Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
October 26, 2018

Exercise Sheet 2*

Deadline: Sheet to be turned in by Friday 2nd of November 2018 by 12 pm in the mailbox next to PH3218.

Exercise 1:
Identities for vector field **3 Points**

Given two smooth vector fields $\vec{A}(\vec{r})$ and $\vec{B}(\vec{r})$, verify the following identities:

(a) $\mathrm{div}(\vec{A} \times \vec{B}) = \vec{B} \cdot \mathrm{rot}\vec{A} - \vec{A} \cdot \mathrm{rot}\vec{B}$,

(b) $\mathrm{rot}(\vec{A} \times \vec{B}) = \vec{A}\,\mathrm{div}\vec{B} - \vec{B}\,\mathrm{div}\vec{A} + (\vec{B} \cdot \vec{\nabla})\vec{A} - (\vec{A} \cdot \vec{\nabla})\vec{B}$,

Hint: Use the rule $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ for the double product. Make sure that the Nabla operator operates on all the fields from the left.

(c) $\mathrm{grad}(\vec{A} \cdot \vec{B}) = \vec{A} \times \mathrm{rot}\vec{B} + \vec{B} \times \mathrm{rot}\vec{A} + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}$

Hint: Use the rule for the double product to reformulate $\vec{A} \times \mathrm{rot}\vec{B} + \vec{B} \times \mathrm{rot}\vec{A}$.

Exercise 2:
Stokes's theorem **3 Points**

Verify Stokes's theorem for the vector field

$$\vec{V}(\vec{r}) = \left(\frac{4x}{3} - 2y\right)\vec{e}_x + (3y - x)\vec{e}_y$$

on the elliptical plane $(x/3)^2 + (y/2)^2 \leq 1$, $z = 0$.

Exercise 3:
Source free **2 Points**

Consider the cylindrical vector field

$$\vec{V}(\vec{r}) = v(\rho)\vec{e}_\rho, \quad \rho = \sqrt{x^2 + y^2} \neq 0, \ \vec{e}_\rho = (\cos\varphi, \sin\varphi, 0)^t.$$

Find $v(\rho)$ such that $\vec{V}(\vec{r})$ is divergence free .

Exercise 4:
Mean value of the potential theory **2 Points**

Prove the mean value property: For charge free space ($\Delta\Phi = 0$) the value of the electrostatic potential is equal to the average of the potential over the surface of any sphere centered on that point.

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Hint: Start with the identity for the mean value

$$\frac{1}{4\pi R^2} \iint_{|\vec{r}\,' - \vec{r}| = R} dF' \, \Phi(\vec{r}\,') = \frac{1}{4\pi} \iint_{S^2} d\Omega \, \Phi(\vec{r} + \vec{n} R),$$

where $\vec{n}$ is the unit outward-pointing normal vector on a sphere of radius R. Use the Gaußtheorem and $\vec{\nabla} \cdot (\vec{\nabla} \Phi) = \Delta \Phi = 0$ to show that the derivative of the right-hand-side expression with respect to R vanishes.