Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
November 18, 2018

Exercise Sheet 5*

Deadline: Sheet to be turned in by Friday 23rd of November 2018 by 12 pm in the mailbox next to PH3218.

Exercise 1:
The potential of a cube with particular boundary conditions **5 Points**

A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at constant potential V. The other four sides are at zero potential.

(a) Find the potential $\phi(x, y, z)$ at any point inside the cube.

(b) Evaluate the potential at the center of the cube numerically, accurate to three significant figures. How many terms in the series is it necessary to keep in order to attain this accuracy? Compare your numerical result with the average value of the potential on the walls.

(c) Find the surface-charge density on the surface $z = a$.

Hint: You may find the discussion in section 2.5 of the lecture notes very useful. But you need to be careful about the different boundary conditions here. Note that we do not assume that

$$\phi(x, y = 0, z = 0) = 0 \text{ or } V, \text{ for } 0 < x < a, \text{ etc.}$$

i.e., boundary conditions on edges are not given.

Exercise 2:
Laplacian in cylindrical coordinates **5 Points**

We study the vacuum Poisson equation in cylindrical coordinates.

(a) Starting from the definition of $\Delta = \nabla \cdot \nabla$ in Cartesian coordinates, arrive to the expression for Δ in cylindrical coordinates, i.e.

$$\rho^2 = x^2 + y^2 \tag{1}$$
$$x = \rho \cos\theta \tag{2}$$
$$y = \rho \sin\theta \tag{3}$$
$$z = z \tag{4}$$

(b) Use the separation of variables method to obtain a general solution for the homogeneous equation $\Delta G(\vec{r}, \vec{r}') = \delta^3(\vec{r} - \vec{r}')$ in these coordinates and with the boundary conditions $G = 0$ at $\rho, \rho' \to \infty$ and $G = 0$ when $|z| \to \infty$. Check that if chosen appropriately the factors of θ and z, already satisfy the jump in the derivative.

---

*Responsible for the sheet: Juan S. Cruz, Office 1112, juan.cruz@tum.de

(c) Match the solutions obtained in the previous item appropriately to solve for the general Green's function (non-homogeneous).

Hint: Bessel functions will be useful, you don't need to solve the Bessel differential equation yourself. These special functions can be found in any standard textbook on mathematical methods in physics. Of relevance for the present problem are the solutions $J_m(x)$ and $Y_m(x)$. Make use of the fact that $J_m(x)$ is regular at the origin whereas $Y_m(x)$ diverges. Finally, employ that for the Wronskian matrix

$$W(x) = \begin{pmatrix} J_m(x) & Y_m(x) \\ J'_m(x) & Y'_m(x) \end{pmatrix} \tag{5}$$

the Wronskian determinant is $\det W(x) = 2/(\pi x)$.