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Technische Universität München
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Group Theory in Physics WS 2019/2020
Exercise Sheet 10

Problems will be discussed in the tutorial sessions every Friday at 2:00p.m. in the Minkowski Room

1 Drawing roots

The bracket introduced in the previous homework can be used to define

$$\langle\langle \alpha, \beta \rangle\rangle := 2\frac{\langle \beta, \alpha \rangle}{\langle \beta, \beta \rangle} \tag{1}$$

this quantity can only be an integer and in fact can be used to define angles between roots. Let us consider root systems for semi-simple complex Lie algebras of rank 2.

a) Find all the possible angles allowed by the above conditions between roots using the quantity $\langle \alpha, \beta \rangle \langle \beta, \alpha \rangle$.

b) Draw diagrams corresponding to these possibilities.

c) Which diagram corresponds to the case $\mathfrak{su}(3)$ (can be deduced from the previous homework)

2 Cartan Matrix and Dynkin diagrams

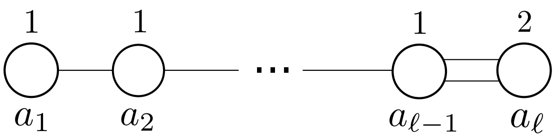

Figure 1: Dynkin diagram associated to the simple complex Lie algebras C_ℓ.

Using the Dynking diagram for the class of Lie algebras C_ℓ depicted in Fig. 1, find the Cartan matrix associated to it.

3 Chevalley Basis

Given a semi-simple complex Lie algebra $\mathfrak{g}$, with a Cartan subalgebra given by $\mathcal{H} = \{h_i\}$ with $i = 1, \ldots, r$ and other generator $\mathcal{E} = \{E_\alpha\}$ with $\alpha \in \Delta$ (non-zero roots). Let us define

$$\alpha^\vee := \frac{2\alpha}{\langle \alpha, \alpha \rangle} \tag{2}$$

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and a new basis for the Cartan subalgebra given by

$$H_\alpha = \sum_{i=1}^{r} \alpha_i^\vee h_i \tag{3}$$

Compute the commutation relations for the generators of the algebra in this new basis.