

GROUP THEORY IN PHYSICS WS 2019/2020 EXERCISE SHEET 9

Problems will be discussed in the tutorial sessions every Friday at 2:00p.m. in the Minkowski Room

The Road to Root Systems

Consider the Lie algebra $\mathfrak{sl}(3)$, of 3×3 traceless matrices of real entries. Repeat the computations as in example II of section 11.4 of Cornwell's book.[†] Namely:

1. Determine a basis for this algebra and compute the structure constants.
2. Compute the Killing bilinear form, K , in general (in terms of generic structure constants) and then use your results from 1) to determine whether $\mathfrak{sl}(3)$ is semi-simple.
3. Find a Cartan subalgebra \mathcal{H} for $\mathfrak{sl}(3)$. (Hopefully your base from the previous item already included generators for this explicitly)
4. Find the roots α (should be real for an appropriate Cartan subalgebra).
5. Compute all the h_α using the correspondence given by the Killing form K :

$$K(h_\alpha, h) = \alpha(h) \tag{1}$$

for all $h \in \mathcal{H}$.

6. Determine all the "inner-products" between the roots, i.e.

$$\langle \alpha, \beta \rangle = K(h_\alpha, h_\beta) \tag{2}$$

for all possible pairs of roots. Check they are rational and real.

*Responsible for the sheet: Juan S. Cruz, Office 1112, juan.cruz@tum.de

[†]Steps 1 and 2 are not in the example in the book