and (x_2, y_2) connected by a cartesian curve $y(x)$. Find the function $y(x)$ that minimizes the surface $J\{y(x)\}$ obtained by rotating the latter around the y-axis.

Aufgabe 4:
A special case: no explicit coordinate-dependence **2.5 Punkte**

Given the functional:

$$J\{y(x)\} = \int_{x_1}^{x_2} f(x, y, y')$$

1. What equation should the function $y(x)$ that extremizes it obey?
2. Consider the case in which f does not depend explicitly on the coordinate x, i.e. $f = f(y, y')$. Prove the following relation:

$$g(y, y') = f - y' \frac{\partial f}{\partial y'} = const$$

(These points are a repetition of what has been discussed in the lectures.)

Bearing in mind the relation proved in the previous point, consider a system in which an unstretchable rope of length l is laying on the xy-plane, and its extrema are fixed in $P_1 = (-d, 0)$ and $P_2 = (d, 0)$. What functional shape of the rope maximizes the area F between the x-axis and the rope itself?

Hint: This is a variational problem that is constrained by the fact that the length of the rope cannot change. It is useful then to use a Lagrange multiplier and impose:

$$\delta(F - \lambda l) = 0$$