Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
December 14, 2018

Exercise Sheet 9*

Deadline: Sheet to be turned in by Friday 21st of December 2018 by 12 pm in the mailbox next to PH3218.

Exercise 1:
Uniformly charged rotating sphere **4 Points**

A charged ball of radius R and total charge Q is charged homogeneously, so that the charge is distributed inside the whole ball (not only on its surface). Consider the situation in which it is rotating with a constant angular velocity $\vec{\omega}$ around the center of the sphere (i.e. $\vec{v} = \vec{\omega} \times \vec{r}$).

(a) (3 Punkte) Derive the pertaining current density $\vec{j}(\vec{r})$ and verify that $\nabla \cdot \vec{j} = 0$. Calculate the vector potential $\vec{A}(\vec{r})$ as well as the the magnetic field $\vec{B}(\vec{r})$ inside and outside the sphere.

Hint: Make use of the spherical symmetry

$$\int_{|\vec{r}'|<R} \mathrm{d}^3\vec{r}' \frac{\vec{r}'}{|\vec{r} - \vec{r}'|} = C(r)\vec{r} \tag{1}$$

and derive $C(r)$ by taking the scalar product of both sides with $\vec{r}/|\vec{r}|^2$.

(b) (1 Punkt) Calculate the magnetic dipole moment $\vec{m}$ of the sphere. Compare the dipole field with the result from part (a).

Exercise 2:
Force between circular circuits **6 Points**

Consider two generic wires along curves $\vec{r}_1$ and $\vec{r}_2$ carrying the currents I_1 and I_2 respectively.

(a) (1 Punkt) Express the total force between the wires through a double integral along the two curves.

(b) (5 Punkte) Assume the wires are two circular loops radius a that lie on parallel planes to the $z = 0$ plane, with their centers at the z-axis and with a distance d between them, such that $d \gg a$. Let the lower loop carry the current I_1 and the upper loop the current I_2. Calculate the force $\vec{F}_{21} = -\vec{F}_{12} \sim \hat{z}$, between the two circuits. Express the result in terms of a power series in the small parameter $s = (a/d)^2$ up to order s^4.

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