Theoretische Physik 2 (Elektrodynamik) WS 2018/2019
Technische Universität München
October 19, 2018

Introduction Problems
Exercise Sheet 1*

Deadline: Sheet to be turned in by Friday 26 of October 2018 by 12 pm in the mailbox next to PH3218.

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1 The symbol ε_{ijk}

(a) Express ε_{ijk} in terms of a determinant. *Hint:* Use Leibniz' formula:

$$\det M = \sum_{\sigma \in S_n} \operatorname{sign} \sigma \prod_{i=1}^{3} M_{i\sigma(i)}.$$

(b) Show that

$$\varepsilon_{klm}\varepsilon_{pqn} = \begin{vmatrix} \delta_{kp} & \delta_{kq} & \delta_{kn} \\ \delta_{lp} & \delta_{lq} & \delta_{ln} \\ \delta_{mp} & \delta_{mq} & \delta_{mn} \end{vmatrix}.$$

2 Vector analysis and Gauß theorem

Show that

$$\int_V d^3x\, \mathbf{u}(\mathbf{x}) \cdot (\nabla \times \mathbf{v}(\mathbf{x})) = \int_V d^3x\, \mathbf{v}(\mathbf{x}) \cdot (\nabla \times \mathbf{u}(\mathbf{x})) - \int_{\partial V} d\mathbf{a}\, (\mathbf{u}(\mathbf{x}) \times \mathbf{v}(\mathbf{x})).$$

3 Taylor-Series

Expand the following scalar fields in their Taylor expansions around $\vec{r} = 0$:

(a) $\varphi(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}$ for $\vec{k}$ a constant vector. Do the expansion to arbitrary order.

(b) $\varphi(\vec{r}) = |\vec{r} - \vec{r}_0|^{3/2}$, with $\vec{r}_0$ a constant vector. Expand up to second order.

4 Flux of vector fields

Compute the flux produced by the following vector fields across the surface of a sphere of radius R:

(a) $\vec{a}(\vec{r}) = 3\frac{\vec{r}}{r^2}$,

(b) $\vec{a}(\vec{r}) = \dfrac{(x, y, z)}{\sqrt{\alpha + x^2 + y^2 + z^2}}$,

(c) $\vec{a}(\vec{r}) = (3z, x, 2y)$.

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