Theoretische Physik 2 (Elektrodynamik)

Wintersemester 2016/17

Abgabe bis Freitag, 02.12.16, 12:00 neben PH 3218.

Übungsblatt Nr. 6 Dieses Blatt wird in den Übungen vom 05.12. - 09.12.16 besprochen.

Aufgabe 1:
The potential of a cube with particular boundary conditions — 6 Punkte

A hollow cube has conducting walls defined by six planes $x = 0$, $y = 0$, $z = 0$, and $x = a$, $y = a$, $z = a$. The walls $z = 0$ and $z = a$ are held at constant potential V. The other four sides are at zero potential.

(a) Find the potential $\phi(x, y, z)$ at any point inside the cube.

(b) Evaluate the potential at the center of the cube numerically, accurate to three significant figures. How many terms in the series is it necessary to keep in order to attain this accuracy? Compare your numerical result with the average value of the potential on the walls.

(c) Find the surface-charge density on the surface $z = a$.

Hint: You may find the discussion in section 2.5 of the lecture notes very useful. But you need to be careful about the different boundary conditions here. They are:

$$\begin{aligned}
\phi(x = 0, y, z) &= 0, \text{ for } 0 < y < a, \ 0 < z < a \\
\phi(x, y = 0, z) &= 0, \text{ for } 0 < x < a, \ 0 < z < a \\
\phi(x = a, y, z) &= 0, \text{ for } 0 < y < a, \ 0 < z < a \\
\phi(x, y = a, z) &= 0, \text{ for } 0 < x < a, \ 0 < z < a \\
\phi(x, y, z = 0) &= V, \text{ for } 0 < x < a, \ 0 < y < a \\
\phi(x, y, z = a) &= V, \text{ for } 0 < x < a, \ 0 < y < a.
\end{aligned}$$

Note that we do not assume that

$$\phi(x, y = 0, z = 0) = 0 \text{ or } V, \text{ for } 0 < x < a, \text{ etc.}$$

i.e., boundary conditions on edges are not given.

Aufgabe 2:
A conducting spherical shell in a uniform field — 4 Punkte

An insulated, spherical, conducting shell of radius a is in a uniform electric field E_0 (pointing along the z axis). When the sphere is cut into two hemispheres by a plane perpendicular to the field, find the force required to prevent the hemispheres from separating

(a) if the shell is uncharged;

(b) if the total charge on the shell is Q.

Hint: You can obtain the homogeneous electric field by placing point charges $\pm Q$ at $z = \mp R$ and take the limit $R \to \infty$ while keeping Q/R^2 fixed. The response of the sphere on the electric field can then be represented by image charges as it has been discussed in the lectures. After placing the image charges, we can calculate the potential and induced charge density of the hemispheres and therefore the force upon them coming from the (modified) electric field.