Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
December 19, 2018

Exercise Sheet 10*

Deadline: Sheet to be turned in by Friday 11th of January 2019 by 12 pm in the mailbox next to PH3218.

Exercise 1:
Radiation of a point charge on a circular orbit **5 Points**

Point charges traveling on circular orbits appear in different contexts (*e.g.* in accelerators or in astrophysical magnetic fields). Their radiation limits the ultimate energy up to which the particles can be accelerated. A point-like model for the electron would suffer from this feature when electrons travel on circular orbits. Therefore, such a classical model of atomic physics would not allow for stable energy levels, which had been one of the main problems of classical physics leading to the discovery of quantum mechanics.

(a) Let a particle of charge q be moving at an angular velocity ω on a circle of radius R. (Assume $R \ll c/\omega$, such that the speed is non-relativistic.) This leads to the time-dependent charge-density

$$\rho(\mathbf{r},t) = q\,\delta(x - R\cos\omega t)\,\delta(y - R\sin\omega t)\,\delta(z)\,.$$

Calculate the pertaining dipole moment $\mathbf{p}(t)$ and express this through a complex vector $\mathbf{p}$ satisfying the relation $\mathbf{p}(t) = \mathrm{Re}[\mathbf{p}\exp(-i\omega t)]$. Make use of the leading order result for the radiation in the far field zone and derive the angular distribution of the differential radiation power $dP/d\Omega$ and integrate this to obtain the total power of the radiation. (2.5 Punkte)

(b) Following classical mechanics and the Coulomb force, an electron would move on a stable circular orbit around the proton. Express the angular frequency ω and the energy (sum of kinetic and potential contributions) of the electron as a function of the radius r. The loss of radiation power would then lead to an orbital radius $r(t)$ that decreases with time. Set up a differential equation and integrate it with the initial condition $r(0) = a_{\mathrm{B}} \simeq 5.29 \times 10^{-9}$cm (Bohr radius). After what time τ does the electron reach the nucleus?(2.5 Punkte)

(*Hints*: The differential equation is of the form $\dot{r}r^2 = \text{const.}$. The final answer is $\tau \simeq 1.56 \times 10^{-11}$s.)

Exercise 2:
Induction in a rotating circular ring **5 Points**

A conducting circular ring ($z = 0$ and $x^2 + y^2 = r_0^2$) rotates with constant angular velocity ω around the x-axis. There is a homogeneous magnetic field $\vec{B} = B\hat{z}$.

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(a) What is the voltage $V(t)$ induced in the ring?

(b) A small lamp is installed in the ring, corresponding to a resistance R. The lamp has a power of $P = U^2/R$. What is the time averaged torque, $\langle M \rangle$, needed to keep the ring rotating with a constant angular velocity? (Ignore any mechanical friction).