Theoretische Physik 2 (Elektrodynamik)
WS 2018/2019
Technische Universität München
December 21, 2018

Mock Exam

Instructions: This mock exam has the purpose of self training for the exam, you should try answering all the questions at home in 90 min without consulting any written material, however it will have no impact on the final grade (i.e. the eligibility for the bonus). Please hand this back by January 11th, 12pm like a regular homework sheet. Solutions will be discussed in the tutorial sessions taking place between the 14th and the 18th of January.

Exercise 1:
Short Questions **10 Points**

As long it is not required otherwise by terms such as *derive*, *prove* or *show that*, it is sufficient to give answers to the short questions without presenting derivations.

1.1) State concisely what Dirichlet and Neumann boundary conditions in electrostatics are. (1 Punkt)

1.2) What is the force on the point charge q_1 at $\mathbf{x}_1$ in presence of a charge q_2 at $\mathbf{x}_2$? (1 Punkt)

1.3) What is $\Delta_x \frac{1}{|\mathbf{x}-\mathbf{x}'|}$? (1 Punkt)

1.4) State Gauß' law in its differential and integral form. (2 Punkte)

1.5) Show that in electrostatics, the work necessary to carry a charge from a point $\mathbf{x}_A$ to a point $\mathbf{x}_B$ is independent of the path. (2 Punkte)

1.6) Derive the energy that is carried by an electric field created by a sphere of radius R and uniformly distributed surface charge Q. (2 Punkte)

1.7) Prove $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}\,(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}\,(\mathbf{a} \cdot \mathbf{b})$. You may use the identities for contractions of the ε-tensor. (1 Punkt)

Exercise 2:
Charged ring **7 Points**

Consider a circular ring with radius a and homogeneously distributed charge Q. Express the potential in terms of a series of Legendre polynomials.

Hint:

$$\frac{1}{|\mathbf{x}-\mathbf{x}'|} = \sum_{l,m} \frac{4\pi}{2l+1} \left[\theta(x'-x)\, \frac{x^l}{x'^{l+1}} + \theta(x-x')\, \frac{x'^l}{x^{l+1}} \right] \times Y^*_{lm}(\theta', \varphi') Y_{lm}(\theta, \varphi)\,,$$

$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi} \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta)\, e^{im\phi}\,.$$

Exercise 3:
Two perpendicular conducting planes **10 Points**

Let the volume $V = \{\mathbf{x} : 0 \le x_1 \le \infty, 0 \le x_2 \le \infty, -\infty \le x_3 \le \infty\}$ be confined by grounded metal plates.

(a) Determine the potential ϕ of a point charge q placed at $\mathbf{x} = (a, b, 0)^T$, where $a, b > 0$. (3 Punkte)

(b) What is the force acting on q? (3 Punkte)

(c) What is the integral form of the potential of a general charge distribution $\rho(\mathbf{x})$ inside of V when the potential ϕ is an arbitrary function on the (now non-conducting) plates? (4 Punkte)

Exercise 4:
Multipole field of homogeneously charged rod **13 Points**

Consider a rod of length l oriented in z-direction and centered at the origin. Let it carry the homogeneously distributed charge Q.

(a) What are the monopole and dipole moment q and $\mathbf{p}$? (4 Punkte)

(b) What is the quadrupole tensor Q_{ij}? (4 Punkte)

(c) Further, determine all multipole moments q_{lm} up to $l = 2$. (5 Punkte)

Hint:

$$Y_{00} = \frac{1}{\sqrt{4\pi}}\,,$$

$$Y_{11} = -\sqrt{\frac{3}{8\pi}} \sin\theta\, e^{i\phi}\,,$$

$$Y_{10} = \sqrt{\frac{3}{4\pi}} \cos\theta\,,$$

$$Y_{22} = \frac{1}{4}\sqrt{\frac{15}{2\pi}} \sin^2\theta\, e^{2i\phi}\,,$$

$$Y_{21} = -\sqrt{\frac{15}{8\pi}} \sin\theta\, \cos\theta\, e^{i\phi}\,,$$

$$Y_{20} = \sqrt{\frac{5}{4\pi}} (\frac{3}{2}\cos^2\theta - \frac{1}{2})\,.$$