Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
November 2, 2018

Exercise Sheet 3*

Deadline: Sheet to be turned in by Friday 9th of November 2018 by 12 pm in the mailbox next to PH3218.

Exercise 1:
The normal derivative of the electric field at the surface of a curved charged conductor **3 Points**

Use Gauß' theorem to prove that at the surface of a curved charged conductor (with no edges or far away from them), the normal derivative of the normal component of the electric field is given by

$$\frac{1}{E}\frac{\partial E}{\partial n} = -\left(\frac{1}{R_1} + \frac{1}{R_2}\right),$$

where R_1, R_2 are the principal radii of curvature at the point of interest at the surface. Recall *principal radii* at a point p refers to the maximum and minimum radii of curvature at p of curves going through p in a surface. If the surface given as a height function on the two-dimensional plane, the inverse of the radii of principal curvature are given by the eigenvalues of the Hessian of the height function.

Hint:

Option1: Consider a surface element $d\sigma = R_1 d\theta_1 R_2 d\theta_2 \equiv R_1 R_2 d\Omega$, as well as another surface element, where the boundaries are shifted by an amount ε along the normal vectors. Apply Gauß' theorem to the volume between these surfaces. A sketch of the two surface elements may also be helpful.

Option2: Consider a surface resembling a cylinder such that the bottom cap is tangent to the conductor at every point and the top cap is an arbitrarily small displacement of the bottom cap in the normal direction. Consider the flux integrals at both caps and use Gauß's Theorem.

Exercise 2:
Point charges and self-energies **2 Points**

Consider a static situation where two electrical charges q_1 and q_2, are located at positions given by $\vec{r}_1$ and $\vec{r}_2$ respectively.

(a) Write down the expression for the electric field $E(\vec{r})$ generated by this configuration.

(b) Compute the energy density associated to the electric field. Identify the different types of contributions appearing.

(c) Compute the energy stored in the electric field from this configuration, separating a finite term and divergent terms, compute the finite term.

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(d) Argue why they divergent terms do so, that is prove they diverge by assuming the charge is uniformly distributed on a ball of radius R, then take the limit $R \to 0$. How can we interpret the divergences?

Exercise 3:
Delta distribution **2 Points**

Do the integrals below:

(a) $\int_0^\infty x^2 \delta(x^2 - 3x + 2) dx$

(b) $\int_0^\infty \ln x \, \delta'(x - 2) dx$

(c) $\int_0^\pi \sin^3 \theta \, \delta \left(\cos \theta - \cos \frac{\pi}{3} \right) \, d\theta.$

Hint: For a function $f(x)$ with roots x_i all simple, we have

$$\delta(f(x)) = \sum_i \frac{\delta(x - x_i)}{|f'(x_i)|}.$$

Exercise 4:
The flux of the vector field **3 Points**

Consider the field $\vec{V}(\vec{x}) = \frac{c}{\rho} \vec{e}_\rho$, where c is constant and $\vec{e}_\rho = (\cos \varphi, \sin \varphi, 0)^t$.

(a) For a cylinder of length l and radius R that is symmetric around the z axis, calculate the flux of $\vec{V}$ through its surface, i.e.

$$F = \int_{\partial \, \text{cylinder}} d\vec{a} \cdot \vec{V}(\vec{x}).$$

(b) Calculate $\vec{\nabla} \cdot \vec{V}(\vec{x})$ including the singular part by use of Gauß' law.

(c) An infinitely long straight wire (at $z = 0$) bears the charge κ per unit length. Find the electric field using

(i) the result from part (b)

(ii) the general formula valid for a 3-dimensional distribution.