Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
November 30, 2018

Exercise Sheet 7*

Deadline: Sheet to be turned in by Friday 7th of December 2018 by 12 pm in the mailbox next to PH3218.

Exercise 1:
Conducting sphere held at a given potential **5 Points**

Two concentric spheres have radii a, b ($b > a$), and each is divided into two hemispheres by the same horizontal plane. The upper hemisphere of the inner sphere and the lower hemisphere of the outer sphere are maintained at potential V. The other hemispheres are at zero potential. Determine the potential in the region $a \leq r \leq b$ as a series in Legendre polynomials. Include terms at least up to $l = 3$. Check your solutions against known results in the limiting cases $b \to \infty$, and $a \to 0$, respectively.

Hint: You may need to calculate $\int_0^1 \mathcal{P}_l(x)\, dx$. You may do it by first integrating the generating function

$$\frac{1}{\sqrt{1 - 2xt + t^2}} = \sum_{l=0}^{\infty} \mathcal{P}_l(x)\, t^l$$

from $x = 0$ to 1 in both sides. After that, you can expand the result in series of t and identify the coefficients of t^l. Alternatively you could use the identity:

$$(2\ell + 1)P_\ell(x) = \frac{\mathrm{d}}{\mathrm{d}x}(P_{\ell+1}(x) - P_{\ell-1}(x)) \qquad \text{for } \ell \geq 1 \tag{1}$$

And the fact that

$$P_\ell(1) = 1 \qquad \forall \ell \in \mathbb{N} \tag{2}$$

Exercise 2:
Gluing method for Green's Functions and its spectral sum **5 Points**

Consider the differential operator $-\Delta = \partial_x^2$ in one dimension, over the interval $[0, L]$.

(a) Find a corresponding Green's function, with vanishing boundary conditions, that is, find $G(x, y)$ such that it satisfies:

$$-\Delta_x G(x, y) = \delta(x - y) \qquad \text{with} \quad G(0, y) = G(x, 0) = G(L, y) = G(x, L) = 0 \tag{3}$$

by means of holding y fixed, finding a solution for $x < y$, another one for $x > y$ and matching them appropriately.

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(b) Find the eigenfunctions of the operator $-\Delta$ satisfying the same boundary conditions. Write an expression for the Green's function in terms of its eigenfunctions and verify that it satisfies the Green function equation. Evaluate the resulting sum for $y = L/2$ and verify it coincides with the result obtained in (a) (also evaluated at $y = L/2$).